

General radar transmission codes

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Overview

Coherent target:

- ▶ Polyphase coding improves the measurement
- ▶ Amplitude modulation is necessary for perfect coding

Incoherent target:

- ▶ Polyphase coding improves the measurement
- ▶ Amplitude modulation improves measurements even further
- ▶ Amplitude modulation allows one to “focus” radar power on important lags allowing us to measure a subset of lags better than alternating codes

General radar transmission codes that minimize measurement error of a static target

Juha Vierinen, Markku Lehtinen, Mikko Orispää, and Baylie Damtie

Abstract—The variances of matched and sidelobe free mismatched filter estimators are given for arbitrary coherent targets in the case of aperiodic transmission. It is shown that mismatched filtering is often better than matched filtering in terms of estimation accuracy. A search strategy for finding general transmission codes that minimize estimation error and satisfy constraints on code power and amplitude range is then introduced. Results show that nearly perfect codes, with performance close to a single pulse with the same total power can be found. Also, finding these codes is not computationally expensive and such codes can be found for all practical code lengths. The estimation accuracy of similar length found codes are compared to binary phase codes of similar length and found to be 5–40% better in terms of estimator variance. Similar transmission codes might be worth investigating also for sonar and telecommunications applications.

Index Terms—radar codes, matched filter, mismatched filter, general modulation codes, target estimation

phases and amplitudes defined by parameters ϕ_k and a_k . These parameters obtain values $\phi_k \in [0, 2\pi]$ and $a_k \in [a_{\min}, a_{\max}]$, where $k \in \{1, \dots, L\}$; $k \in \mathbb{N}$. The reason why one might want to restrict the amplitudes to some range stems from practical constraints in transmission equipment. In most traditional work, the amplitudes have been set to 1 and often the number of phases has also been restricted, eg., in the case of binary phase codes to $\phi_k \in \{0, \pi\}$.

Defining $\delta(t)$ with $t \in \mathbb{Z}$ as

$$\delta(t) = \begin{cases} 1 & \text{when } t = 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

we can describe an arbitrary baseband radar code $\epsilon(t)$ as

$$\epsilon(t) = \sum_{k=1}^L a_k e^{i\phi_k} \delta(t - k + 1). \quad (2)$$

In addition to this, we restrict the total transmission code power to be constant for all codes of similar length. Without any loss of generality, we set code power equal to code length

$$L = \sum_{t=1}^L |\epsilon(t)|^2. \quad (3)$$

This will make it possible to compare estimator variances of codes with different lengths and therefore different total transmission powers. Also, it is possible to compare codes of the same length and different transmission power simply by treating L as transmission power.

1. INTRODUCTION

PHASE modulation of a radar transmission is a well known method for increasing radar transmission power, while still maintaining a good range resolution. Such transmission codes can consist of two or more individual phases. The performance of binary, quadri and polyphase codes has been thoroughly inspected in terms of heuristic criteria, such as the integrated sidelobe level (ISL), or peak to sidelobe level (PSL) [1]–[7]. In previous work, binary phase codes have also been investigated in terms of estimation accuracy of a static target for sidelobe free mismatched filter for

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MEASUREMENT EQUATION

describes the basic principle of estimating a target with a linear filter. When the target is moving, the Doppler shift in the returned signal must be accounted for.

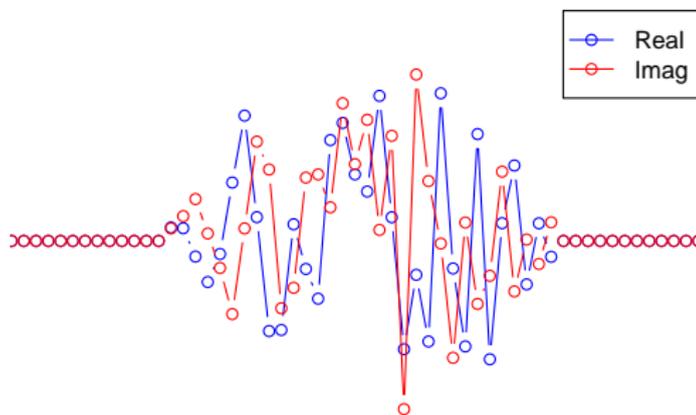
General transmission code

Defining $\delta(t)$ with $t \in \mathbb{Z}$ as

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we define an arbitrary baseband radar code $\epsilon(t)$ of length L as

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Coherent (stationary) target

The measurement can be expressed as a convolution

$$m(t) = \sum_{\tau=-\infty}^{\infty} \underbrace{\epsilon(\tau)}_{\text{transmission}} \underbrace{\sigma(t-\tau)}_{\text{target}} + \underbrace{\xi(t)}_{\text{noise}} = (\epsilon * \sigma)(t) + \xi(t) \quad (3)$$

