Several radar target models

Juha Vierinen

Sodankylä Geophysical Observatory, Finland

November 21, 2008

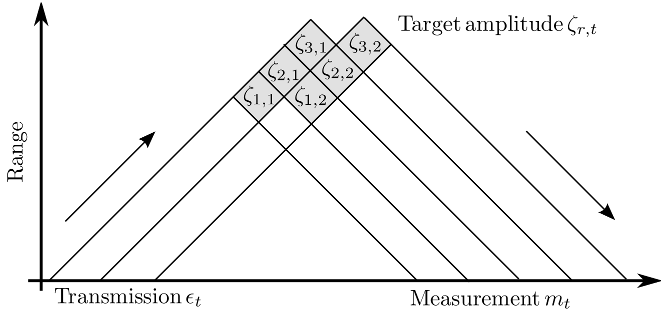
Range spread target

$$m_t = \sum_r \epsilon_{t-r}\,\zeta_r + \xi_t. \tag{1}$$

- Raw voltage measurement m_t
- Transmission envelope ϵ_t
- Backscatter coefficient ζ_r
- Measurement noise $\xi_t \sim N_{\mathbb{C}}(0, \mathrm{SNR}^{-1})$

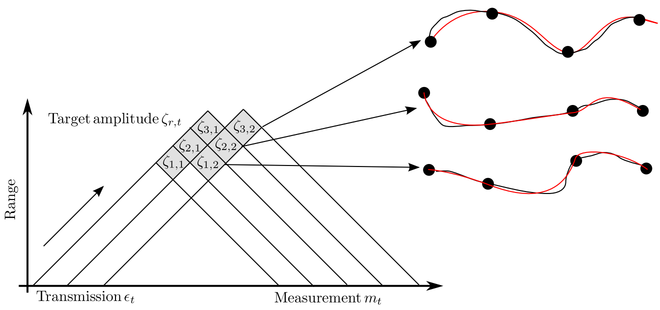
$$\Sigma_t = \lim_{M\to\infty} \frac{1}{M\mathrm{SNR}} \mathcal{F}_M^{-1}\left\{ \left(\mathcal{F}_M \left\{ \sum_{\tau=-\infty}^{\infty} \epsilon_\tau \overline{\epsilon_{t-\tau}} \right\} \right)^{-1} \right\}.$$

Range and Doppler spread target



$$m_t = \sum_r \epsilon_{t-r}\, \zeta_{r,t-\frac{1}{2}r} + \xi_t. \tag{2}$$

Range and Doppler spread target model $\zeta_{r,t}$



- Assume that target backscatter $\zeta_{r,t}$ is band-limited.
- Linear models: B-Spline, Fourier series, ...

$$\hat{\zeta}_{r,t} = S_r^k(t) \tag{3}$$

Range and Doppler spread target

The probability density can be written as ($m \in \mathbb{C}$, $x \in \mathbb{C}$):

$$p(\mathbf{m}|\mathbf{x}) \propto \exp\left(-\frac{1}{\sigma^2}\|\mathbf{m} - \mathbf{A}\mathbf{x}\|^2\right) \tag{4}$$

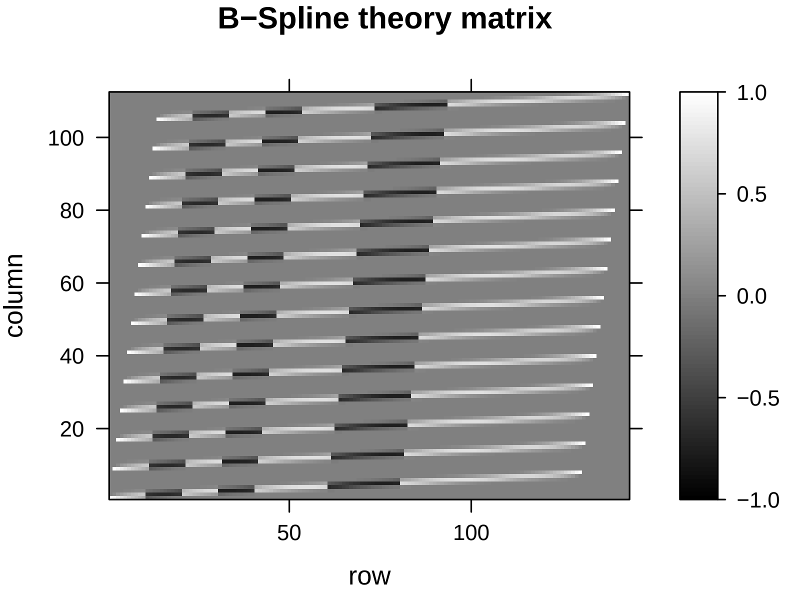
and assuming uniform priors, the maximum *a posteriori* (MAP) estimate, i.e., the peak of $p(\mathbf{m}|\mathbf{x})$ is

