Statistical analysis of ISR experiments using MCMC

Juha Vierinen, Ilkka Virtanen, Markku Lehtinen and Antti Kero

Sodankylä Geophysical Observatory

Juha.Vierinen@sgo.fi

Abstract

The analysis of incoherent scatter measurements is mostly done by finding a maximum of the posterior distribution of the plasma parameters. In this study we have used the Markov chain Monte-Carlo method to study the full distribution of the plasma parameters. As an example, we show how the method can be applied to D and E-layer measurements. We have shown that examining the full distribution of the estimated parameters yields more information than the traditional method and makes the inversion more stable.

1. Introduction

INCOHERENT scatter radar measurements are inherently of stochastic nature. The target itself is assumed to be a random process with a given autocorrelation over some time interval [4]. It is the autocorrelation function that is estimated and fitted to physical theory. Traditionally this has been done by finding the maximum of the *a posteriori* distribution (eg., GUISDAP). In this case, we do not get the full posteriori distribution of parameters, which would be useful in cases where the distribution is wide or multimodal. In this poster, we present initial results that utilize the Markov chain Monte-Carlo [3] method to study the full posterior distribution. We show several different case studies involving D- and E-region of the ionosphere.

2. Theory

The Bayes theorem is the basis of statistical inversion. It allows us to study the probability distribution of physical parameters θ of a physical theory, additionally allowing us to impose information about the parameters using an *a priori* distribution $p(\theta)$. Here D denotes measurement. It is this density that we solve using MCMC.

$$p(\theta|D) = \frac{p(D|\theta)\,p(\theta)}{p(D)}, \tag{1}$$

$$\propto p(D|\theta)\,p(\theta). \tag{2}$$

The measured autocorrelation functions are described in terms of a physical theory of incoherent scatter [1, 2]. Probability is defined by assuming measurement errors to be independent and normal distributed. The parameter vector contains all plasma parameters and several calibration related constants $\theta = \{C, a, N_e, T_i, T_e/T_i, \nu_i, v, \phi, \sigma^2, r\}$. These are: scaling constant, DC offset, electron density, ion temperature, electron-ion temperature ratio, ion-neutral collision frequency, bulk velocity and ion species concentrations, measurement variance and target range. Figure 1. shows a D-region ACF measurement and the maximum *a posteriori* model.

