10th Inverse Days, Sodankylä Geophysical Observatory
University of Oulu
15th–17th December 2004
Organized by the Finnish Inverse Problems Society

Local organizing committee, Sodankylä Geophysical Observatory

prof. Markku Lehtinen / Mr. Jyrki Manninen / Mr Tero Raita / Mr. Johannes Kultima / Ms. Nina Riipi / Ms. Anna-Liisa Piippo

List of registered participants

Auvinen, Harri, M.Sc. University of Helsinki Finland
Calvetti, Daniela, Dr. Case Institut USA
Dahl, Matias, Dr. Helsinki University of Technology Finland
Damtie, Baylie, Dr. Bahir Dar University Ethiopia
Enell, Carl-Fredrik, Dr., Sodankylä Geophysical Observatory, Finland
Francini, Elisa, Dr. Consiglio Nazionale delle Ricerche Italy
Haario, Heikki, Prof. Tech. Univ. of Lapeenranta Finland
Harju, Markus, M.Sc. University of Oulu Finland
Heikkinen, Lasse, Mr University of Kuopio Finland
Heino, Jenni, M.Sc. Helsinki University of Technology Finland
Helin, Tapio, Grad. student Helsinki University of Technology Finland
Huuskonen, Asko, Dr Finnish Meteoroligical Institute Finland
Hyvönen, Nuutti, D.Sc. (Tech.) University of Helsinki Finland
Kaasalainen, Mikko, Dr. University of Helsinki Finland
Kaipio, Jari, Prof University of Kuopio Finland
Karttunen, Kyösti, Dr. University of Oulu Finland
Khudukon, Boris, Dr. Polar Geophysical Institute Russia
Kiili, Hanna, Dr. University of Oulu Finland
Kolehmainen, Ville, Dr. University of Kuopio Finland
Kress, Rainer, Professor University of Göttingen Germany
Lampinen Jouko, Helsinki University of Technology, Finland
Lasanen, Sari, Dr. University of Oulu Finland
Lassas, Matti, Prof. Helsinki University of Technology Finland
Lehikoinen, Anssi, Mr University of Kuopio Finland
Lehtinen, Markku, Professor Sodankylä Geophysical Observatory Finland
Markkanen, Markku, Dr. Eigenor oy Finland
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Nickel, Julia, Dr. University of Osnabrück Germany
Nygrén Tuomo, Prof. University of Oulu Finland
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Osipov, Andrey, Dr. Scientific-Research Institute for System Studies, RAS Russia
Peltonen, Kirsi, Dr. Helsinki University of Technology Finland
Perkkiö, Juha-Matti, Dr. University of Helsinki Finland
Piiroinen, Petteri, Dr. University of Helsinki Finland

Pikkarainen, Hanna Katariina, Dr. Helsinki University of Technology Finland
Pryse, S Eleri, Dr University of Wales Aborystwyth U.K.
Pursiainen, Sampsa, Dr. Helsinki University of Technology Finland
Päivärinta, Lassi, Prof. University of Helsinki Finland
Rantala, Maaria, Dr. GE Healthcare Finland
Roininen, Lassi, Dr Kavaro Oy Finland
Salo, Mikko, Dr., University of Helsinki, Finland
Serov, Valery, Dr. University of Oulu Finland
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Sofieva, Viktoria, Dr. Finnish Meteorological Institute Finland
Somersalo, Erkki, Prof. Helsinki University of Technology Finland
Tamminen, Johanna, Dr. Finnish Meteorological Institute Finland
Tereshchenko, Evgeny D., Prof. Polar Geophysical Institute Russia
Tietäväinen, Juha-Pekka, Grad. Student Helsinki University of Technology Finland
Toivonen, Riitta, University of Helsinki Finland
Trottier, Philippe, Dr. Eigenor Oy Finland
Vallinkoski, Matti, Dr. Micropolis Oy, Ii, Finland
Vänskä, Simopekka, Dr. University of Helsinki Finland
Zacharopoulos, Athanasios, University College london, UK