$$\mathbf{x}_{\mathrm{MAP}} = (\mathbf{A}^{\mathrm{H}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{m} \tag{5}$$

and the *a posteriori* covariance is:

$$\mathbf{\Sigma_p} = \sigma^2(\mathbf{A}^{\mathrm{H}}\mathbf{A})^{-1}. \tag{6}$$

Range and Doppler spread target



Range and Doppler spread target

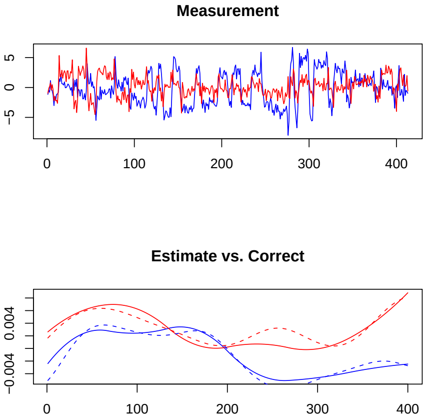
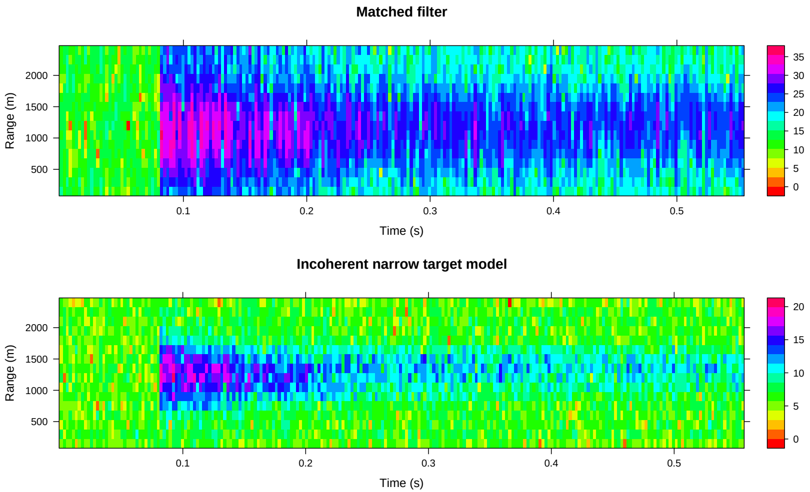


Figure: A simulated radar echo from a range and Doppler spread target. The range spread is four range gates and the Doppler spread is Gaussian with 10 kHz spectrum half-width at each range gate. SNR is 10 dB. The transmission code is a non-uniform baud-length code optimized for a 10

F-region ion-line overshoot

Comparison of target power estimates (dB scale)

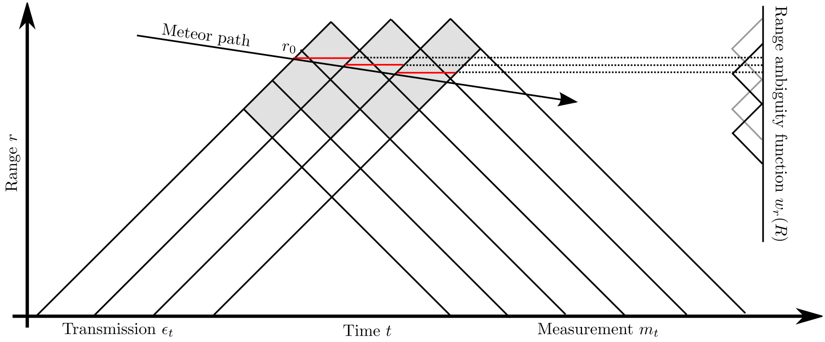


Meteor head echo

- How accurately can a meteor range and Doppler shift be estimated?
- Range ambiguity corrected moving point model.
- In principle, better range resolution than sampling rate possible.

