General radar transmission codes

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August 6, 2007

Overview

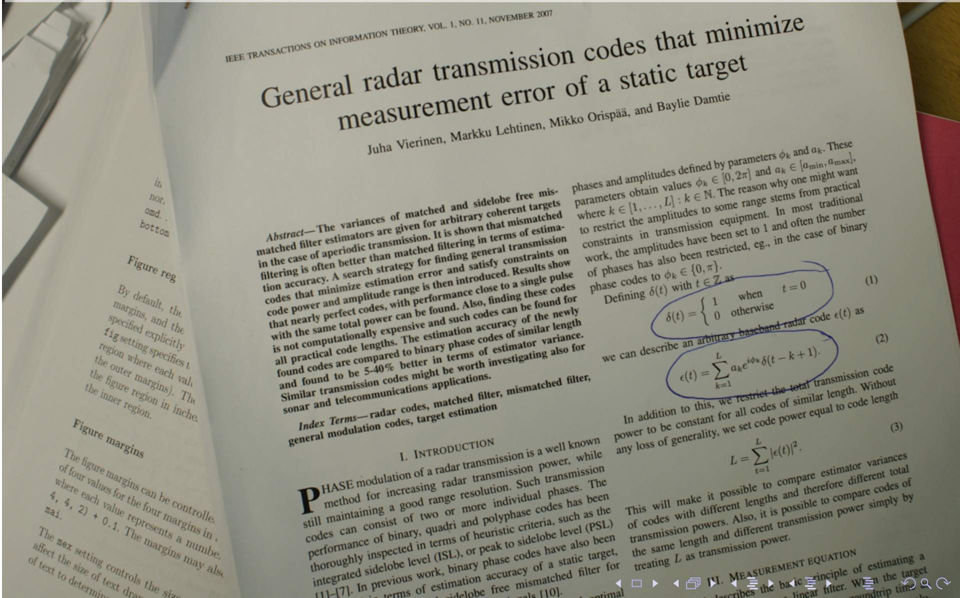
Coherent target:

- Polyphase coding improves the measurement
- Amplitude modulation is necessary for perfect coding

Incoherent target:

- Polyphase coding improves the measurement
- Amplitude modulation improves measurements even further
- Amplitude modulation allows one to “focus” radar power on important lags allowing us to measure a subset of lags better than alternating codes

arxiv.org/abs/physics/0612040



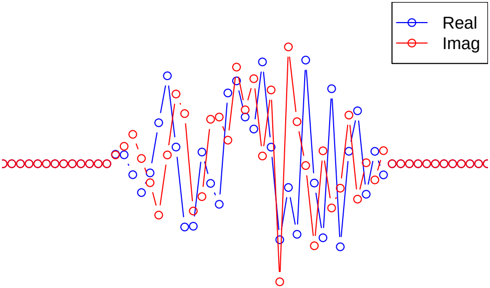
General transmission code

Defining $\delta(t)$ with $t \in \mathbb{Z}$ as

$$\delta(t) = \begin{cases} 1 & \text{when} \quad t = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

we define an arbitrary baseband radar code $\epsilon(t)$ of length L as

$$\epsilon(t) = \sum_{k=1}^{L} a_k e^{i\phi_k} \delta(t - k + 1). \tag{2}$$



Coherent (stationary) target

The measurement can be expressed as a convolution

$$m(t) = \sum_{\tau=-\infty}^{\infty} \underbrace{\epsilon(\tau)}_{\text{transmission}} \overbrace{\sigma(t-\tau)}^{\text{target}} + \underbrace{\xi(t)}_{\text{noise}} = (\epsilon * \sigma)(t) + \xi(t) \quad (3)$$

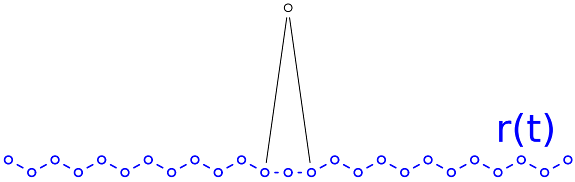
- Assuming target coherent, ie., scattering amplitude stays constant $\sigma(r,t) = \sigma(r, t+l) = \sigma(r)$, while transmission travels through
- Use round-trip time as range index $\sigma(t)$
- Assume that target is infinite length
- This can now be solved easily in frequency domain