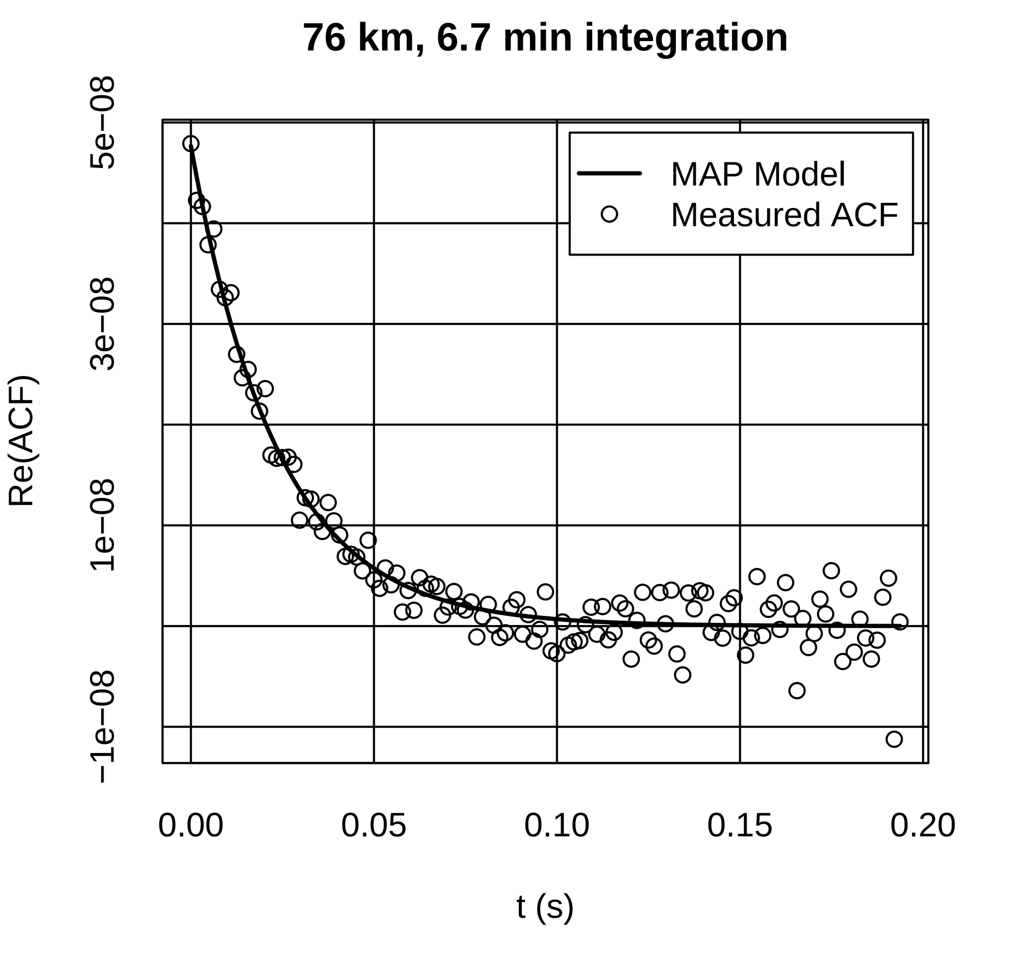

Figure 1: *An example MAP model and measurements of a D-region experiment.*

All parameters can be freely distributed. In some cases we have used MSIS, IRI and SIC-models to obtain prior information about the parameters. This has been done by setting the priori distribution of a certain parameter to be a Gaussian distribution with a sufficiently large variance and the mean given by the experimental ionosphere model (eg. MSIS).

3. Marginal distributions

MCMC gives us a set of points from the joint probability distribution of all parameters, but this has a too large dimensionality to be visualized as is. Thus, it is useful to study marginal distributions of one or two parameters. This can be easily estimated using a histogram. Fig. 2. depicts estimates of marginal distributions of the velocity and temperature parameters of a certain range. From this distribution, one can also extract a confidence interval for some parameter range, which is useful for determining the quality of a measurement of a parameter.

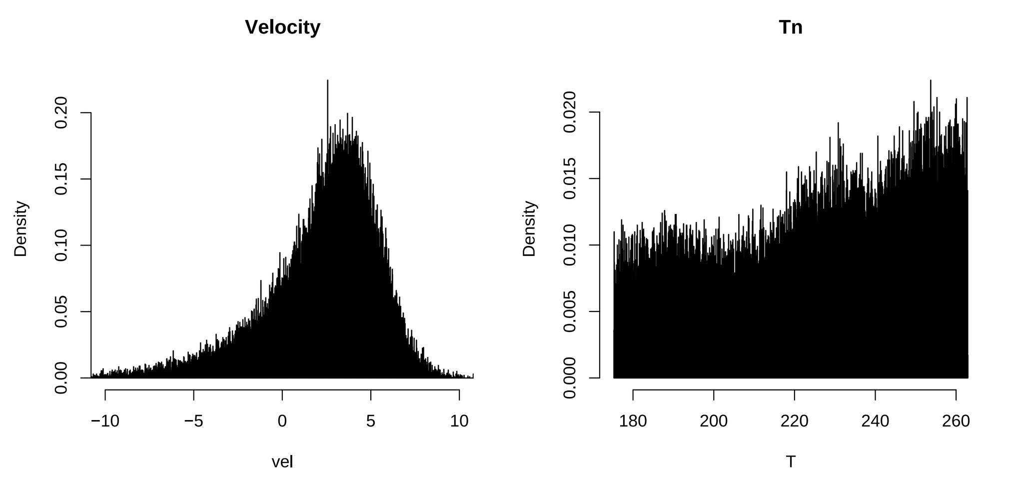

Figure 2: *Marginal distributions of the velocity and neutral temperature parameters in a D-region experiment.*

4. Example: D-region velocities

On 24/11/2006, we conducted D-region measurements during PMWE conditions. We used this data for studying ionospheric heating, but we also noticed that the velocities could be determined nicely from the data. Fig. 3. shows the radial velocities during a 20 minute interval. Here we used the width of the marginal distribution of the velocity to avoid plotting too noisy data.