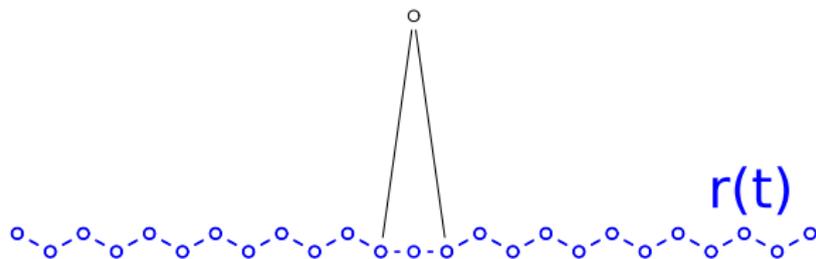
- ▶ Assuming target coherent, ie., scattering amplitude stays constant $\sigma(r, t) = \sigma(r, t + l) = \sigma(r)$, while transmission travels through
- ▶ Use round-trip time as range index $\sigma(t)$
- ▶ Assume that target is infinite length
- ▶ This can now be solved easily in frequency domain

Maximum likelihood estimate

- ▶ For a distributed target, the so called inverse filter is the maximum likelihood estimate $h_\lambda(t) = \mathcal{F}_D^{-1} \{L/\hat{\epsilon}(\omega)\}$
- ▶ For a point-like target, the matched filter is the maximum likelihood estimate $h_m(t) = \overline{\epsilon(-t)}$

Shown by eg. (Ruprecht 1989)

Matched filter in terms of the inverse filter



If we subtract the sidelobes, we can relate the matched filter $h_m(t)$ to the inverse filter $h_\lambda(t)$

$$(\epsilon * h_m)(t) - r(t) = L\delta(t) \quad (4)$$

$$h_m(t) = h_\lambda(t) + \frac{1}{L}(h_\lambda * r)(t), \quad (5)$$

We can use this to solve the convolution equation for a matched filter. The latter also gives some insight on perfect codes.

Filter output

After filtering, the matched filter is

$$m_m(t) = L\sigma(t) + \underbrace{(r * \sigma)(t)}_{\text{sidelobe term}} + \underbrace{(\xi * h_m)(t)}_{\text{measurement error}} . \quad (6)$$

and the inverse filter is

$$m_\lambda(t) = L\sigma(t) + \underbrace{(\xi * h_\lambda)(t)}_{\text{measurement error}} . \quad (7)$$

Total noise power is

$$B_{\text{mat}} = \sum_{t=-\infty}^{\infty} |(\xi * h_m)(t)|^2 = L\text{SNR}^{-1} \quad (8)$$

$$B_{\text{inv}} = \sum_{t=-\infty}^{\infty} |(\xi * h_\lambda)(t)|^2 \geq L\text{SNR}^{-1} \quad (9)$$

Stochastic target

For a spread stochastic target $\mathbf{E} \sigma(t) \overline{\sigma(t')} = x(t) \delta(t - t')$, the target estimation variance is:

$$\mathbf{Var} \hat{x}_{\text{mat}}(t) = \frac{1}{N} \left[x(t)^2 + \frac{2B_{\text{mat}} x(t)}{L^2} + \frac{2S(t)x(t)}{L^2} + \frac{B_{\text{mat}}^2}{L^4} + \frac{S(t)^2}{L^4} + \frac{2B_{\text{mat}} S(t)}{L^4} \right] \quad (10)$$

and the inverse filter has variance:

$$\mathbf{Var} \hat{x}_{\text{inv}}(t) = \frac{1}{N} \left[x(t)^2 + \frac{2B_{\text{inv}} x(t)}{L^2} + \frac{B_{\text{inv}}^2}{L^4} \right] \quad (11)$$

Code optimality

- ▶ There are many things that one can take into account.
- ▶ One is to minimize the total noise power B_{inv} (Lehtinen & Dantie 2004).
- ▶ In the case of a stochastic target, this is also the only code-dependent term.

