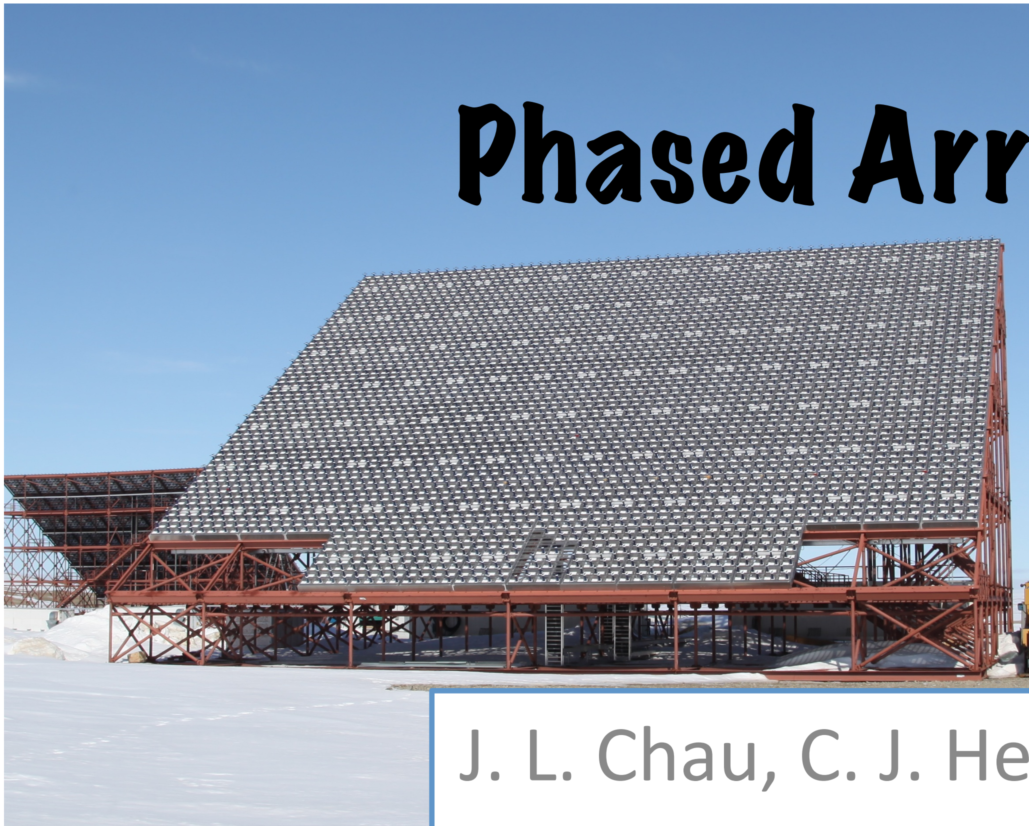
Phased Array Radars



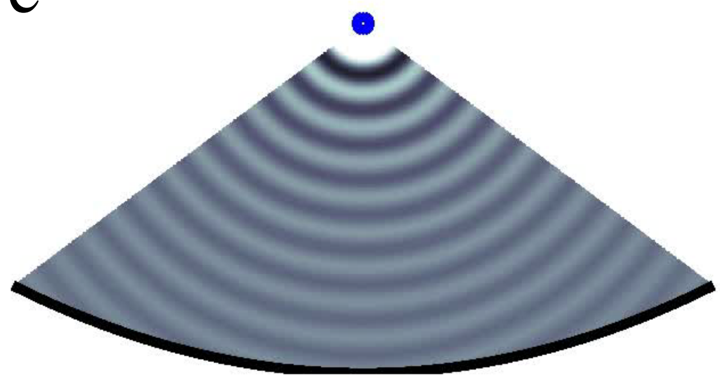
J. L. Chau, C. J. Heinselman, M. J. Nicolls

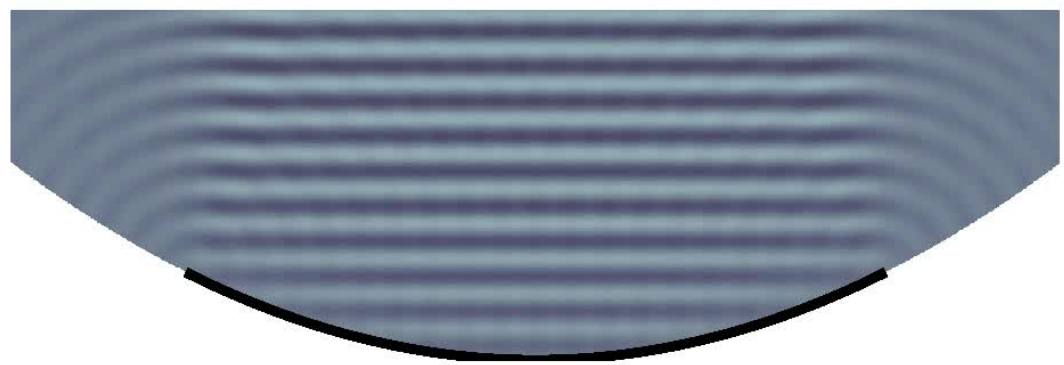
EISCAT Radar School, Sodankyla, August 29, 2012

Contents

Dish Antennas

$$E_\theta \propto \frac{1}{r} e^{j(\omega t - k_0 r)}$$





What is a Phased Array?

- A phased array is a group of antennas whose effective (summed) radiation pattern can be altered by phasing the signals of the individual elements.

- By varying the phasing of the different elements, the radiation pattern can be modified to be maximized / suppressed in given directions, within limits determined by
    - (a) the radiation pattern of the elements,
    - (b) the size of the array, and
    - (c) the configuration of the array.