Maximum likelihood estimate

- For a distributed target, the so called inverse filter is the maximum likelihood estimate $h_\lambda(t) = \mathcal{F}_D^{-1}\left\{L/\hat{\epsilon}(\omega)\right\}$
- For a point-like target, the matched filter is the maximum likelihood estimate $h_m(t) = \overline{\epsilon(-t)}$

Shown by eg. (Ruprecht 1989)

Matched filter in terms of the inverse filter



If we subtract the sidelobes, we can relate the matched filter $h_m(t)$ to the inverse filter $h_\lambda(t)$

$$(\epsilon * h_m)(t) - r(t) = L\delta(t) \tag{4}$$

$$h_m(t) = h_\lambda(t) + \frac{1}{L}(h_\lambda * r)(t), \tag{5}$$

We can use this to solve the convolution equation for a matched filter. The latter also gives some insight on perfect codes.

Filter output

After filtering, the matched filter is

$$m_m(t) = L\sigma(t) + \underbrace{(r * \sigma)(t)}_{\text{sidelobe term}} + \underbrace{(\xi * h_m)(t)}_{\text{measurement error}} . \quad (6)$$

and the inverse filter is

$$m_\lambda(t) = L\sigma(t) + \underbrace{(\xi * h_\lambda)(t)}_{\text{measurement error}} . \quad (7)$$

Total noise power is

$$B_{\mathrm{mat}} = \sum_{t=-\infty}^{\infty} |(\xi * h_m)(t)|^2 = L\mathrm{SNR}^{-1} \quad (8)$$

$$B_{\mathrm{inv}} = \sum_{t=-\infty}^{\infty} |(\xi * h_\lambda)(t)|^2 \geq L\mathrm{SNR}^{-1} \quad (9)$$

Stochastic target

For a spread stochastic target $\mathbf{E}\,\sigma(t)\overline{\sigma(t')} = x(t)\delta(t-t')$, the target estimation variance is:

$$\mathbf{Var}\,\hat{x}_{\mathrm{mat}}(t) = \frac{1}{N}\Big[x(t)^2 + \frac{2B_{\mathrm{mat}}\,x(t)}{L^2} + \frac{2S(t)x(t)}{L^2} + \frac{{B_{\mathrm{mat}}}^2}{L^4} + \frac{S(t)^2}{L^4} + \frac{2B_{\mathrm{mat}}\,S(t)}{L^4}\Big] \tag{10}$$

and the inverse filter has variance:

$$\mathbf{Var}\,\hat{x}_{\mathrm{inv}}(t) = \frac{1}{N}\left[x(t)^2 + \frac{2B_{\mathrm{inv}}\,x(t)}{L^2} + \frac{{B_{\mathrm{inv}}}^2}{L^4}\right] \tag{11}$$

Code optimality

- There are many things that one can take into account.
- One is to minimize the total noise power B_{inv} (Lehtinen & Damtie 2004).
- In the case of a stochastic target, this is also the only code-dependent term.

Optimization searches

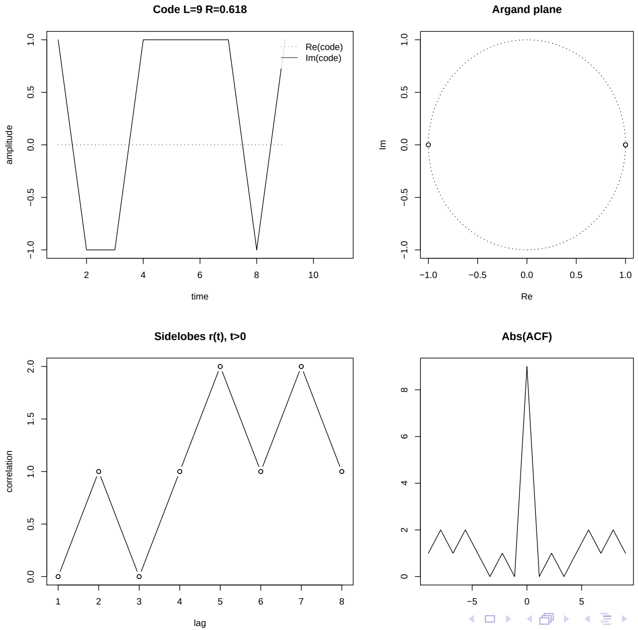
- Codes were searched using a method similar to simulated annealing
- Different restrictions were applied to the code amplitudes
- Code better than the best binary phase codes can be found this way
- It turns out that amplitude modulation allows nearly perfect finite length codes