Optimization searches

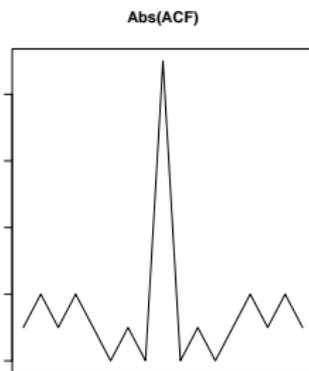
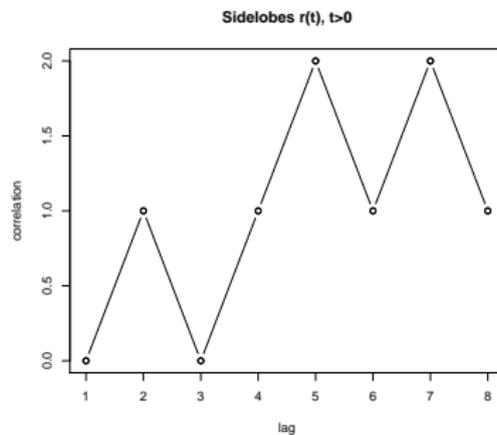
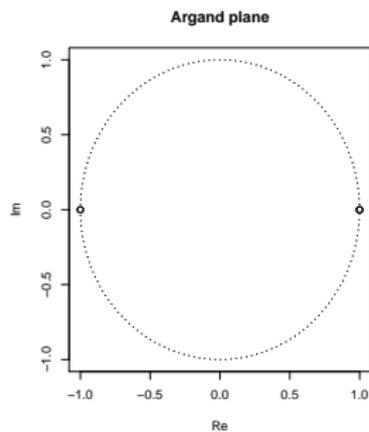
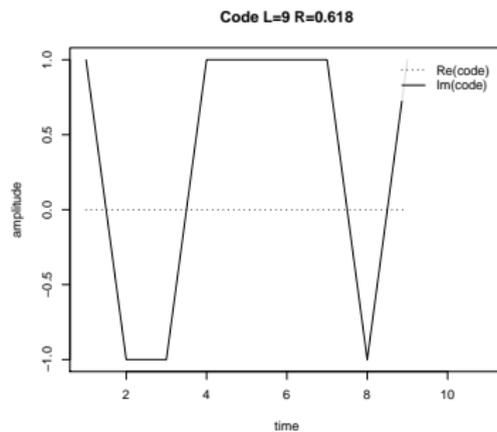
- ▶ Codes were searched using a method similar to simulated annealing
- ▶ Different restrictions were applied to the code amplitudes
- ▶ Code better than the best binary phase codes can be found this way
- ▶ It turns out that amplitude modulation allows nearly perfect finite length codes

We use the following parametrization for an arbitrary baseband radar code $\epsilon(t)$ of length L as:

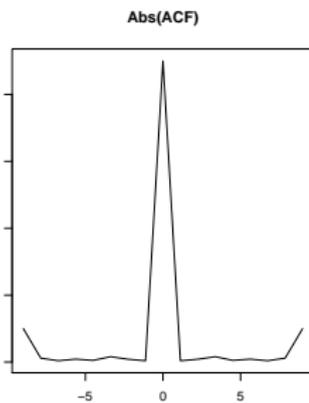
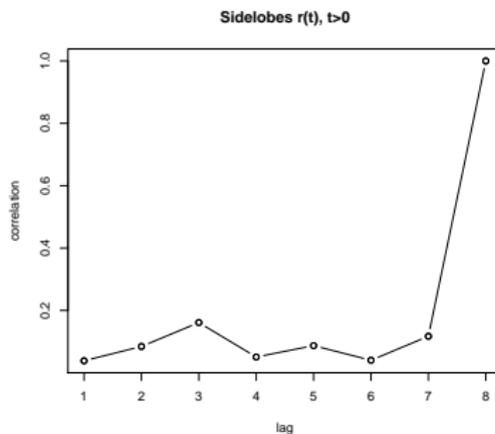
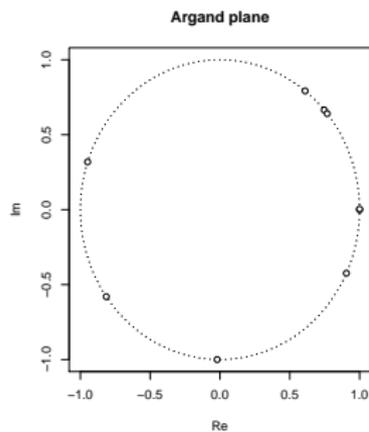
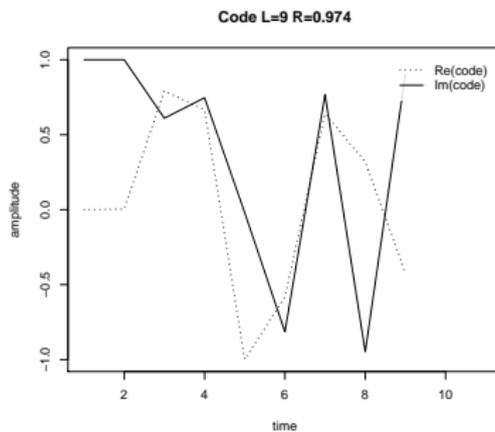
$$\epsilon(t) = \sum_{k=1}^L a_k e^{i\phi_k} \delta(t - k + 1). \quad (12)$$

where $a_k \in [a_{\min}, a_{\max}]$ constrained to some interval.