Some Benefits of Phased Arrays

- Does not require moving a large structure around the sky for pointing. (Less infrastructure)
- Fast steering. (Pulse-to-pulse)
- Distributed, solid-state transmitters as opposed to single RF sources. (Less warm-up time, no need for complex feed system, elimination of single-point failures)

- These features allow for:
- Remote operations
- Graceful degradation / continual operations

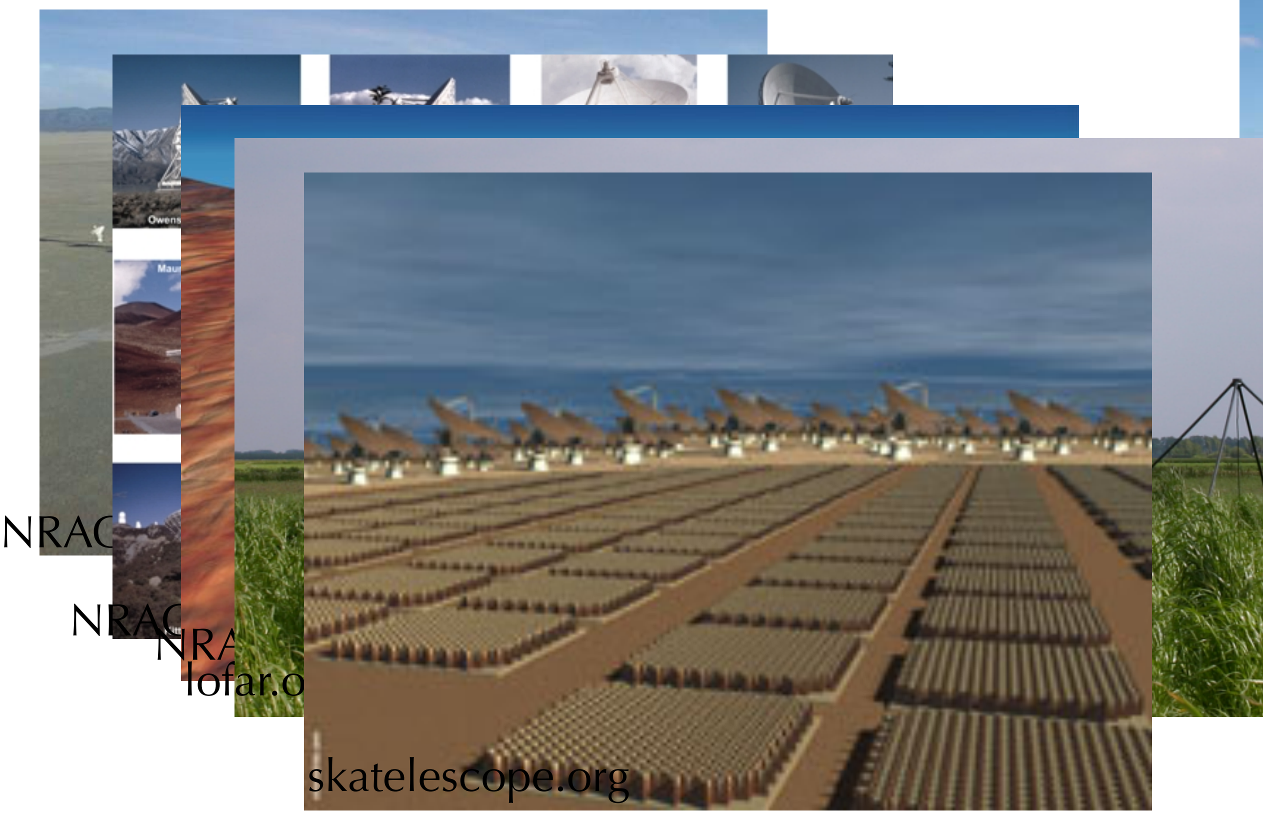
- Impact on ionospheric research:
- Elimination of some time-space ambiguities
- Ability to "zoom-in" in time
- Long durations runs (e.g., IPY)

Some Benefits of Phased Arrays (2)

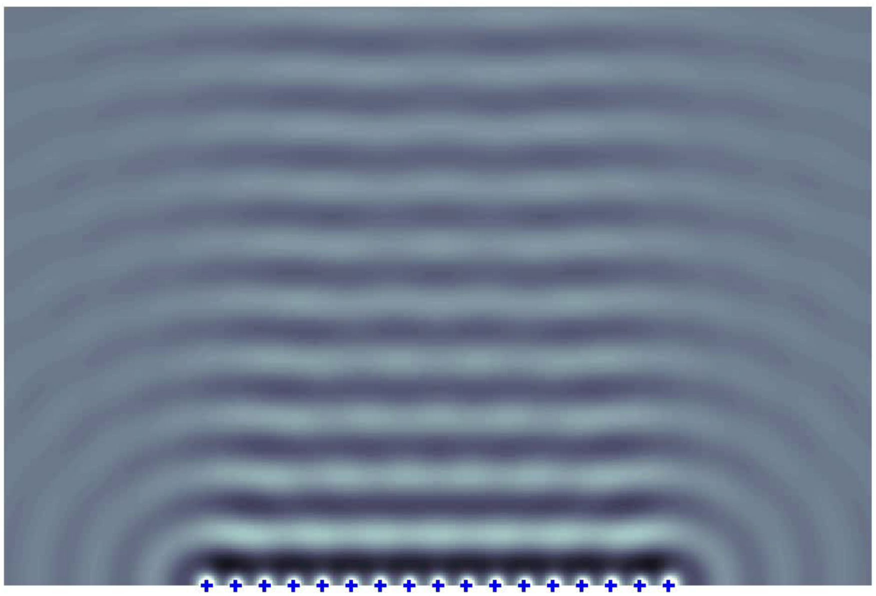
- Non-ionospheric scientific benefits
  - Radio astronomy – affordable way to achieve spatial resolutions of a few arc minutes or better
  - Aperture real-estate – directly associated with cost of system. E.g., consider a square kilometer dish versus a square kilometer array
- Non-scientific benefits
  - Conformity of a phased array to the “skin” of a vehicle/aircraft
  - Surveillance/tracking – can both survey and track 1000s of objects
  - Communication/downlink? – small satellites

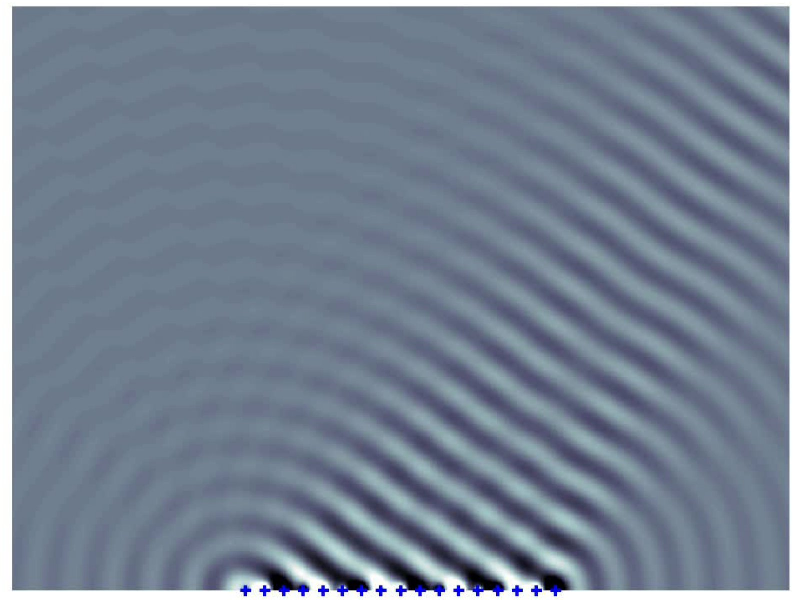
History / Technology

- Originally developed during WWII for aircraft landing
- Now used for a plethora of military applications
- Applied to radio astronomy in 1950’s