Range ambiguity corrected moving point model

Model the spreading of the target when it moves down.



Range ambiguity corrected moving point measurement

- One "point" travelling at velocity $v \in \mathbb{R}$, starting from range $r_0 \in \mathbb{R}^+$, following radial trajectory $R = vts^{-1} + r_0$. Sample rate s. Doppler shift is $\omega = vf/c$.
- Range ambiguity function $w_r(R)$ gives contribution of target for each measurement sample m_t. True range: $R \in \mathbb{R}^+$, range gate $r \in \mathbb{N}$.

$$m_t = \underbrace{\sum_r w_r(vts^{-1} + r_0)\epsilon_{t-r} c_r \exp(i\omega ts^{-1})}_{f_t(\theta)} + \xi_t \qquad (7)$$

Solution method

- Examine the *a posteriori* probability distribution, the probability of model parameters given data:

$$p(\theta \mid D) = \frac{p(D \mid \theta)p(\theta)}{\int d\theta p(D \mid \theta)p(\theta)}$$

- The probability distributions solved using Markov chain Monte-Carlo (Hastings 1970).
- Other faster methods also possible for finding the peak of the probability distribution.

Likelihood function $p(D|\theta)$ and priors $p(\theta)$

- Measurement $D = (m_1, ..., m_N) \subset \mathbb{C}^N$
- Point-target parameters: $\theta = (\sigma, c, r_0, \omega) \subset \mathbb{R} \times \mathbb{C} \times \mathbb{R} \times \mathbb{R}$

Likelihood function, the probability of data given parameters:

$$p(D \mid \theta) = \prod_{t \in R} \frac{1}{\pi\sigma^2} \exp\left\{-\frac{|m_t - z_t(\theta)|^2}{\sigma^2}\right\}$$

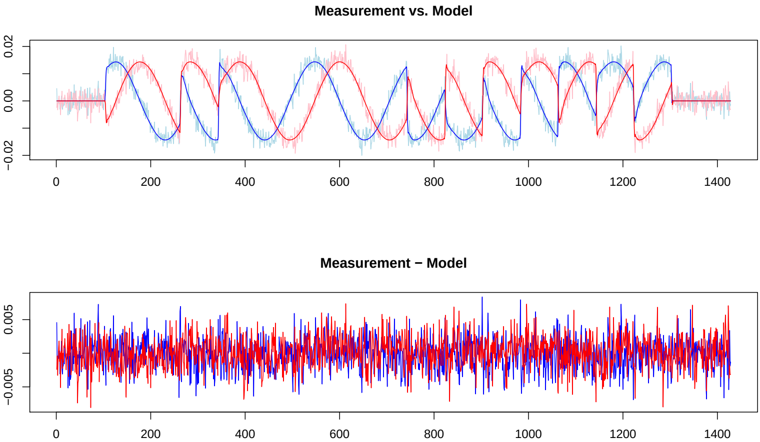
Priors, the probability distribution of model parameters:

- Measurement noise close to known system noise power $P = kTB$.

$$\sigma^2 \sim N_T(P, 0.1P)$$

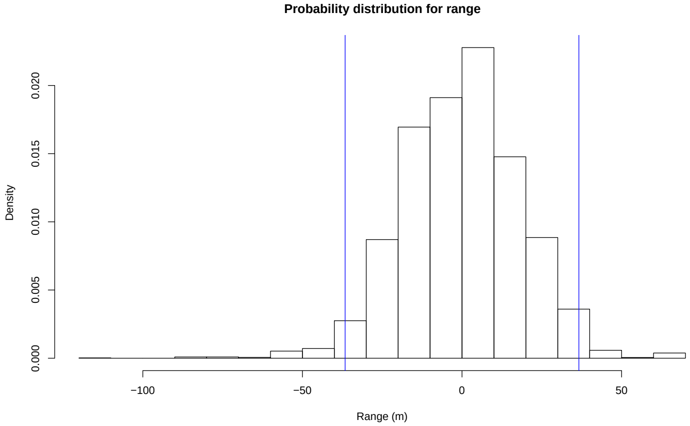
- Other parameters c_r, v, and r_0 uniformly distributed.