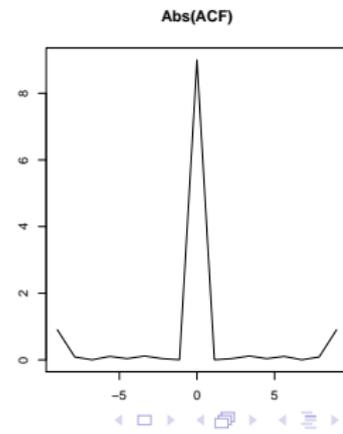
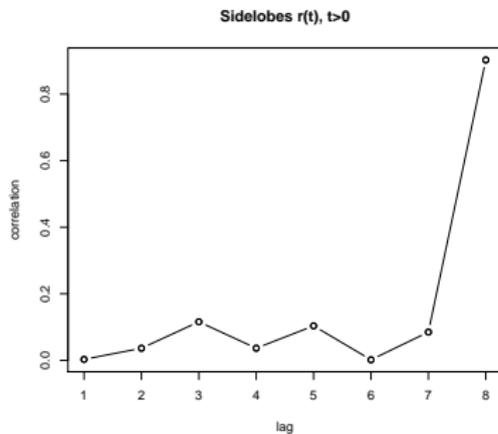
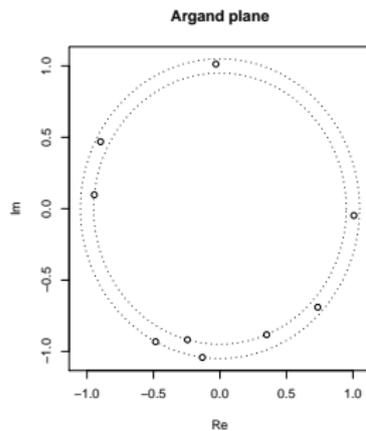
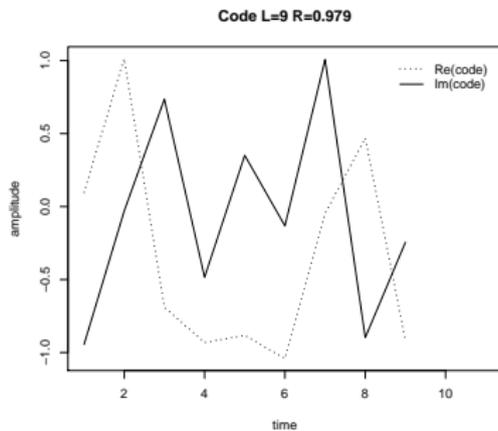
Binary phase code L=9



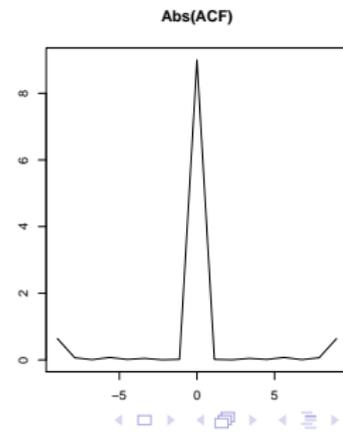
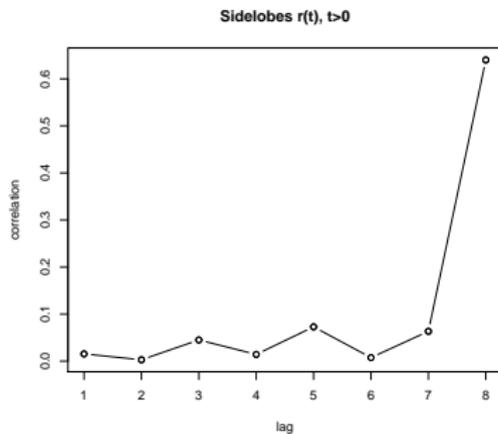
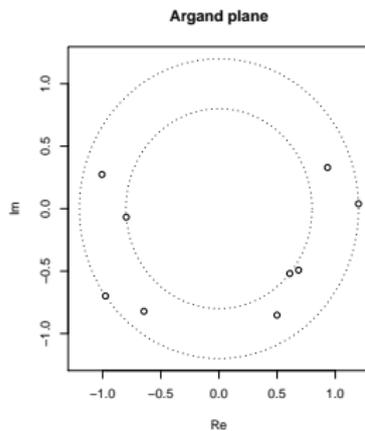
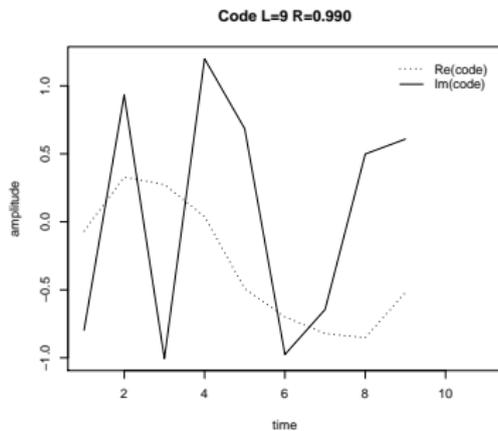
Polyphase $a_k = 1$ code L=9