Programme

12:00–13:00 Lunch at SGO canteen
13:00–13:30 registration near to SGO main lecture hall

Session 1, Wednesday 13:30–16:00 (all talks 25 min + 5 min pause)

1) Conference opening, followed by Markku Lehtinen: Juha Pirttilä in memoriam

The Sodankylä inverse problems group has implemented statistical inversion based new signal analysis and measurement methods in many different practical applications. These include new methods for ionosphere radars, applications of these methods in weather radars and sonars as well as tomography in scientific, industrial and medical applications. Juha Pirttilä was usually the inital worker in implementing these ideas in each new field of applications, laying foundation for work then continued by other scientists in the group. His completely unexpected death was a heavy blow to our group and to applied inversion science in general. In this talk I show some of the fantastic representation material left by him. This talk works well as a general introduction to all the work the Sodankylä group has done during it's existence.

2) Baylie Damtie, Bahir Dar University
Incoherent Scatter Radar data analysis by means of inversion

In the incoherent scatter radar measurements, one uses the signal ACF in order to estimate the physical parameters of the ionosphere, such as electron density, electron and ion temperatures, ion composition, collision frequency and line-of-sight plasma velocity. This is done by fitting the theoretical plasma ACF to the measured ACF. In some cases the sampled signals, which are used to calculate the measured ACF, contain contributions from several range gates. These kinds of measurements create an unwanted ambiguity problem and hence measurements with such kinds of range ambiguities are usually considered useless. Here we demonstrate how one can infer non-ambiguous data from this kind of ambiguous measurements by employing linear inversion.

3) S Eleri Pryse, University of Wales, Aberystwyth
Imaging near-Earth space by radio tomography

The application of radio tomography to image the ionised atmosphere was first proposed in the mid-1980s. Since then the idea has developed into an operational experimental technique used for scientific study of near-Earth space. Developments have been made in both the inversion methods for this limited-geometry application and in the experimental verification and exploitation of the technique. The method provides a relatively inexpensive means of routinely monitoring the large-scale features of the ionised atmosphere of interest in scientific research and also for radio applications where the density structures can lead to degradation of the signals.

The University of Wales Aberystwyth monitors phase coherent radio signals from the polar orbiting satellites of the Navy Ionospheric Monitoring System (NIMS) at a chain of ground receivers in the high Arctic that are spaced in latitude but aligned in longitude. This provides line integral measurement along intersecting satellite-to-receiver ray paths, with inversion of the data yielding images of the spatial distribution of plasma density in the ionised atmosphere. The observations have roles in studies of signatures of Space Weather processes, in validation and development of ionospheric models, and for mapping and characterising large-scale density features of interest to radio applications. Recent developments have seen the combination of observations from tomography chains in different longitude sectors to investigate the distribution of plasma over extended areas of the high-latitude sector. This multi-chain configuration has a potential role in the forthcoming International Polar Year in studies of the plasma processes influencing the high-latitude ionosphere.

4) Kirsi Peltonen, Helsinki University of Technology
Finsler geometry through Hilbert's Fourth problem

This is a survey lecture on Finsler geometry for general audience. Hilbert's Fourth Problem entitled *Problem of straight line as the shortest distance between two points* provides a famous open inverse problem and natural approach to look at manifolds modelled with a norm (Finsler setting) instead of an inner product (Riemannian setting). We will compare basic geometric objects like geodesics and Riemann curvature in different settings. We discuss on some known solutions to Hilbert?s Fourth problem.

5) Johanna Tamminen, Finnish Meteorological Institute
Adaptive Markov chain Monte Carlo algorithms for GOMOS inverse problems

We discuss the inversion of gas profiles (ozone, NO3, NO2, aerosols, neutral density) in the upper atmosphere from the GOMOS measurements onboard the Envisat satellite. The instrument measures the attenuation of stellar light spectra at various horizontal paths from about 100 km down to 10 km. A direct inversion leads to a nonlinear model with hundreds of parameters to be estimated.

