

Calculation of signal spectrum by means of stochastic inversion

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The conventional method of calculating signal spectra is based on Fourier transform, which makes use of time series taken at fixed intervals. Since the length of the time series is limited, the samples actually represent the true signal multiplied by a window function, and the Fourier transform is the convolution of the Fourier transforms of the window function and the signal itself. Hence the spectral resolution is determined by the length of the window function. With n samples in the time series, the number of frequencies in the spectrum (including zero frequency) is $n/2+1$ when n is even and $(n+1)/2$ when n is odd. It is possible to increase the number of frequency bins by padding the time series with zeros, but this does not improve the spectral resolution; it only results into interpolation of the convolution of the two Fourier transforms.

In this paper a different method of calculating the spectrum is presented. Just like the Fourier transform, the present method represents the signal in terms of a set of sinusoidal signals with different amplitudes and phases. This makes a set of linear equations which pose a linear inversion problem. The best values of the amplitudes and phases are then obtained as the well-known solution of the problem. There are two main differences in comparison with the standard Fourier method. The first one is that the method works even if the samples are not taken at equal intervals. This is useful for instance in the case of missing data points in observations. Secondly, if the experimental variances of samples are known, the method also gives error limits for the spectrum. This helps in distinguishing meaningful spectral peaks from noise. The method also allows a free choice of frequency bins. This is not of major importance, however, since, the frequency step can be reduced to any value both in the Fourier method and in the present one.

The working principle of the method is explained and its performance is demonstrated by means of both synthetic and geophysical data. It is also pointed out that the method is suitable for interpolation, especially in the case of non-equal sampling intervals.