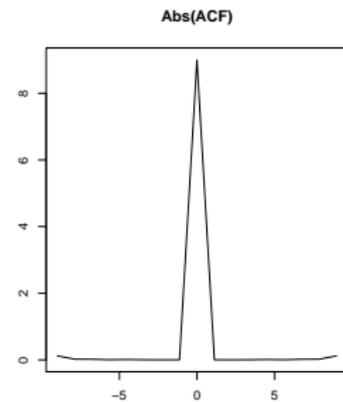
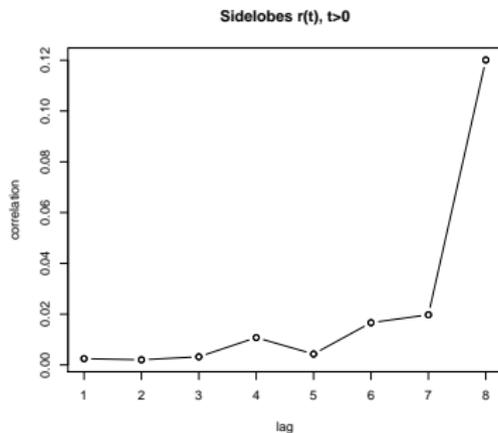
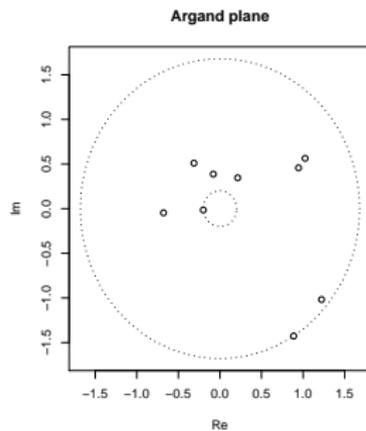
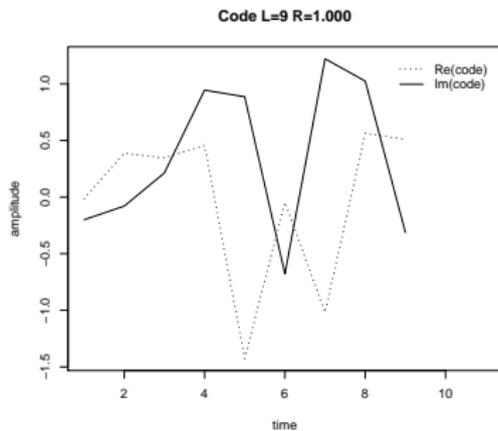
$a_k \in [0.95, 1.05]$ code L=9



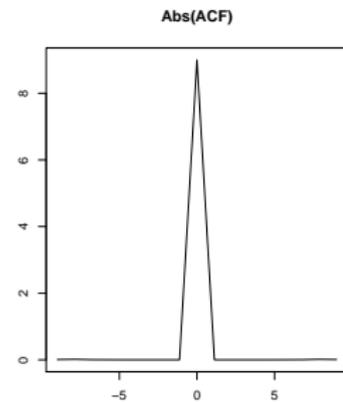
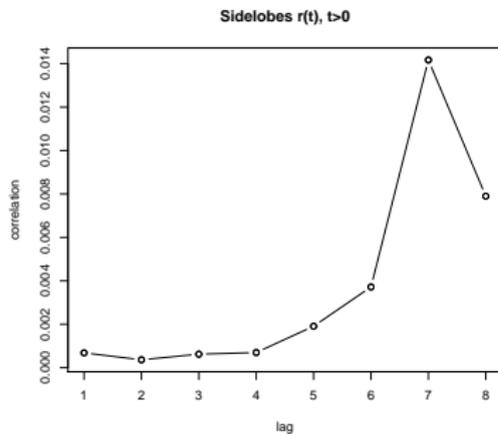
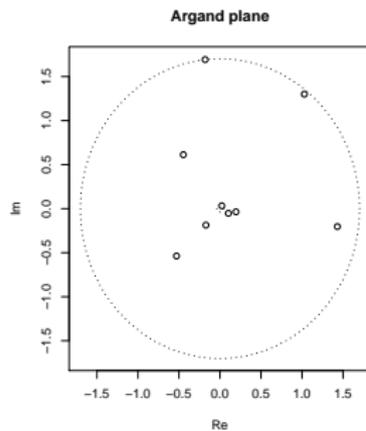
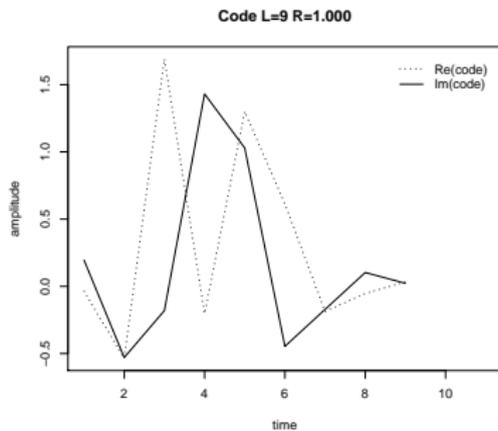
$a_k \in [0.80, 1.20]$ code L=9