This problem can be solved with an adaptive single step MCMC algorithm called SCAM. Another approach is to divide the problem in several nonlinear smaller dimensional problems, run parallel adaptive MCMC chains for them, and solve the gas profiles in repetitive linear steps.

It is shown that compared to the operational inversion some advantages and flexibility can be achieved by using the MCMC technique. We demonstrate how the MCMC technique can be used in the validation of the fast operational inversion algorithms. The adaptive algorithms that are used makes the implementation of the MCMC technique effortless. Some highlights of the first GOMOS measurements will also be shown.

16:00–16:45 FIPS business meeting
16:45 Bus to hotel
17:00–18:45 CoE planning meeting (FIPS board members only)
19:00–21:00 Sodankylä town reception at City Hall (walking distance from hotel)

Session 2, Thursday 10:00–12:00 (9:30 Bus transport from Hotel to SGO)

6) Rainer Kress, Institut für Numerische und Angewandte Mathematik Universität Göttingen
Umpteen ways for solving inverse boundary value problems for the Laplace equation

We present a survey on some methods for solving inverse boundary value problems for harmonic functions arising in electrostatic imaging. Special emphasis is given to a method based on conformal mapping techniques and another method based on the reciprocity gap and nonlinear integral equations.

7) Simopekka Vänskä, University of Helsinki
Beltrami fields and scattering

We continue the discussion of Beltarmi fields scattering that was started last year at Iso-Syöte. A Beltrami field is a solution of

$$\nabla \times u = ku,$$

where $k > 0$ is constant (right handed and linear). If u is a Beltrami field, then u is also a solution for the Helmholtz equation.

In the talk, we first outline the proof for the solvability of the direct obscatle scattering problem where the normal component of the total field vanishes on the boundary of the scatterer. Then we consider the inverse obstacle scattering problem.

The direct problem is solved "traditionally". Obtain the uniqueness with the help of the Rellich's lemma. The existence follows from Fredholm theory by making a suitable ansatz.

For the inverse problem, we show that the method of singular sources can be applied. The method of singular sources follows quite carefully the idea of the uniqueness proof for the inverse scattering problem. We also discuss, what kind of procedure one should use in choosing the approximation domain.

8) V.S. Serov, Department of Mathematical Sciences, University of Oulu
Weierstrass' solutions to certain nonlinear equations

A method is presented for finding a subset of the exact (mainly travelling-wave) solutions of various nonlinear wave and evolution equations (NLWEE). By using an appropriate ansatz the NLWEE is transformed into nonlinear ordinary differential equation. The solutions of this equation can be expressed in terms of Weierstrass' elliptic functions and include (real, bounded) periodic and solitary-wave-like solutions.

Solitary-wave-like solutions are obtained by a specific choice of the parameters of the NLWEE. This choice can be compactly represented by the discriminant and the invariants of Weierstrass' functions, leading to algebraic conditions for solitary wave solutions of the different NLWEE.

This method is applied to the nonlinear cubic-anticubic-quintic Schrödinger equation (NLCAC-QSE), the generalized compound Korteweg-de Vries equation (gcKdVE), the generalized modified Kadomtsev-Petviashvili (gmKPE), the sine-Gordon equation and the double sine-Gordon equation, the Ginzburg-Landau equation and many others. Some results for further nonlinear wave and evolution equations are listed.

Some applications to the TE-polarized waves quided by Kerr-nonlinear three-layer structure as well as the problem of reflection and transmission of a plane TE-wave at Kerr-nonlinear three-layer structure are considered.

9) Julia Nickel, University of Osnabrück
Elliptic and (linear) superposition solutions to certain nonlinear wave and evolution equations

A method is presented to enlarge a solution set of various nonlinear wave and evolution equations (NLWEEs) by linear superposition.