Phased Array, λ/2 spacing





Hertzian Dipole

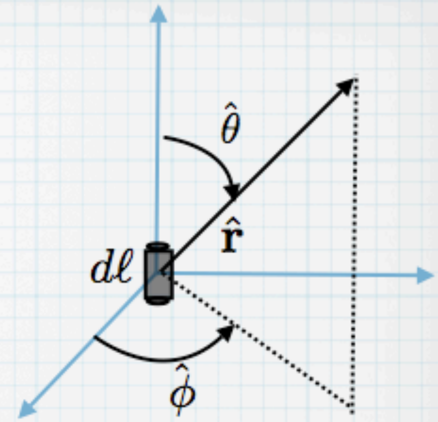
far field　　near field

$$H_\phi = I d\ell \sin\theta \frac{1}{4\pi}\left[\frac{jk_0}{r} + \frac{1}{r^2} + 0\right] e^{j(\omega t - k_0 r)}$$

$$E_r = I d\ell \cos\theta \frac{jz_0}{2\pi k_0}\left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

$$E_\theta = I d\ell \sin\theta \frac{jz_0}{4\pi k_0}\left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

Spherically expanding wavefront



$$z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$$

$$k_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \omega\sqrt{\mu_0\epsilon_0}$$

For $r \gg \lambda$, keep terms only linear in r - **far field approximation.**

$$E_\theta \perp H_\phi \perp \hat{\mathbf{r}} \qquad \frac{E_\theta}{H_\phi} = z_0 \qquad r_{\text{far-field}} \geq \frac{D^2}{\lambda}$$

Power flow represented by Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \qquad \langle P_r \rangle = \frac{1}{2}\Re\{P_r\} = \frac{1}{2}\Re\{\mathbf{E} \times \mathbf{H}\} \cdot \hat{\mathbf{r}} \quad \mathrm{W/m^2}$$

Far-field vs. Near-field: Power

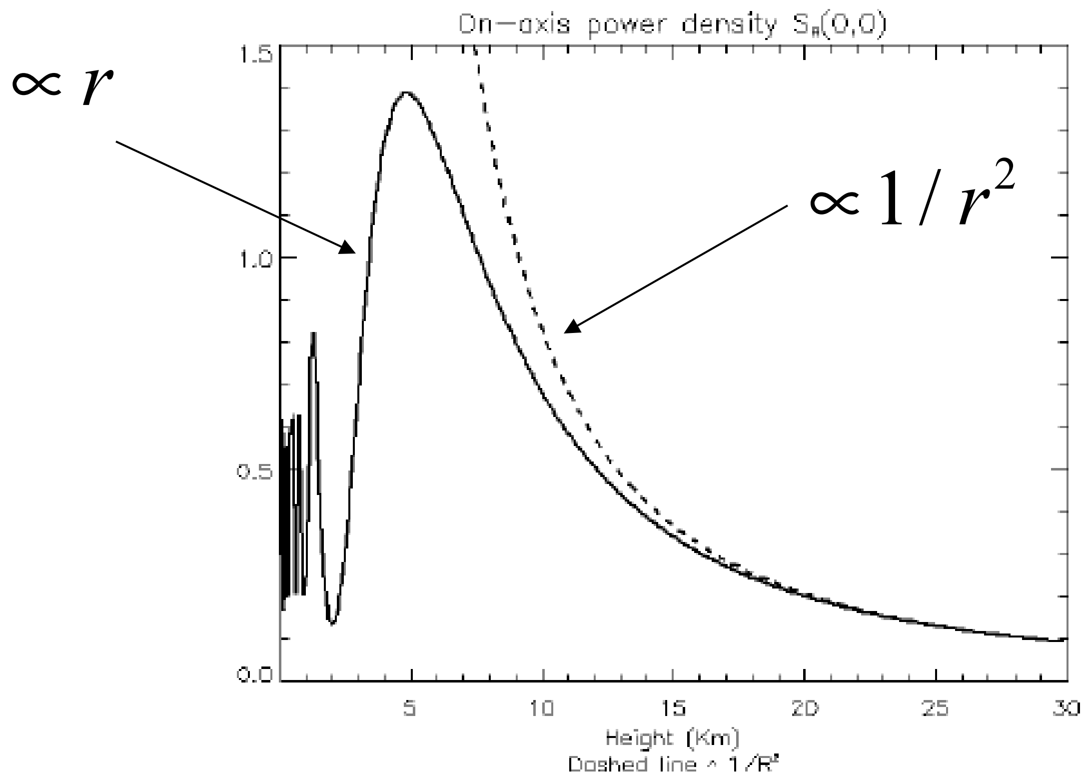


Figure B.4: Comparison of power density under the far-field approximation (dashed line) with the computed value (solid line) along the axis of the antenna beam in the vertical direction. For $z > 10$ km, the power density based on the far-field approximation differs from the actual value by 10 % or less.