$a_k \in [0.20, 1.80]$ code L=9



$a_k \in [0, 2]$ code L=9



Incoherent target (less stationary)

- ▶ Measurements are lagged products of measured receiver voltage
- ▶ Lagged product transmissions are usually groups of codes, which we will index as $\epsilon^c(t)$
- ▶ Under certain assumptions, lagged product measurements can be stated as a convolution equation involving target ACF and lagged product envelope:

$$m(t)\overline{m(t+\tau)} \equiv m_\tau(t) \quad (13)$$

$$\epsilon^c(t)\overline{\epsilon^c(t+\tau)} \equiv \epsilon_\tau^c(t) \quad (14)$$

$$m_\tau(t) = (\epsilon_\tau^c * \sigma_\tau)(t) + \xi_\tau(t) \quad (15)$$

Noise term

The normalized measurement “noise power” of a certain lag is:

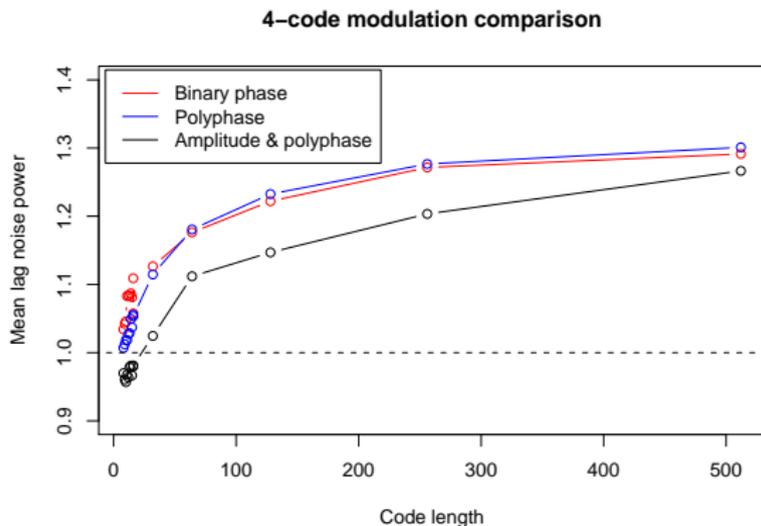
$$\mathcal{F}_D^M\{\epsilon_\tau^c(t)\} = \hat{\epsilon}_\tau^c(\omega) \quad (16)$$

$$P_\tau \approx \int_0^{2\pi} \frac{N_c(N_b - \tau)}{\sum_{c=1}^{N_c} |\hat{\epsilon}_\tau^c(\omega)|^2} d\omega \quad (17)$$

For alternating codes $P_\tau = 1$ for all τ . But when amplitude modulation is used, this is not the lower limit, because in some cases, more radar power can be used on certain lags, even though the average transmission power is the same.

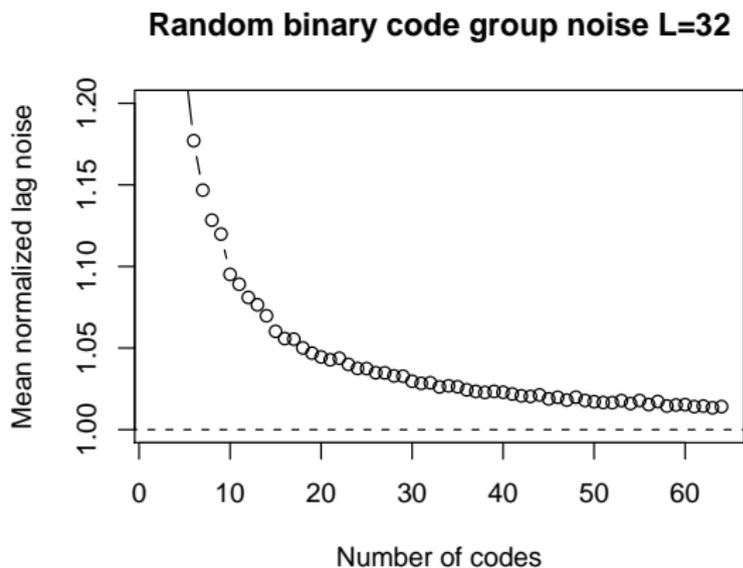
Code length

- ▶ Polyphase code groups have better theoretical properties than binary phase code groups
- ▶ Amplitude modulation improves them further
- ▶ As code length is increased, the modulation becomes less important