Point-target example (EISCAT VHF)



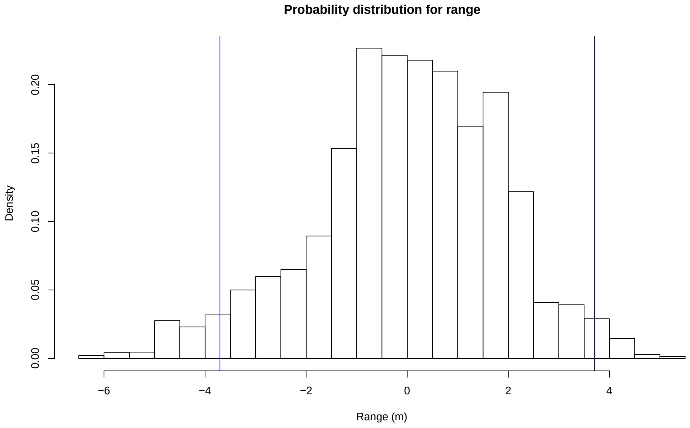
Marginal range distribution $p(r_0|D)$

Weak echo

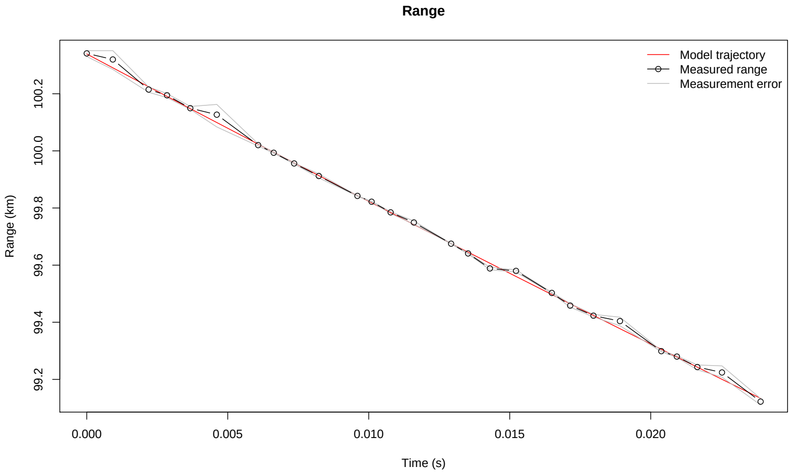


Marginal range distribution $p(r_0|D)$

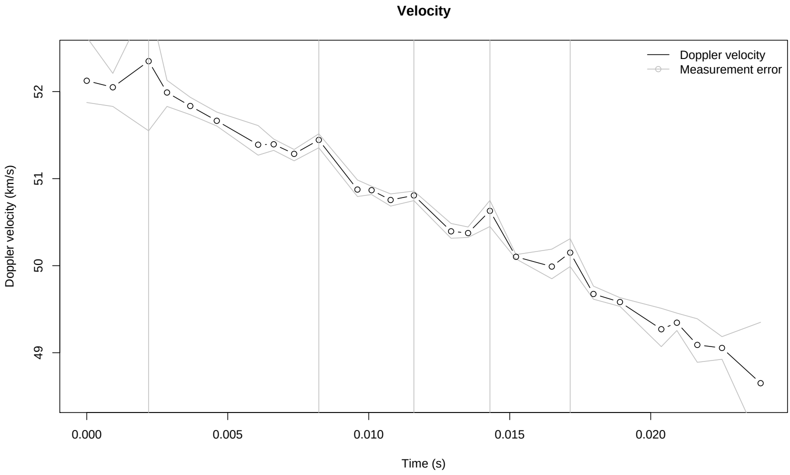
Strong echo



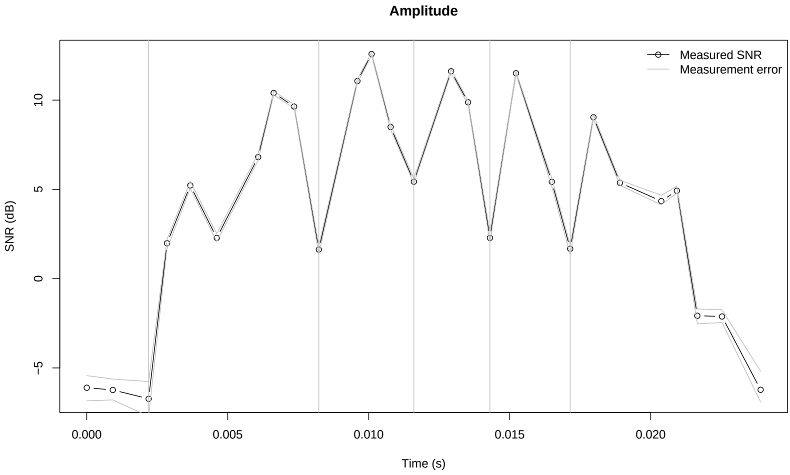
Point-target trajectory (Range)