Jicamarca example

Far-field vs. Near-field: Phase

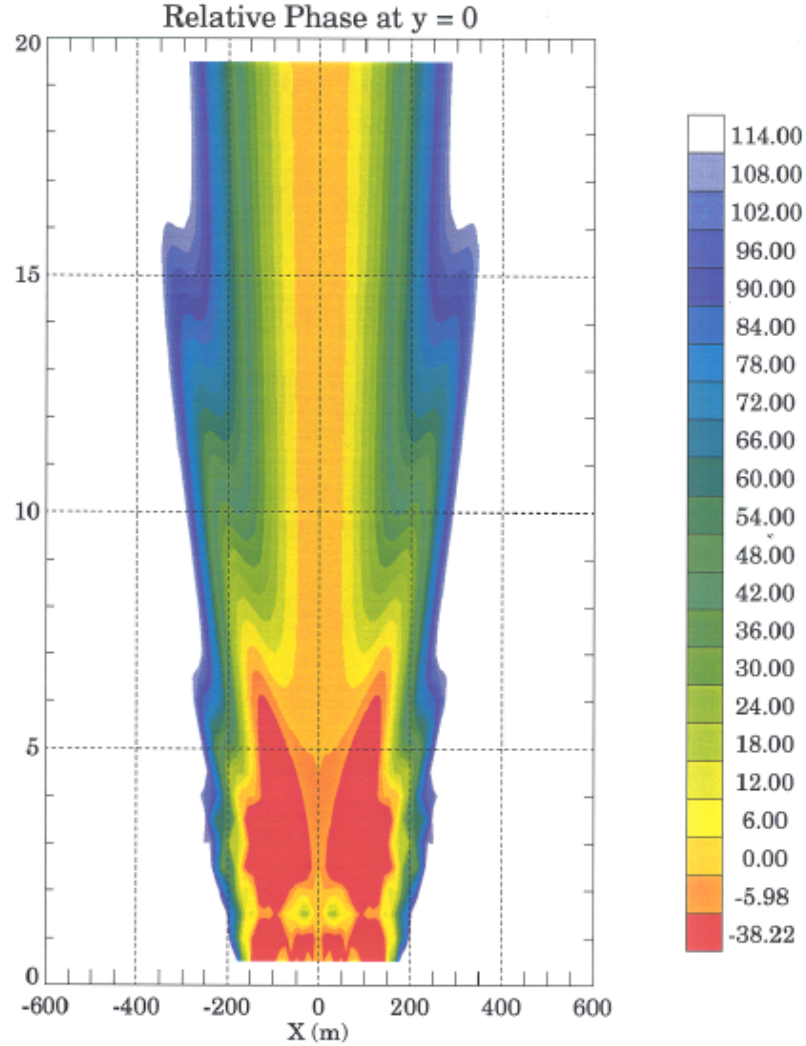


Figure B.5: Relative phase as a function of height. The smaller the value of relative phase, the similar is the wave front to a plane wave.

Near -field
Need focusing

Jicamarca example

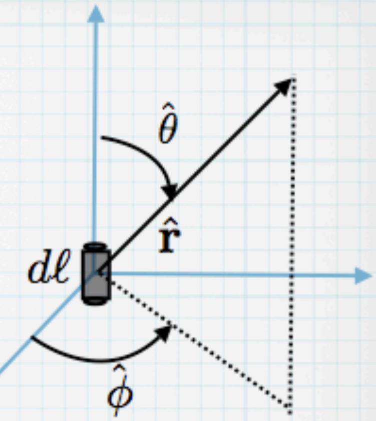
Hertzian Dipole (2)

far field — near field — Spherically expanding

$$H_\phi = I d\ell \sin\theta \frac{1}{4\pi}\left[\frac{jk_0}{r} + \frac{1}{r^2} + 0\right] e^{j(\omega t - k_0 r)}$$

$$E_r = I d\ell \cos\theta \frac{j z_0}{2\pi k_0}\left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

$$E_\theta = I d\ell \sin\theta \frac{j z_0}{4\pi k_0}\left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$



Directivity pattern:

$$D(\theta,\phi) = \frac{\text{Power Density Radiated In } (\theta,\phi) \text{ Direction}}{\text{Average Power Density}} = 4\pi R^2 \frac{\text{Power Density In } (\theta,\phi)}{\text{Total Power Radiated}}$$

$$\langle P_r \rangle = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\}\cdot\hat{\mathbf{r}} = I^2 z_0 (d\ell)^2 k_0^2 \sin^2\theta \frac{1}{32\pi^2 r^2} \ \mathrm{W/m^2}$$

$$P_{total} = \int_0^{2\pi} d\phi \int_0^{\pi} \langle P_r\rangle r^2 \sin\theta d\theta = z_0 \frac{\pi}{3}\left(\frac{I d\ell}{\lambda}\right)^2 \ \mathrm{W}$$

$$P_{total} = \frac{1}{2} I^2 R_{rad}$$

$$D(\theta,\phi) = \frac{3}{2}\sin^2\theta$$

Directivity Patterns for Dipoles

Hertzian Dipole

$$D(\theta, \phi) = \frac{3}{2}\sin^2\theta$$

$$\text{HPBW} = 90^\circ$$

Half-Wave Dipole

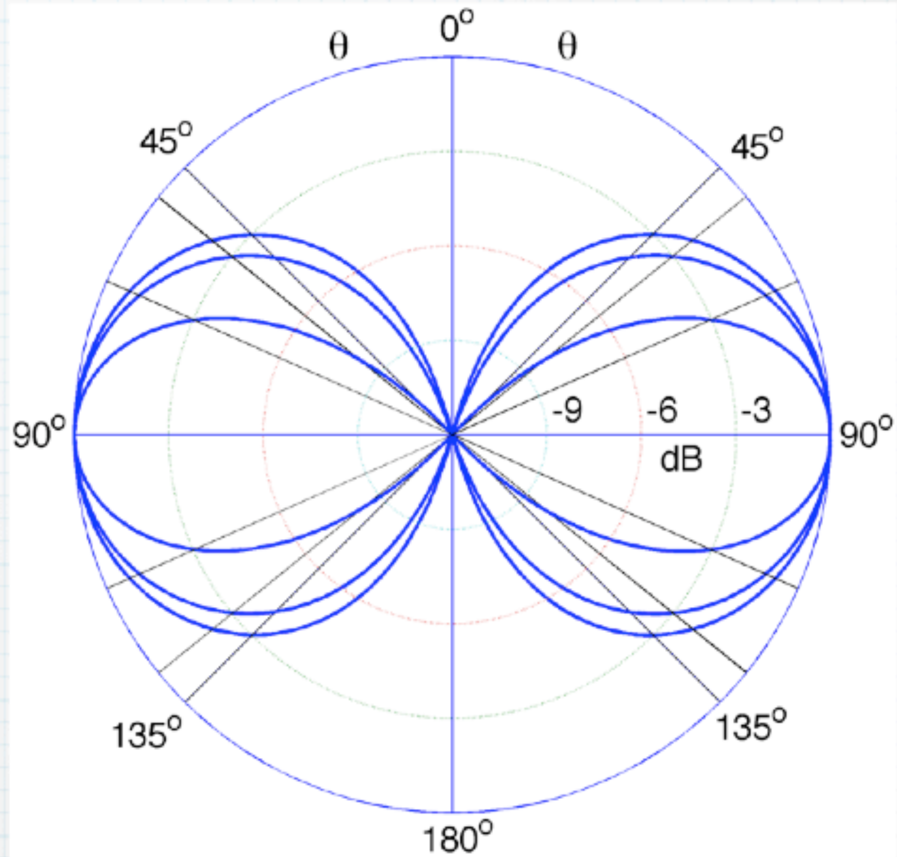
$$D(\theta, \phi) = 1.64\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

$$\text{HPBW} \approx 78^\circ$$

Full-Wave Dipole

$$D(\theta, \phi) = 2.41\left|\frac{\cos(\pi\cos\theta) - 1}{\sin\theta}\right|^2$$