Code group length

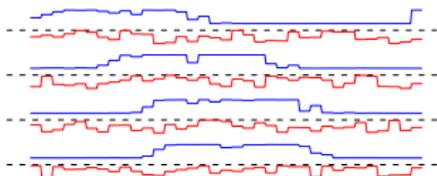
- ▶ As code group size is increased, the noise performance gets better



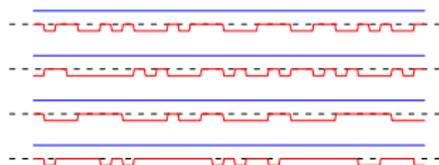
Amplitude and phase modulated code group

- ▶ Total power of one pulse is set to be constant (in our considerations similar to constant amplitude pulse of same length)
- ▶ Amplitude modulation makes it possible to use more radar power on important lags
- ▶ A subset of lags can be extracted with lower noise power than with alternating codes

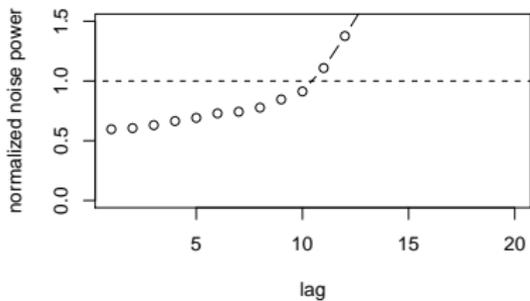
35-baud code group with 4 codes



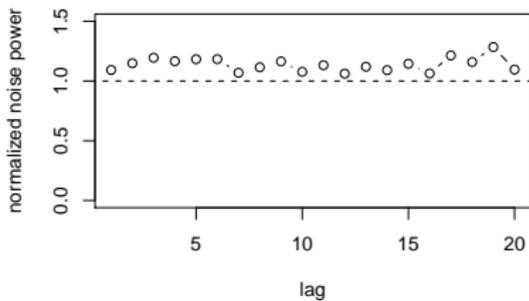
35-baud code group with 4 codes



Code group normalized noise power

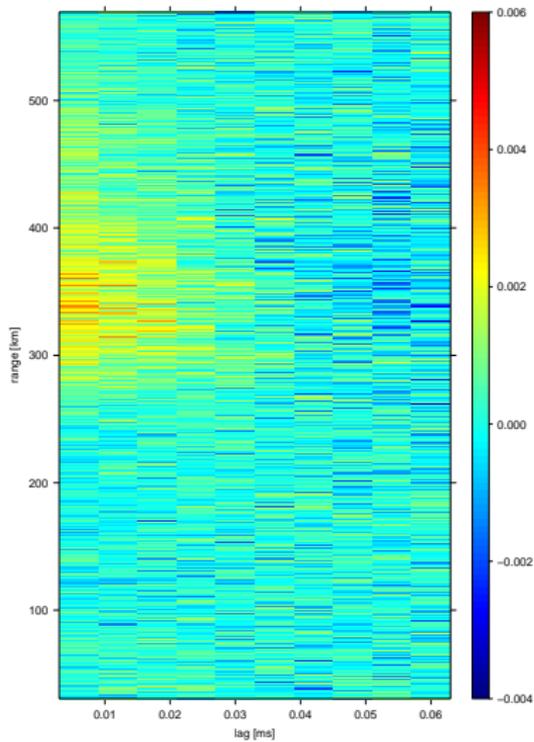


Code group normalized noise power



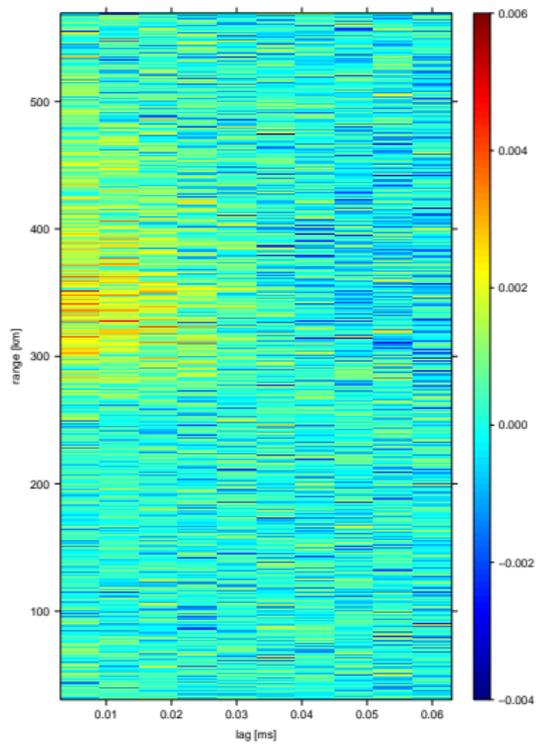
AMPP

real part

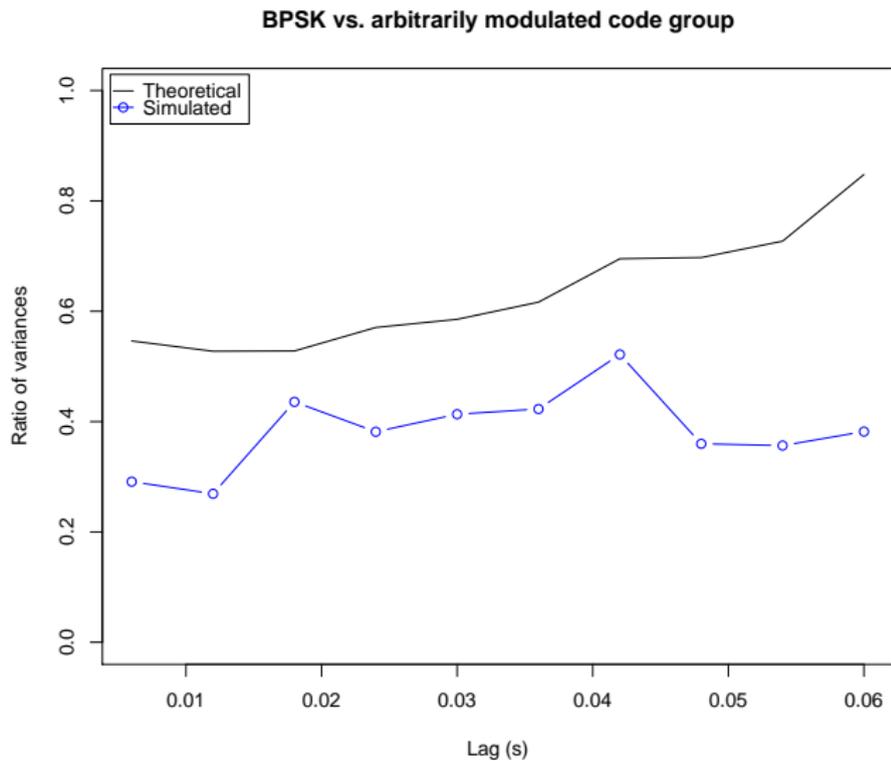


Binary

real part

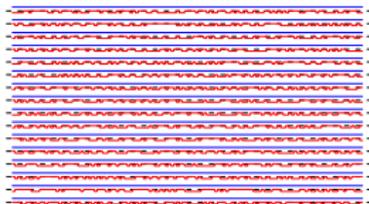


Simulation results...