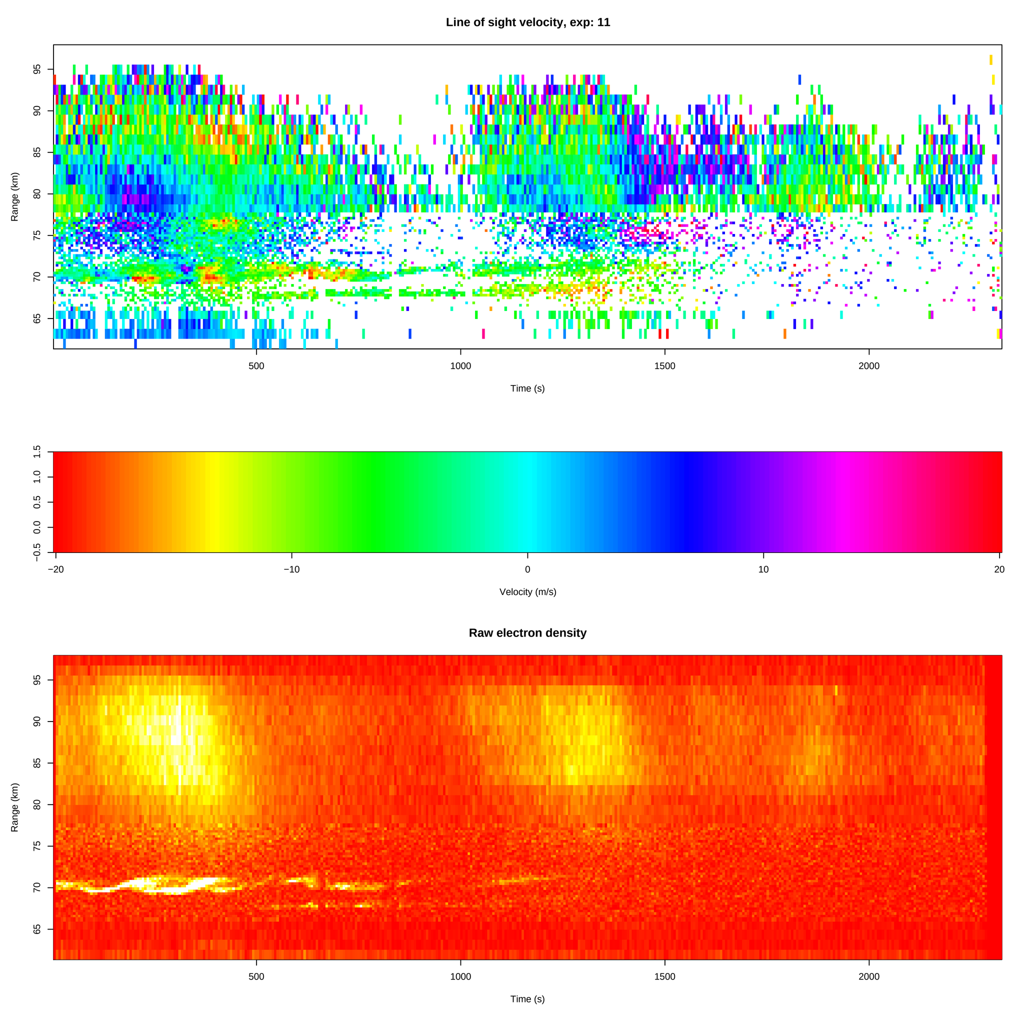

Figure 3: *D-region radial velocities during PMWE with EISCAT VHF on 24.11.2006. For purely visualization purposes, velocities considered too noisy have been removed and replaced with white. This results in a more "smooth" colormap of the velocities albeit with missing values.*

It has been proposed that the theory for weakly ionized plasma does not apply to PMWE, which is thought to be turbulent scattering. Unrealistically large Ne in the PMWE region supports this. In determining the bulk velocity of the medium this is not that critical, as another interpretation is that we are simply fitting a Lorentzian velocity spectrum to the region.

5. Example: E-region ion composition

E-region measurements were conducted with a new code pair experiment called SIPPI-E. In this case, we used three ion masses and examined the posterior distributions of their concentrations at a single range gate using a 2s integration time. The estimated marginal distributions are shown in Fig. 4. The distributions are extremely wide, which means that there is a large uncertainty in determining these parameters.