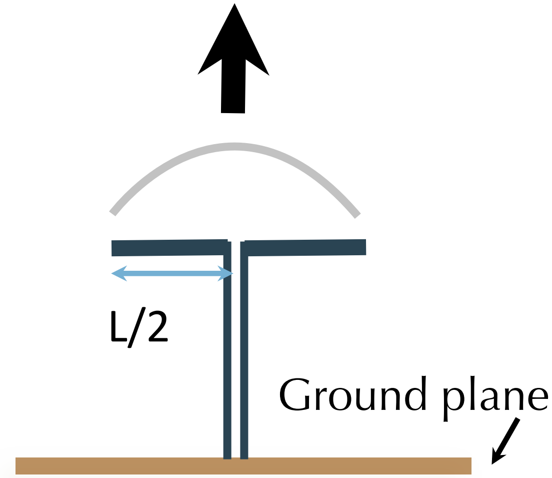
$$\text{HPBW} \approx 48^\circ$$

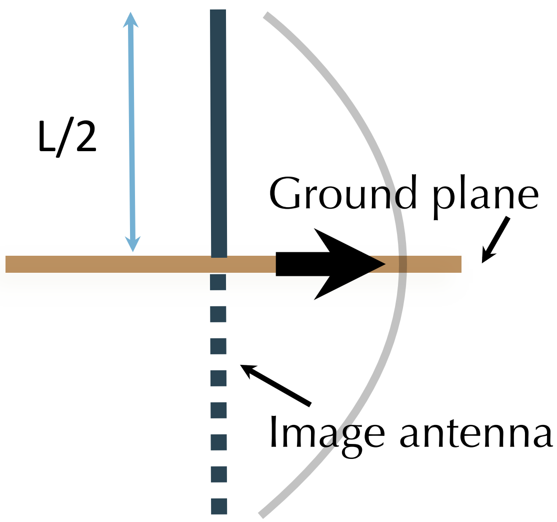


Other Elemental Antennas

Folded Dipoles

Current distribution on element is ~standing wave
- analogous to open-ended transmission line



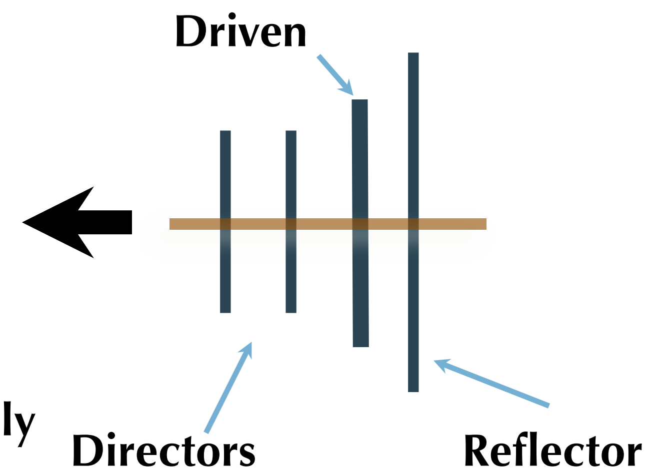


Monopole

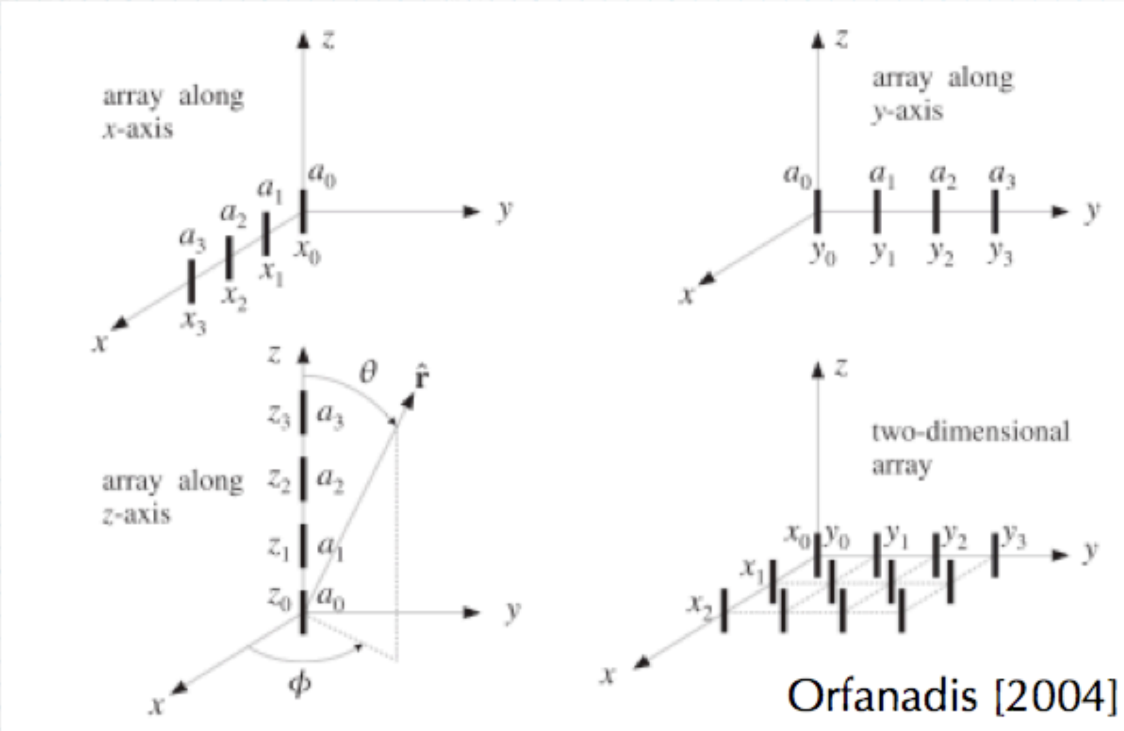
Same concept, twice the directivity
(radiation resistance halved)
E.g., AM Radio