We use the following parametrization for an arbitrary baseband radar code $\epsilon(t)$ of length L as:

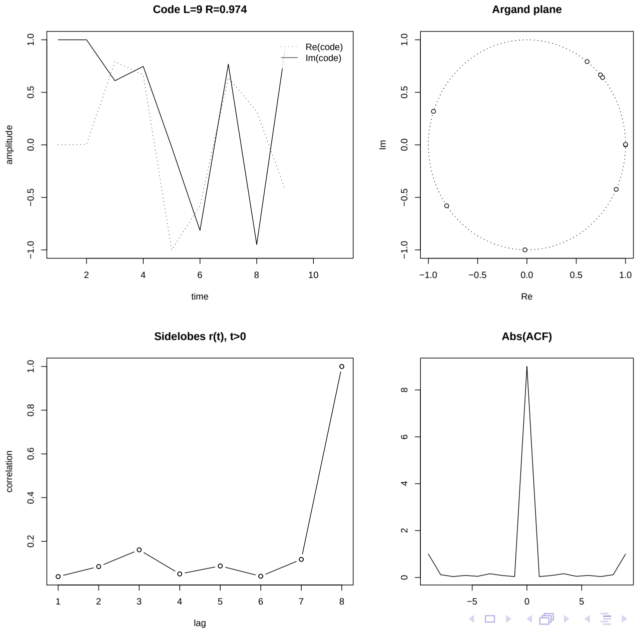
$$\epsilon(t) = \sum_{k=1}^{L} a_k e^{i\phi_k} \delta(t - k + 1). \tag{12}$$

where $a_k \in [a_{\min}, a_{\max}]$ constrained to some interval.