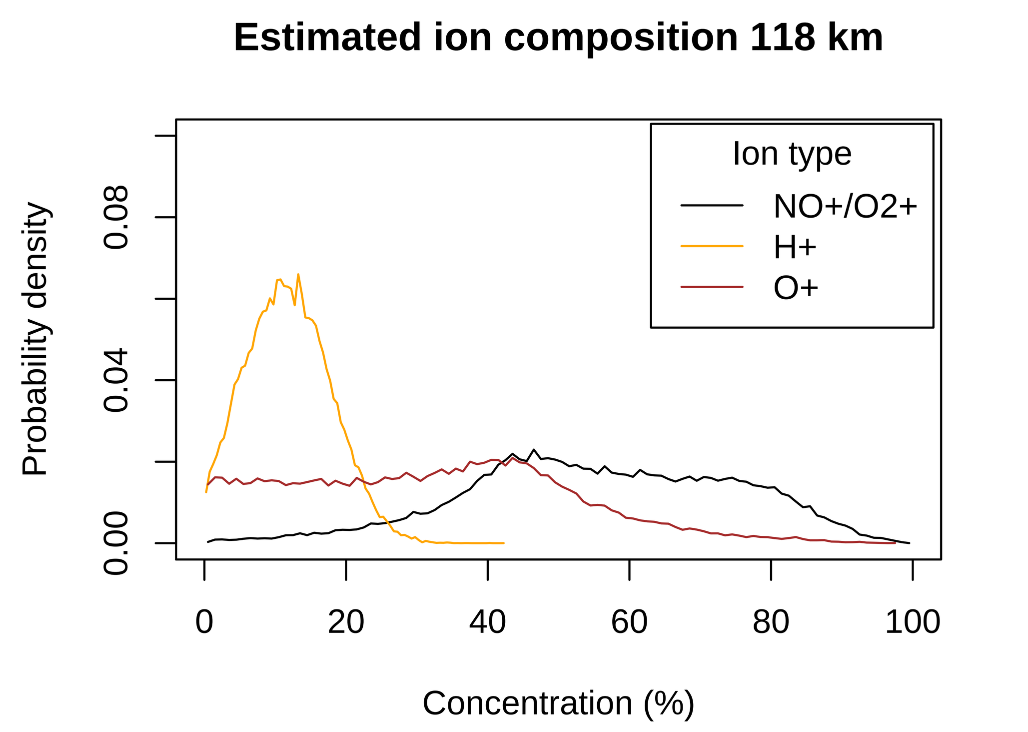

Figure 4: *The marginal probability densities of ion composition concentrations in F-region.*

To narrow down these distributions one would need an independent temperature measurement. Another way would be to combine measurements from neighboring range gates and time instances, assuming that the concentration parameters are a fairly smooth function of time and range.

6. Conclusions

Statistical inversion offers a consistent and robust framework for comparing measurements to models. Often one is confronted with a problem that cannot be solved using a linear model (in a stochastic sense) and one needs to examine the possibly complex shaped posterior distribution by some means. The maxima of a distribution can often be found using gradient-type methods, but the shape of the distribution is often also important – one might have several nearly equally large peaks in the distribution and the widths of the peaks may vary. MCMC makes it possible to study the posteriori distribution numerically, albeit with an increase in computer processing needs.

References

[1] J.P. Dougherty and D.T. Farley. A theory of incoherent scattering of radio waves by a plasma. *Proc. Roy. Soc. A*, 259, 1960.

[2] K. Fukuyama and W. Kofman. Incoherent scattering of an electromagnetic wave in the mesosphere: a theoretical consideration. *J. Geomagn. Geoelectr.*, 32:67–81, 1980.

[3] W. R. Gilks, S. Richardson, and D. J. Spiegelhalter, editors. *Markov Chain Monte Carlo in Practice.* Chapman & Hall/CRC, Boca Raton, 1996.

[4] M. Lehtinen. *Statistical theory of incoherent scatter measurements.* EISCAT Tech. Note 86/45, 1986.

13th EISCAT International Workshop and Radar School, 6-10 August 2007, Aland Islands