Point-target trajectory (Doppler)

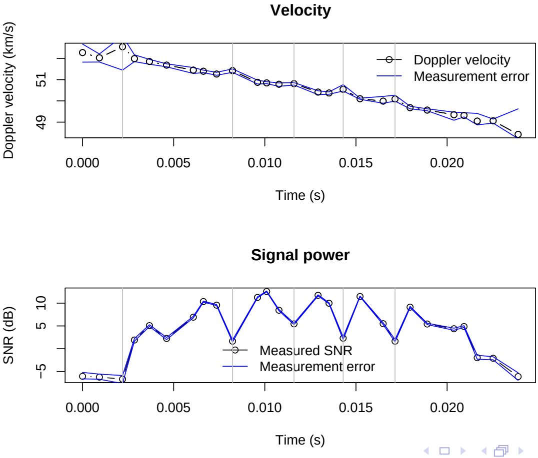


Point-target trajectory (Amplitude)



Forward expanding plasma?

- The order of 10^{-2} m forward expansion of sufficient to explain Doppler shift.
- YORP-effect
- A $\approx$ 350 Hz rotating meteor with irregular surface?



Comparison of measurements

- What is optimal receiver bandwidth?
- Range spread target with known uniform Doppler shift
- Point-like target measurement equation.

Optimal receiver bandwidth

- "Wouldn't a narrow bandwidth measurement m_1 be better than the wide bandwidth measurement m_2 that you have done, as there would be less noise?"
- No. A wideband measurement m_1 can be used to simulate m_2 (by filtering m_1). But *usually not* the other way around.

$$m_2(t) = \int m_1(t-\tau)h(\tau)d\tau \tag{8}$$

Known Doppler shift in range spread target

If we assume that each range is Doppler shifted by ω and that the complex valued ζ_r contains the independent phase and amplitude for each range, the measurement equation is

$$m_t = \sum_r \epsilon_{t-r} \zeta_r \exp\left(i\omega(t-r)\right) + \xi_t. \tag{9}$$

By writing

$$\exp(-i\omega r)\exp(i\omega t) = \exp(i\omega(t-r)), \tag{10}$$

we can write the meteor head echo equation as

$$m_t = \sum_r \epsilon_{t-r} \zeta_r' \exp\left(i\omega t\right) + \xi_t, \tag{11}$$

using $\zeta_r' = \zeta_r \exp\left(-i\omega r\right)$. We can now divide by $\exp\left(i\omega t\right)$ to get

$$m_t \exp\left(-i\omega t\right) = \sum_r \epsilon_{t-r} \zeta_r' + \exp\left(-i\omega t\right) \xi_t, \tag{12}$$

which is the measurement equation of a coherent target with the exception that the measurement and the noise are modulated by a complex sinusoid. In terms of the theory of measurements, the measurement $m_t \exp\left(-i\omega t\right)$ is equivalent to m_t in the sence that they can both be simulated from each other. If we now assume that ω is known, and we examine the estimation error covariance variance for ζ_r, we see that it is the same as the one obtained in section **??**.

Point-target ambiguity corrected estimation variance

Let us examine the sub-baud resolution estimation accuracy for a point-target with known backscatter amplitude σ, but high-resolution range r_0.
Assuming that the impulse response of the system is a boxcar. Also, let us assume that the target location is known to be between two range gates r and $r+1$. Let us denote true range $r_0 \in \mathbb{R}$ in range gate scale, we thus know that the target is $r_0 = r + R$, where $R \in [0, 1]$.