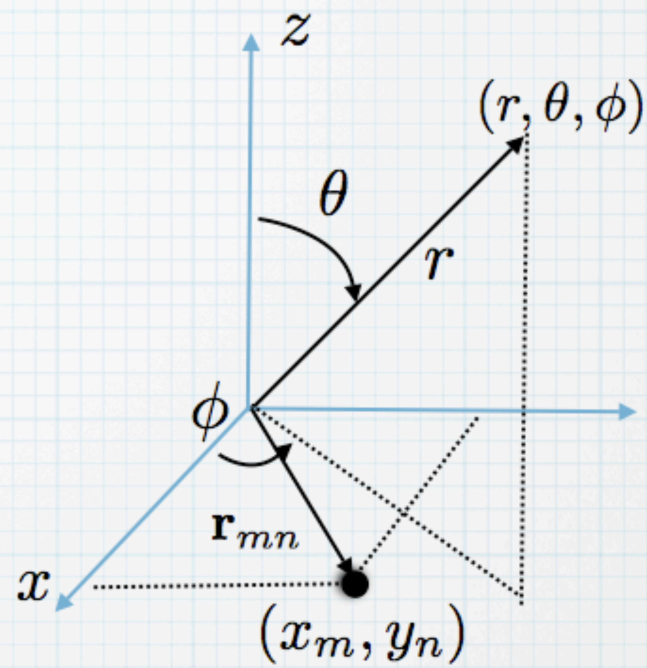
Yagi Antenna

"Parasitic" antenna (coupled elements)
Director(s) slightly shorter, reflector(s) slightly longer than driven element - higher gain
Current distributions must in general be solved for numerically



Antenna Arrays



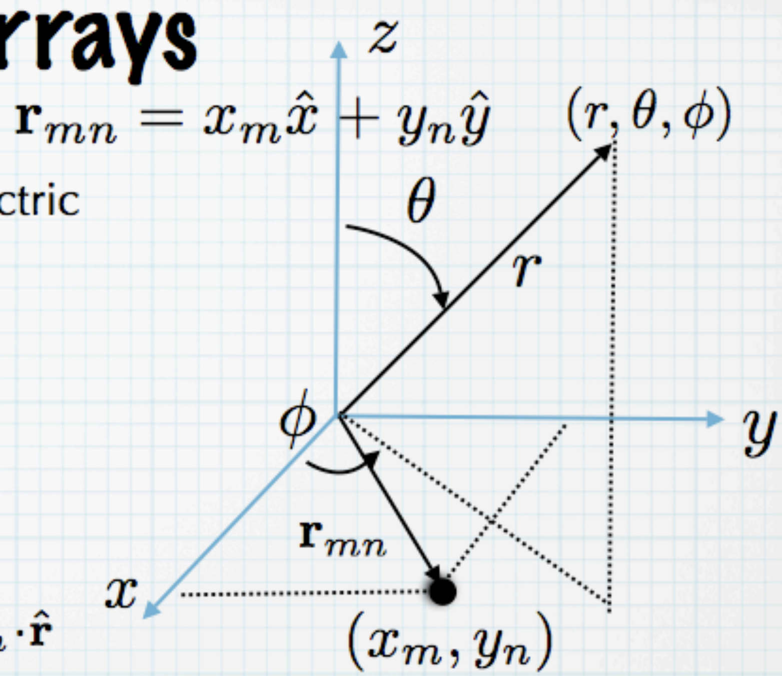


$$\mathbf{r}_{mn} = x_m\hat{x} + y_n\hat{y}$$

Assumptions:

1. Far field
   - parallel rays, 1/r amplitude dependence
2. No mutual coupling between elements (will discuss later)
3. A "reference" element radiates from the origin
4. All elements/radiators are identical, max radiation in z direction (broadside)

Antenna Arrays



Reference element at origin will produce a vector electric field at point (r, θ, ϕ)

$$\mathbf{E}_{00} = I_{00}(E_\theta\hat{\theta} + E_\phi\hat{\phi})$$

Constant

Fields due to *mn*th element is:

$$\mathbf{E}_{mn} = I_{mn}(E_\theta\hat{\theta} + E_\phi\hat{\phi})e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$
$$= I_{mn}(E_\theta\hat{\theta} + E_\phi\hat{\phi})e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)}$$

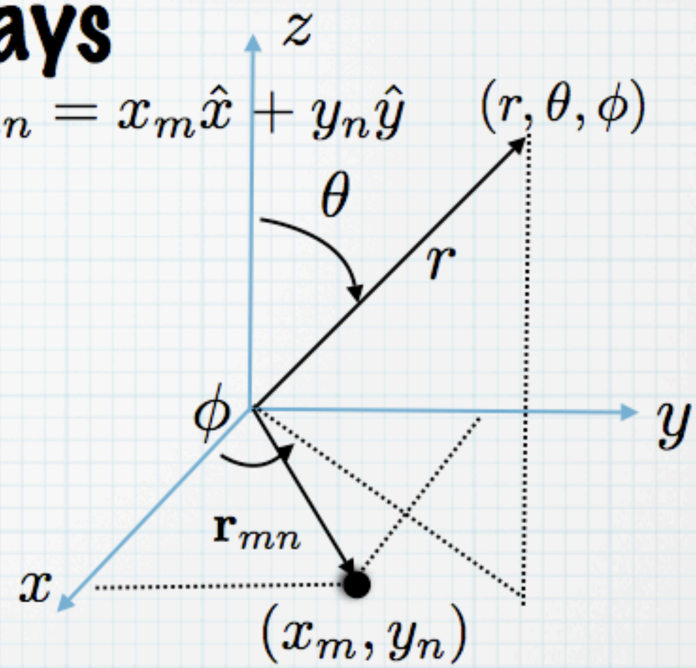
Total vector field at (r, θ, ϕ)

$$\mathbf{E} = (E_\theta\hat{\theta} + E_\phi\hat{\phi}) \sum_m \sum_n I_{mn} e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$

Element Factor — Array Factor

Antenna Arrays

$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$



$$F_{array}(\theta, \phi) = \sum_m \sum_n I_{mn} e^{jk\mathbf{r}_{mn} \cdot \hat{\mathbf{r}}}$$