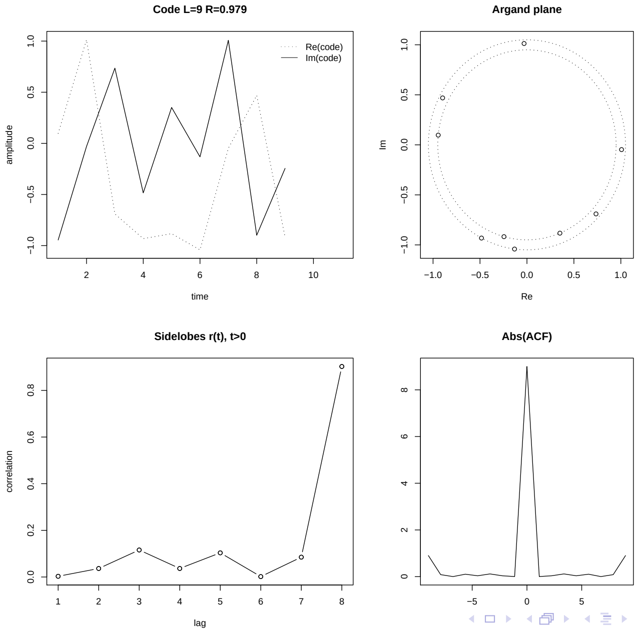
Binary phase code L=9



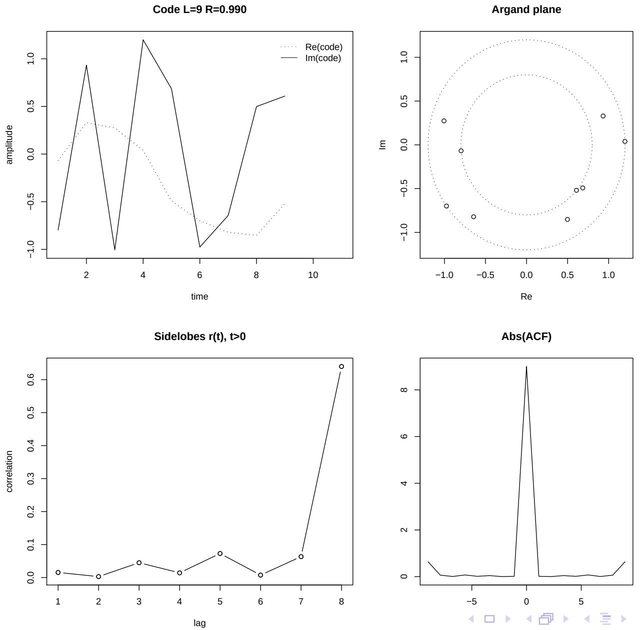
Polyphase $a_k = 1$ code L=9



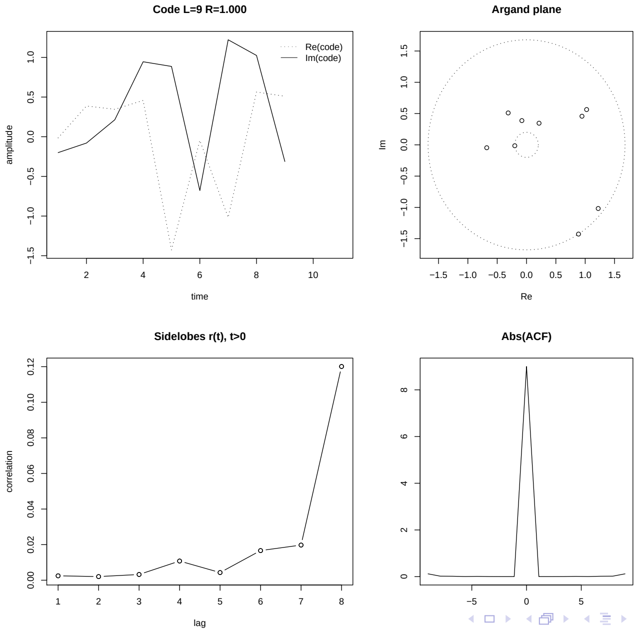
$a_k \in [0.95, 1.05]$ code L=9



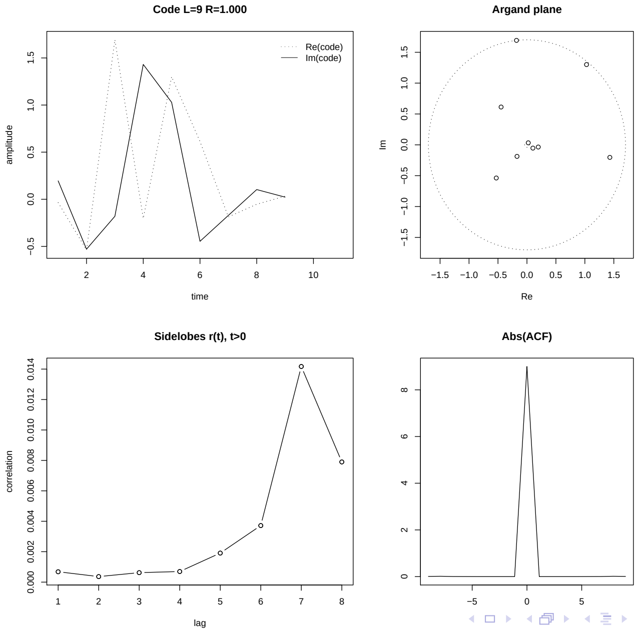
$a_k \in [0.80, 1.20]$ code L=9



$a_k \in [0.20, 1.80]$ code L=9



$a_k \in [0, 2]$ code L=9



Incoherent target (less stationary)

- Measurements are lagged products of measured receiver voltage
- Lagged product transmissions are usually groups of codes, which we will index as $\epsilon^c(t)$
- Under certain assumptions, lagged product measurements can be stated as a convolution equation involving target ACF and lagged product envelope:

$$m(t)\overline{m(t+\tau)} \equiv m_\tau(t) \quad (13)$$

$$\epsilon^c(t)\overline{\epsilon^c(t+\tau)} \equiv \epsilon^c_\tau(t) \quad (14)$$

$$m_\tau(t) = (\epsilon^c_\tau * \sigma_\tau)(t) + \xi_\tau(t) \quad (15)$$

Noise term

The normalized measurement "noise power" of a certain lag is:

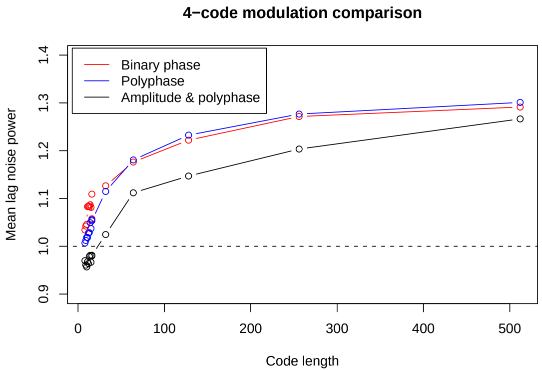
$$\mathcal{F}_D^M\{\epsilon_\tau^c(t)\} = \hat{\epsilon}_\tau^c(\omega) \quad (16)$$

$$P_\tau \approx \int_0^{2\pi} \frac{N_c(N_b - \tau)}{\sum_{c=1}^{N_c} |\hat{\epsilon}_\tau^c(\omega)|^2} d\omega \quad (17)$$

For alternating codes $P_\tau = 1$ for all τ. But when amplitude modulation is used, this is not the lower limit, because in some cases, more radar power can be used on certain lags, even though the average transmission power is the same.

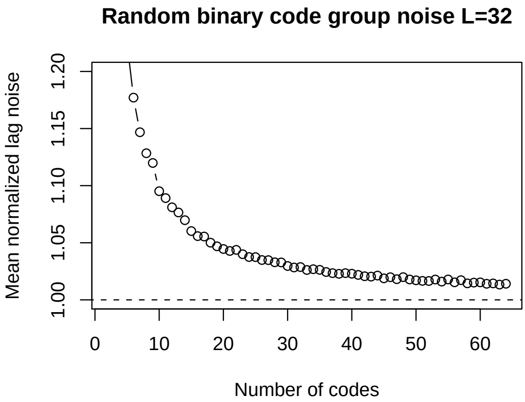
Code length

- Polyphase code groups have better theorethical properties than binary phase code groups
- Amplitude modulation improves them further
- As code length is increased, the modulation becomes less important



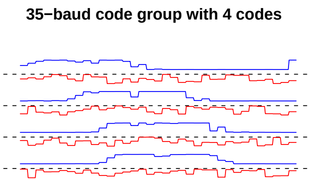
Code group length

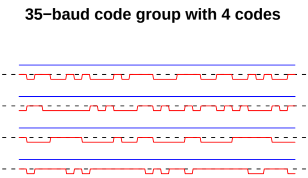
- As code group size is increased, the noise performance gets better

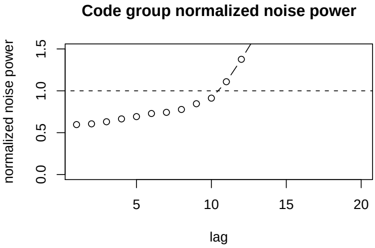


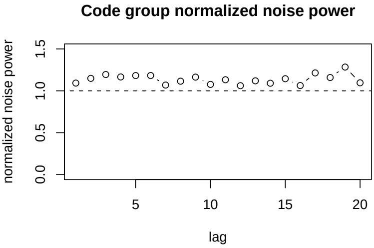
Amplitude and phase modulated code group

- Total power of one pulse is set to be constant (in our considerations similar to constant amplitude pulse of same length)
- Amplitude modulation makes it possible to use more radar power on important lags
- A subset of lags can be extracted with lower noise power than with alternating codes