The using a range ambiguity resulting from a boxcar impulse response, the measurement equation can be written as

$$m_t = \sigma R(\epsilon_{t+1} - \epsilon_t) + \sigma \epsilon_t + \xi_t, \tag{13}$$

where ϵ_t is the transmission code and ξ_t is a normal complex random variable. Reorganising the terms, we get a linear form:

$$m_t - \sigma \epsilon_t = \sigma R(\epsilon_{t+1} - \epsilon_t) + \xi_t. \tag{14}$$

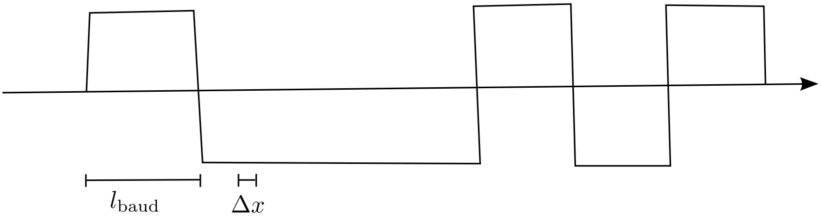
The variance for the range parameter R is:

$$\operatorname{var} R = \frac{1}{\sigma^2 \sum_t |\epsilon_{t+1} - \epsilon_t|^2}. \tag{15}$$

The result is quite interesting. The variance is inverse square proportional to the sum of absolute baseband changes within a code. Thus, a code with the a largest possible number of bauds is optimal. In addition to this, π phase changes are the most optimal. This that a binary phase code, with the largest possible number of phase changes is the most optimal code for sub-baud resolution.

This in conflict with code optimality for spread target (above baud-length range resolution). What if we combine these two?

Transmission code optimality



- Radar transmission bandwidth is limited.
  - How do you optimize sub-baud range resolution?
  - This can be determined by looking at the code-dependent estimation covariance $\Sigma_p(\epsilon_t) \propto (\overline{\mathbf{A}(\epsilon_t)}^{\mathrm{T}} \mathbf{A}(\epsilon_t))^{-1}$.
- Coding with **non-uniform baud-lengths**

Radio Science, Volume ???, Number , Pages 1–7,

Fractional baud-length coding

J. Vierinen

Sodankylä geophysical observatory, Sodankylä, Finland

We present a novel approach for timing radar transmission envelopes in order to improve target range and Doppler resolution. This is achieved by using non-uniform baud lengths. With this method, it is possible to significantly increase sub-baud range-resolution of radar measurements while maintaining a narrow bandwidth. We first derive target estimation accuracy in terms of a covariance matrix for arbitrary targets when estimating backscatter in amplitude domain. We define target optimality and discuss different search strategies that can be used to find well performing transmission envelopes. We give several examples and compare the results to conventional uniform baud length transmission codes.

1. Introduction

We have previously described a method for estimating range and Doppler spread radar targets in amplitude domain at sub baud-length range-resolution using linear statical inversion [*Vierinen et al.*, 2007b]. However, we did not use codes optimized for the targets that we analyzed. Also, we only briefly discussed code optimality. In this paper we will focus on optimal transmission codes for a target range resolution that is smaller than the minimum allowed baud-length. We will introduce so called *fractional baud-length codes* that are optimal for range and possibly Doppler spread targets, with a better resolution than the minimum allowed radar transmission envelope baud-length.

In radar systems, there is a limit to the smallest baud length, which arises from available bandwidth

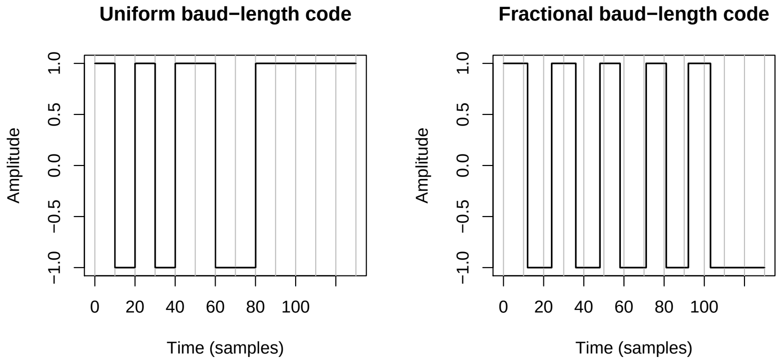
form baud-length radar transmission code with baud lengths that are integer multiples of 1 μs. The reason is that the uniform baud-length will cause a singular or near-singular covariance matrix when analyzing experiments with sub-baud range-resolution.