If a periodic solution ψ of a NLWEE can be expressed as a sum of Jacobian elliptic functions, this solution can be used as start solution for linear superposition. The basis of this superposition procedure is given by some remarkable identities involving Jacobian elliptic functions. The solution $\widetilde{\Psi}$ obtained by linear superposition may also satisfy the NLWEE.

Our approach is based on solutions that can expressed compactly in terms of Weierstrass' elliptic function $\wp$ (cf. talk of Prof. Serov) and include (real, bounded) periodic and solitary-wave-like solutions ("elliptic solutions"). The algebraic conditions for both cases are given in terms of the parameters of the NLWEE and the parameters of the (traveling-wave) transformation. We develop conditions under which these solutions are suitable for linear superposition and show general solutions for special cases. As an example the method is applied to the nonlinear cubic-quintic Schrödinger equation (NLCQSE).

12:00–13:00 Lunch at SGO canteen

Session 3, Thursday 13:30 – 15:30

10) Daniela Calvetti (Case Western Reserve University) and Erkki Somersalo
Statistical compensation of boundary effects in deconvolution

We consider the deconvolution problem of restoring an image from a blurred and noisy version of it. Being an integral operator, the convolution is non-local. Consequently, the pixels near the boundary of the blurred image contain also information of the scenery outside the image area. This phenomenon and its effect on the image restoration is known as the boundary effect. We discuss computational methods, based on Bayesian modelling, for compensating the boundary effects in deconvolution

11) Erkki Somersalo (Helsinki University of Technology), Daniela Calvetti and Fiorella Sgallari
Image inpainting and bootstrap priors

Image inpainting refers in general to image processing techniques of restoring a damaged image by removing holes, scraches or other defects. The proposed inpainting algorithm is based on the construction of a pilot image that is easy to inpaint and a structural prior, or a bootspsrap prior, that facilitates the inpainting of the original image.

12) Elisa Francini, Istituto per le Applicazioni del Calcolo, Firenze
Reconstruction of thin conductivity imperfections

We consider the case of a uniform plane conductor containing multiple inhomogeneities that can be represented as neighborhoods of segments. We address the problem of determining the segments that describe the inhomogeneities from boundary measurements of the steady state voltage potential. We show that the segments (under natural assumptions) depend continuously from one single boundary measurement and obtain Lipschitz stability estimates. This is a joint work with Habib Ammari and Elena Beretta.

13) Athanasios Zacharopoulos, J. Sikora and S. Arridge, University College London
Reconstruction of 3D region boundaries in Optical Tomography using Parametric Surfaces and BEM

A parametric description of closed volumetric surfaces is described and exploited in a shape reconstruction algorithm for Optical Diffusion Tomography. Parts of the human anatomy with interest in the Optical Tomography research of the brain are initially parameterised using a mapping algorithm to a unit sphere and then meshed using regular meshes defined on the sphere. Then a Boundary Element numerical solution of the diffusion equation for the propagation of light in media with homogeneous optical properties is constructed. Finally, the numerical forward solution is utilised in an shape parameter's optimisation Inverse Problem for the reconstruction of region boundaries for simulated non invasive light measurements.

15:30–16:00 Coffee

Session 4, Thursday 16:00–18:00

14) Jouko Lampinen, Helsinki University of Technology
Bayesian aspects in inverse problems - model validation and hierarchical models

Bayesian inference yields the posterior probabilities of target variables, given the data and prior assumptions. Those probabilities are only indirectly related to the ground truth, i.e., the actual values of the target variables in the system producing the data, as the probabilities are conditioned on the prior assumptions, and the results allow only relative comparison of the probabilities of the alternatives included in the model. We consider various techniques for validating the results (including the likelihood, or forward model, and the priors) and discuss the applicability of the techniques in inverse problems.