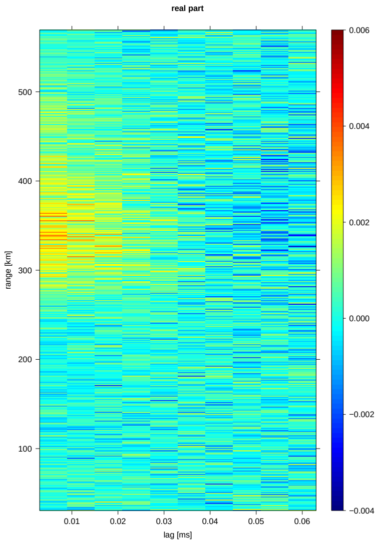

35−baud code group with 4 codes


35−baud code group with 4 codes

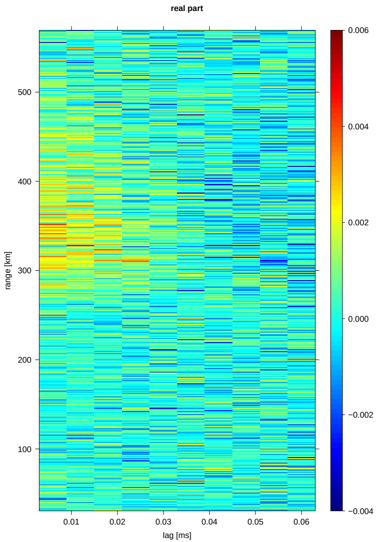

Code group normalized noise power
normalized noise power
0.0
0.5
1.0
1.5
5
10
15
20
lag


Code group normalized noise power
normalized noise power
0.0
0.5
1.0
1.5
5
10
15
20
lag

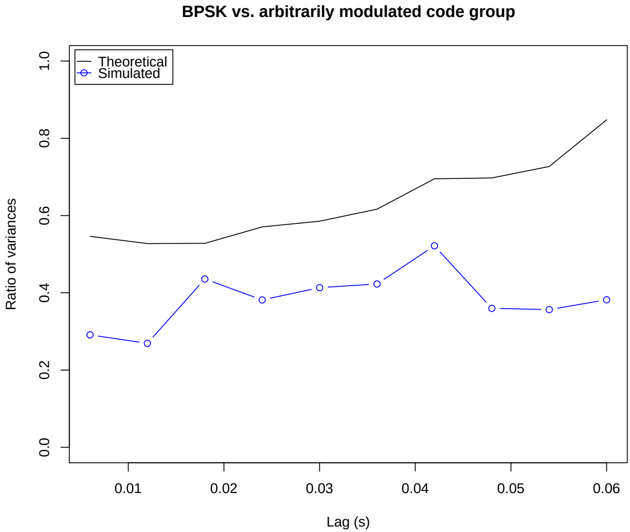
AMPP


real part
range [km]
500
400
300
200
100
0.01
0.02
0.03
0.04
0.05
0.06
lag [ms]
0.006
0.004
0.002
0.000
−0.002
−0.004

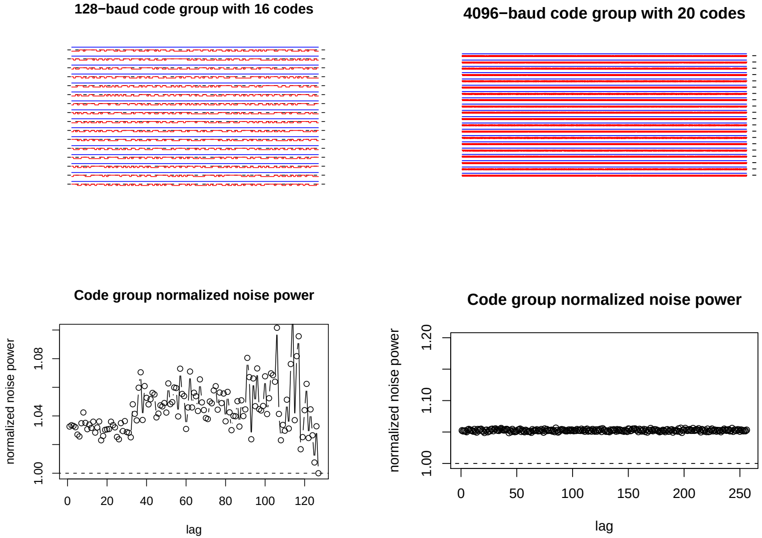
Binary


real part
range [km]
500
400
300
200
100
0.01
0.02
0.03
0.04
0.05
0.06
lag [ms]
0.006
0.004
0.002
0.000
−0.002
−0.004

Simulation results...



Even higher resolution...



Conclusions

- ISR can benefit in many ways from amplitude and polyphase modulation
- Polyphase coding allows shorter code groups while maintaining a thermal noise figure close to theoretical minimum
- Amplitude modulation makes it possible to focus transmission power on important lags, thus making possible better modulations than alternating codes (for a subset of lags)

Conclusions for incoherent targets

- Amplitude and phase modulation allow better measurements than constant amplitude binary phase coded measurements of same transmission power
- We would like try to these ideas in reality, who would want to collaborate?

Questions?