In this paper, we first derive the target parameter estimation covariance for range and Doppler spread radar targets when estimating target parameters in amplitude domain. Then we define transmission code optimality for a given target. After this, we then present two search strategies which can be used to find optimal transmission codes: an exhaustive search algorithm, and an optimization search algorithm. As an example, we study code optimality in the case of a range spread coherent target, and a range and Doppler spread target.

2. Transmission code

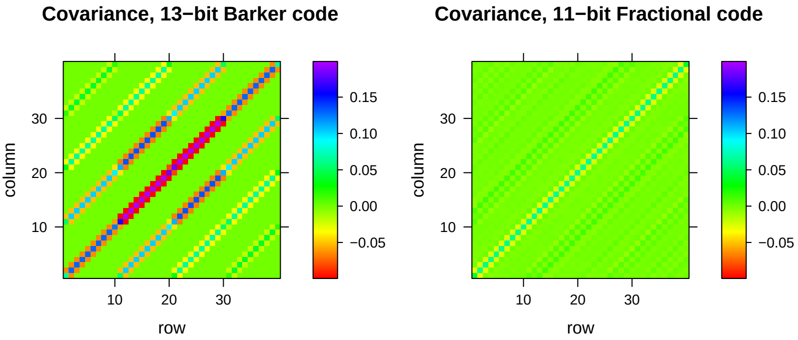
Fractional baud-length code

Baud lengths timed very accurately, but no baud is not shorter than the minimum allowed length.



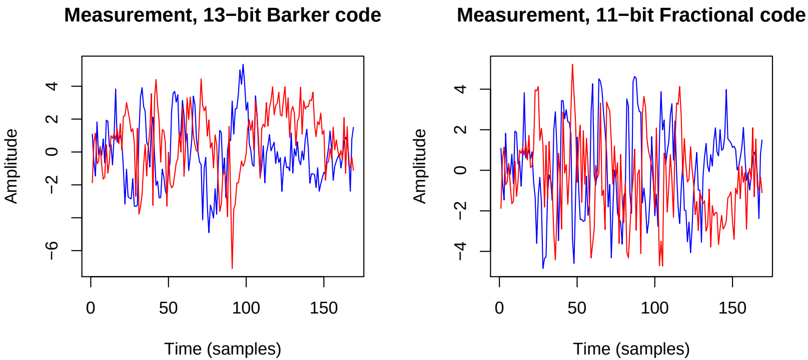
Error covariance matrix

Uniform baud-length code has close to singular covariance. Fractional baud-length code has smaller variance and very low off-diagonal elements.

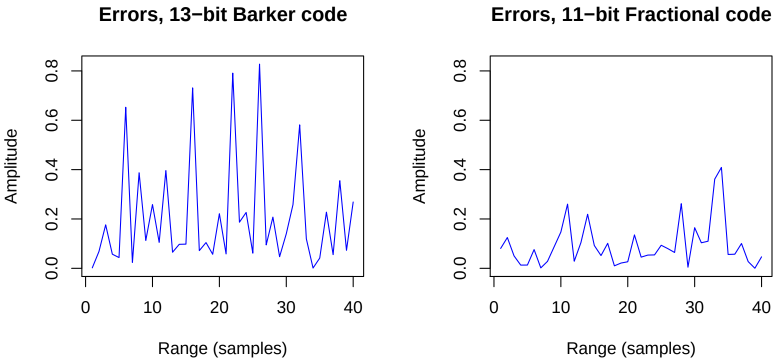


Simulated measurement

SNR ≈ -2 dB. Target range extent 20 samples. Code length 130 samples, with 10 sample bauds.



Simulation Errors



Conclusions

- Heating related strong range and Doppler spread echos can be analyzed in amplitude domain on a single echo basis if they are narrow enough (in range and Doppler spread)
- Meteor head echo parameters can be determined very accurately even for low-bandwidth transmissions. Range resolution only limited by SNR, accuracy of impulse response and system clock. Typically < 10 m for strong echos and < 100 m for weak echos.
- *Fractional baud-length* coding improves sub-baud range resolution estimation accuracy. At EISCAT 15 m range resolution possible.
- Future work will focus on amplitude domain inversion of overspread weak incoherent backscatter.