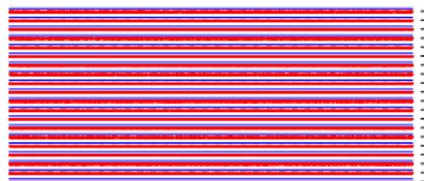


Even higher resolution...

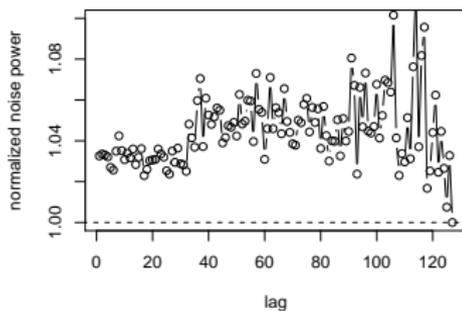
128-baud code group with 16 codes



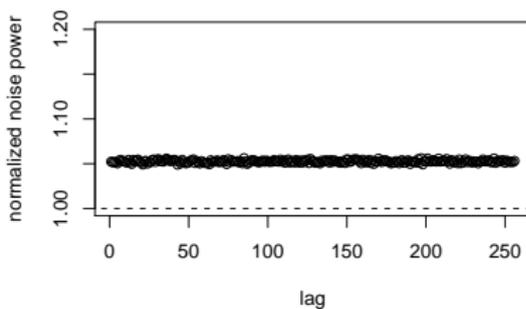
4096-baud code group with 20 codes



Code group normalized noise power



Code group normalized noise power



Conclusions

- ▶ ISR can benefit in many ways from amplitude and polyphase modulation
- ▶ Polyphase coding allows shorter code groups while maintaining a thermal noise figure close to theoretical minimum
- ▶ Amplitude modulation makes it possible to focus transmission power on important lags, thus making possible better modulations than alternating codes (for a subset of lags)

Conclusions for incoherent targets

- ▶ Amplitude and phase modulation allow better measurements than constant amplitude binary phase coded measurements of same transmission power
- ▶ We would like try to these ideas in reality, who would want to collaborate?

Questions?