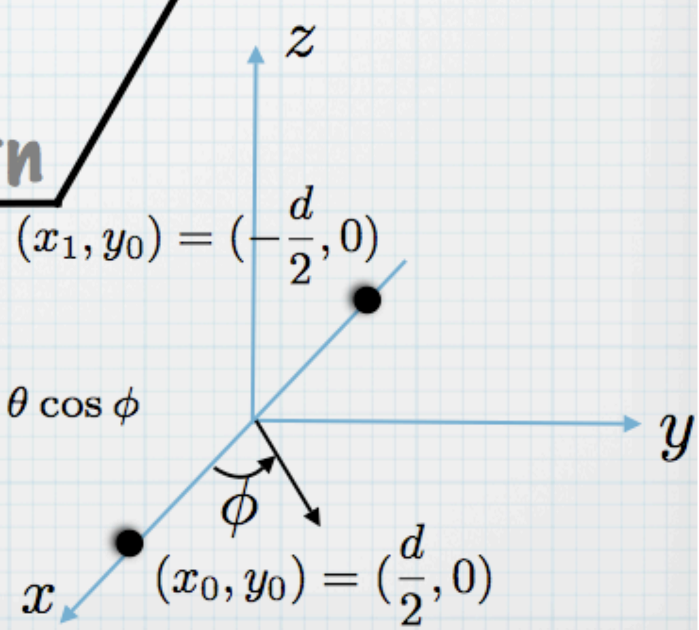
Poynting vector

$$\mathbf{P} = \frac{1}{2} \Re\{\mathbf{E} \times \mathbf{H}\} = \frac{1}{2z_0} |\mathbf{E}|^2 \hat{\mathbf{r}}$$

$$= \frac{1}{2z_0} (|E_\theta|^2 + |E_\phi|^2) |F_{array}|^2 \hat{\mathbf{r}}$$

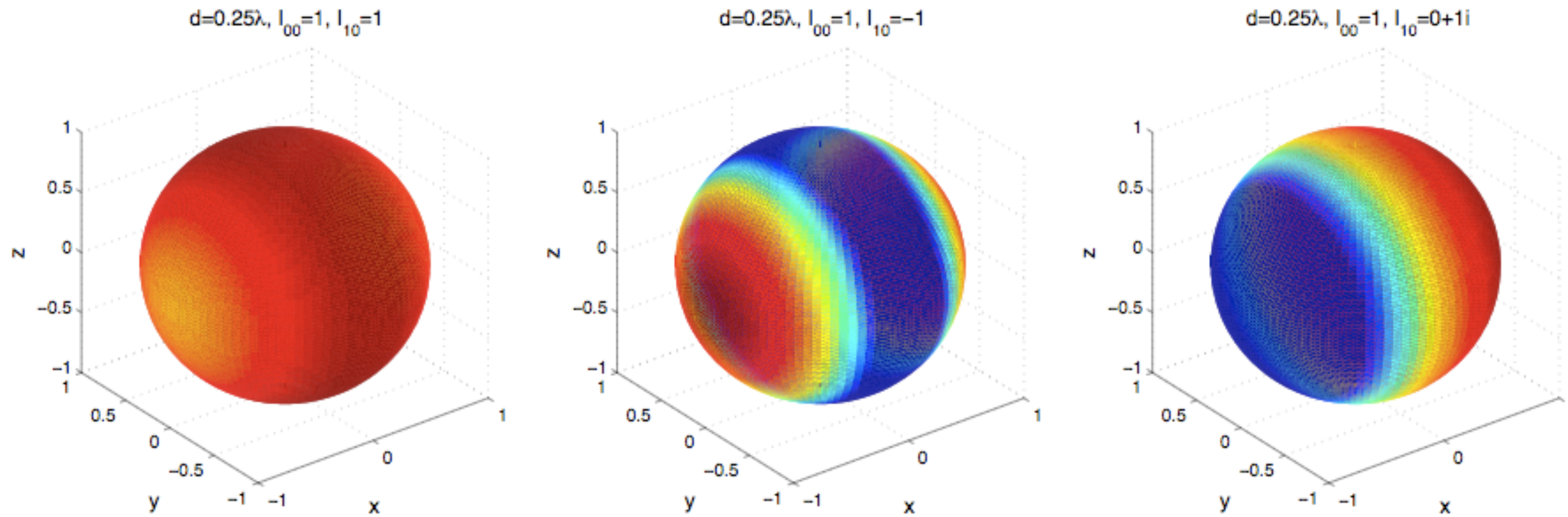
Element Pattern   Array Pattern

Simple Two Element Array

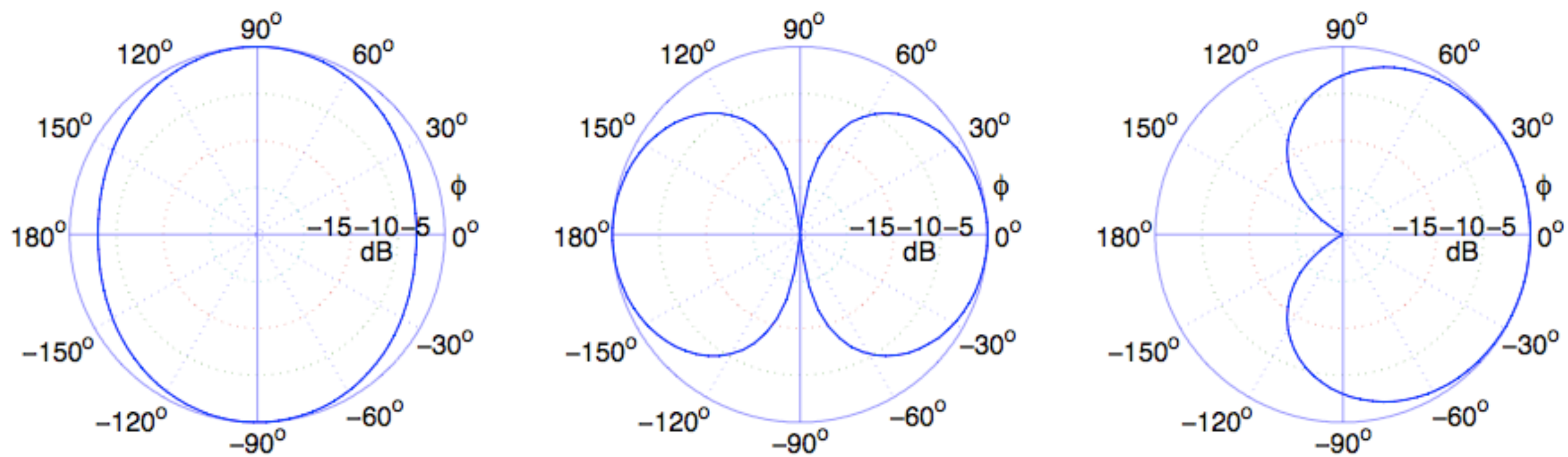


$$F_{array} = I_{00} e^{jk(d/2) \sin\theta \cos\phi} + I_{10} e^{-jk(d/2) \sin\theta \cos\phi}$$