A characteristic feature of Bayesian modelling is hierarchical prior structures. The posterior of the target variables is then computed by integrating over the posterior of the hyperparameters. This allows, for example, treatment of the smoothing parameters in Bayesian manner. We discuss the hierarchical modelling in MEG inverse

15) Hanna Pikkarainen, Helsinki University of Technology
A Mathematical Model for Electrical Impedance Process Tomography

We consider the process tomography problem of imaging the concentration distribution of a given substance in a fluid moving in a pipeline based on electromagnetic measurements on the surface of the pipe. We view the problem as a state estimation problem. The concentration distribution is treated as a stochastic process satisfying a stochastic differential equation referred to as the state evolution equation. The measurements are described in terms of an observation equation containing the measurement noise. The main interest is in the mathematical formulation of the state evolution and observation equations and presenting a discretized state estimation system in which the discretization error is taken into account. The time evolution is modeled by the stochastic convection-diffusion equation. The measurement situation is represented by the most realistic model for electrical impedance tomography (EIT), the complete electrode model.

16) V. F. Sofieva, J. Tamminen, E. Kyrölä – FMI, H. Haario – UoH M. Lehtinen – SGO
Profile smoothness as a priori information in the inversion of limb measurements

In this work we discuss inclusion of a priori information about smoothness of atmospheric profiles in inversion algorithms. The smoothness requirement can be formulated in the form of Tikhonov-type regularization, where the smoothness of atmospheric profiles is considered as a constraint or in the form of Bayesian optimal estimation (maximum a posteriori method, MAP), where the smoothness of profiles can be included as a priori information. We develop further two recently proposed retrieval methods. One of them - Tikhonov-type regularization according to the target resolution - develops the classical Tikhonov regularization. The second method - maximum a posteriori method with smoothness a priori - effectively combines the ideas of classical MAP method and Tikhonov-type regularization. We propose a grid-independent formulation for the proposed inversion methods, thus isolating the choice of calculation grid from the question how strong the smoothing should be.

The discussed approaches are applied to the problem of ozone profile retrieval from stellar occultation measurements by the GOMOS instrument on board the Envisat satellite. The a priori information was created from 10-years analysis of ozone sounding at Sodankylä. The technique for creating a priori information based on correlation analysis is developed. Realistic simulations for the typical measurement conditions and analysis of total retrieval error illustrate the advantages of the proposed inversion methods.

The proposed methods are equally applicable to other profile retrieval problems from remote sensing measurements.

17) Andrey Osipov, Scientific-Research Institute for System Studies, Moscow
On some properties of infinite-dimensional elliptic coordinates

A number of the equations in classical mechanics is solvable by separation of variables. In particular, some Hamiltonian systems in the finite-dimensional case are integrable in elliptic Jacobi coordinates. In recent years a generalization of elliptic coordinates to the infinite case has been offered. We consider this generalization and indicate the connection of infinite-dimensional elliptic coordinates with the inverse problem for the regular Sturm-Liouville operator.

18:15 Bus to hotel
19:00 Bus from hotel to conference dinner at 'Kommattilampi'. Possibility to have a sauna and a bath through a hole in the ice of the lake. Also possible to sit outside by fire. Warm clothing is recommended.

Session 5, Friday, 10:00–12:00 (9:30 Bus transport from hotel to SGO)

18) Mikko Salo, University of Helsinki
Inverse problems in computer vision

I will discuss some inverse problems which arise in computer vision applications. The purpose in the applications is to recognize objects from digital images which are taken from different view angles.

The focus will be on global invariant methods, where one tries to compute features invariant to certain geometric transformations by using the whole image instead of some segmented information. Particularly I will consider the recently introduced Multi-Scale Autoconvolution (MSA) transform, which is a promising new technique for affine invariant recognition.

19) Markku Markkanen, Eigenor Oy, Sodankylä
Gibbs sampling with total variation priori

In Markov chain Monte Carlo (MCMC) methods one generates samples from posteriori distribution and calculates posteriori expectation and covariance from those samples. The most common MCMC methods are Metropolis-Hastings algorithm, which generates one new sample on time and Gibbs sampling, which generates new sample coordinate by coordinate from one dimensional conditional distributions. The drawback of the simplicity of Gibbs sampling is that it is generally very slow when the number of coordinates is large.

In the special case when one uses Total Variation priori (or some other related prioris) as the priori distribution, it is possible to do the Gibbs sampling in an efficient way. The efficiency of the method makes it possible to use Gibbs sampling in tomographic problems having up to a few million unknowns.

20) Asko Huuskonen, Finnish Meteorological Institute, Helsinki
Solution of the Range-Doppler dilemma of weather radars by the SMPRF codes based on statistical inversion

Based on the measurement principles used on Incoherent Scatter Radars, the authors have developed the SMPRF pulse code which solves *the range-Doppler dilemma* and which can be used with modern magnetron radars. The working principle of the code is explained in mathematical terms and with the help of a simplified model. Results from nearly simultaneous SMPRF and traditional fixed PRF weather radar measurements are compared and the reasons for the differences explained. The practical results show that the SMPRF code indeed works in the manner predicted by the theoretical and model calculations.

The SMPRF-code provides enough information to produce a high-resolution measured spectrum for each range gate. The shape of these measured spectra are seldom purely Gaussian. It is possible, that more advanced raw products than just reflectivity, velocity and width can be produced with the help of these high-resolution spectra.

21) Philippe Trottier, Eigenor Oy, Sodankylä
How does processor, compilers and system design react to big problems.

”Big problems that don’t fit in memory are often treated and programmed like normal small problems, since compilers don’t always see the whole picture it is important to know how to help them, understanding the processors and knowing their limits might save time and energy in a final product.”

12:00–12:45 Lunch at SGO canteen

Session 6, Friday 13:00-16:00

22) Lassi Päivärinta, Rolf Nevanlinna Institute, Helsinki
Boundary integral equation for bounded measurable conductivities in the plane.

23) Matti Lassas, Helsinki University of Technology
Inverse conductivity problem with an imperfectly known boundary

We show how to eliminate the error caused by an incorrectly modeled boundary in electrical impedance tomography (EIT). In practical measurements, one usually lacks the exact knowledge of the boundary. Because of this the numerical reconstruction from the measured EIT data is done using a model domain that represents the best guess for the true domain.

However, it has been noticed that the inaccurate model of the boundary causes severe errors for the reconstructions. We introduce a new algorithm to find a deformed image of the original isotropic conductivity based on the theory of Teichmuller spaces and implement it numerically. The results are made in collaboration with Ville Kolehmainen (University of Kuopio) and Petri Ola (Rolf Nevanlinna Institute, University of Helsinki)

14:00–14:30 Coffee

24) Sari Lasanen, University of Oulu
Green's priors in statistical inverse theory

In statistical inverse theory, the direct theory is often formulated in infinite-dimensional space while the Bayesian inversion is carried out in a finite-dimensional space. This raises a question, what happens when dimensionality of the solution space is increased. We study certain function-valued priors called Green's priors and show that their discretized versions satisfy conditions guaranteeing the convergence of corresponding posterior distributions.

25) Jussi Markkanen, Sodankylä Geophysical Observatory
Space debris signal modeling

The EISCAT incoherent scatter radar system in Northern Scandinavia and on Svalbard is one of the most sophisticated of its kind worldwide. The system consists of three separate radars: monostatic VHF radar, located near Tromsø, Norway and operating at 224 MHz, monostatic but two-antenna EISCAT Svalbard Radar in Longyerbyen, Svalbard, operating at 500 MHz, and tristatic EISCAT UHF radar at 928 MHz, with transmitter/receiver in Tromsø and receivers also in Kiruna, Sweden, and Sodankylä, Finland. All the transmitters operate in the MW peak power range and have duty cycles between ten and twenty percents. Following a successful feasibility study in 2000–2001, and a follow-up study in 2002–2004 aimed at boosting our data processing to real-time speed, we at EISCAT are now entering the third phase in our contract work for ESA, with the goal to start conducting routine measurements of centimetre-sized space debris (SD) with the EISCAT radars.

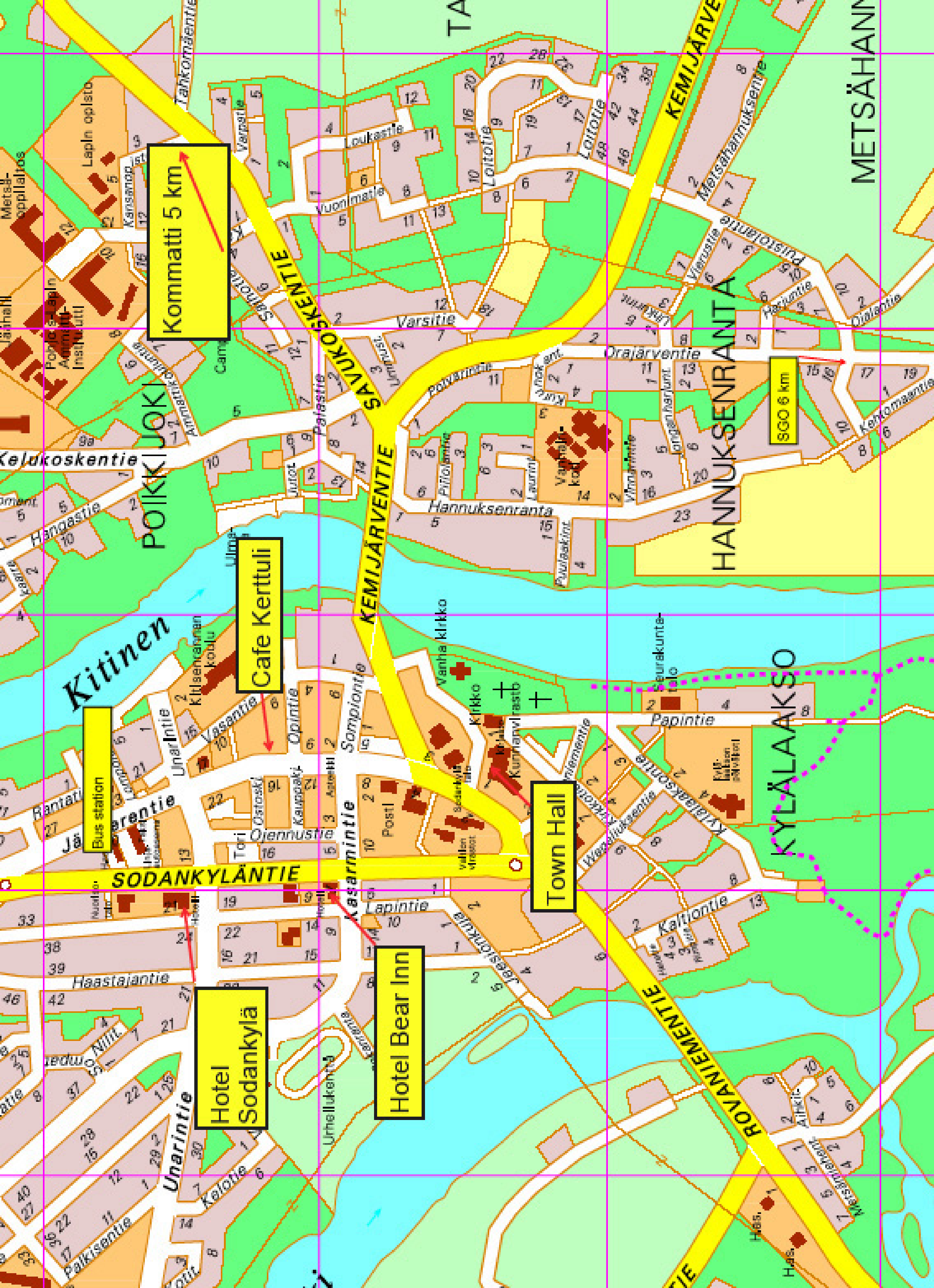
To be free to optimize the SD signal processing for small hard targets, we operate a custom digital receiver in parallel with EISCAT's standard receiver. We sample fast enough to correctly band-pass sample the EISCAT analog frequency band. To increase detection sensitivity, we use amplitude domain pulse-to-pulse coherent integration. The coherent integration is implemented by our method of target parameter estimation, which we call the Match Function (MF) method. The method is motivated by Bayesian statistical inversion, but reduces, with standard assumptions about the noise and the prior, to minimizing the least squares norm $\|z(t) - b\,\chi(R, v, a; t)\|$, where z is the measured signal (-vector) and $\{b\,\chi(t)\}$ is a set of model signals. Because the model signals depends linearly on the amplitude b, it is sufficient to maximize the magnitude of the inner product (cross-correlation) between z and χ. We define $\mathrm{MF}(R, v, a) = |\langle z, \chi(R, v, a)\rangle|/\|\chi\|$. To construct the set of model signals, we assume that the target is a point-like object which moves with constant radial acceleration across the radar beam. Then the target echo is a delayed version of the transmission, $\chi(t) = x(t')$, where t' is the time of reception of a marker transmitted at time t. We get the transmission $x(t)$ by sampling the actual EISCAT transmission, in the same way as we sample the received signal. With several approximations in the MF, due to Markku Lehtinen of SGO, it is possible to achieve efficient numerics so that we can obtain real-time detection speed with a single G5 Mac even with the fairly long data vectors involved in the coherent integration.

26) Mikko Kaasalainen, Rolf Nevanlinna Institute, Helsinki
Grand Unified Projection Operator Scheme (GUPOS)

In science we often study objects whose size is negligible compared to their distance, i.e., no resolved images can be obtained of them. The obtained data (typically intensity of electromagnetic radiation) are then called disk-integrated. However, if we can obtain such data from different sides of the object, we can try to form a complete three-dimensional model of it. This is the case in, e.g., space remote sensing of objects in our solar system.

Almost all current space remote sensing data can be represented in the form of generalized projection operators operating on the target body. The generalization concerns the integration region of the integral operator and the functional form of the operator's kernel: in addition to the simple projection of a surface patch, the kernel includes radiation scattering models and possibly other weight or convolution functions. For example, the photometric operation giving the brightness L of the body in the viewing/illumination directions (ω, ω_0) on $S^2 \times S^2$ is of the form $L(\omega, \omega_0) = \int\int_{A^+(\omega,\omega_0)} P(\omega, \omega_0)\, d\sigma$ integrated over the set $A^+(\omega, \omega_0)$ of the surface patches $d\sigma$ of the body that are both illuminated and visible. The kernel P is the product of the projected patch area and the local scattering intensity of light from direction ω_0 to direction ω. For, e.g., radar or interferometry, P has simple additional functions corresponding to additional data dimensions of L such as delay time, Doppler frequency, or coordinates of plane-of-sky projection. Thus the data sources have a common mathematical form for inverse problem solution, and this can efficiently be used in producing multidatainversion solutions that simultaneously utilize various data modes.

15:40 Transport to Sodankylä, for connection to Rovaniemi trains/planes at 1805 or 1835
16:15 Bus transport to hotel and Café Kerttuli, continues to Rovaniemi at 1830 for plane at 2035 and train at 2100



Kommatti 5 km
Cafe Kerttuli
Bus station
Town Hall
Hotel Bear Inn
Hotel Sodankylä
SGO 6 km
Kitinen
POIKKIJOKI
HANNUKSENRANTA
KYLÄLAAKSO
METSÄHANNU
SODANKYLÄNTIE
KEMIJÄRVENTIE
SAVUKOSKENTIE
ROVANIEMENTIE
Kelukoskentie
Hannuksenranta
Orajärventie
Papintie
Varsitie
Kasarmintie
Lapintie
Haastajantie
Unarintie
Jäämerentie
Ojennustie
Sompiontie
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Rantatie
Kelotie
Palkisentie
Vanha kirkko
Kirkko
Posti
Seurakuntatalo
Urheilukenttä