Two-element Array

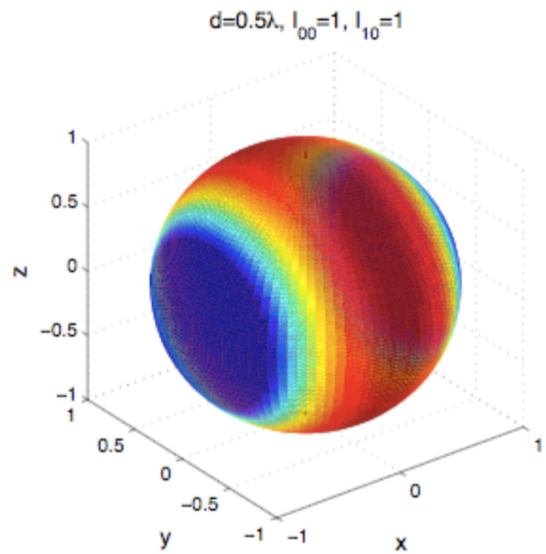


0.25 λ, In phase, Out of phase (180°), 90°

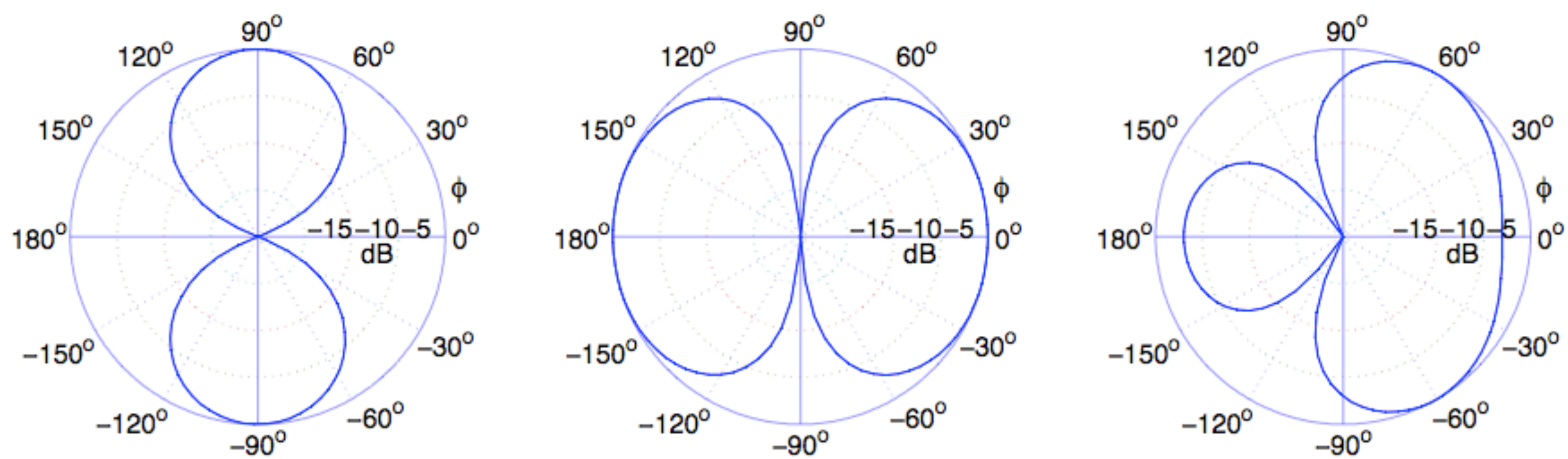


Phase and Separation effects

Two-element Array

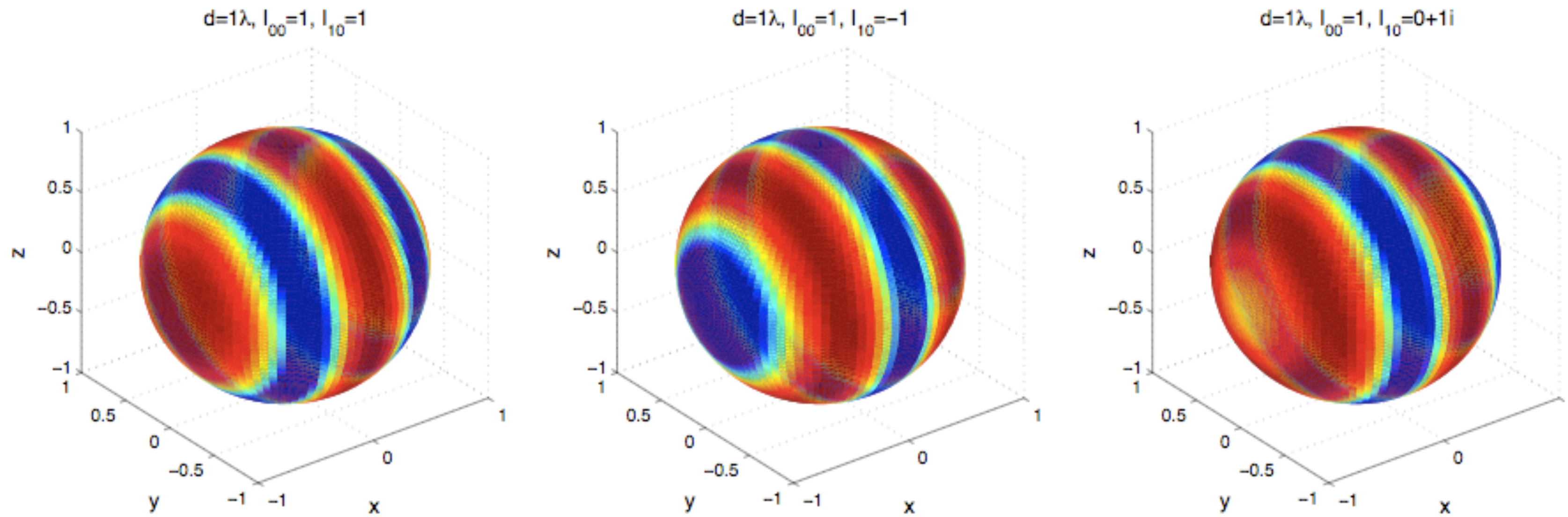


0.50 λ, In phase, Out of phase (180°), 90°

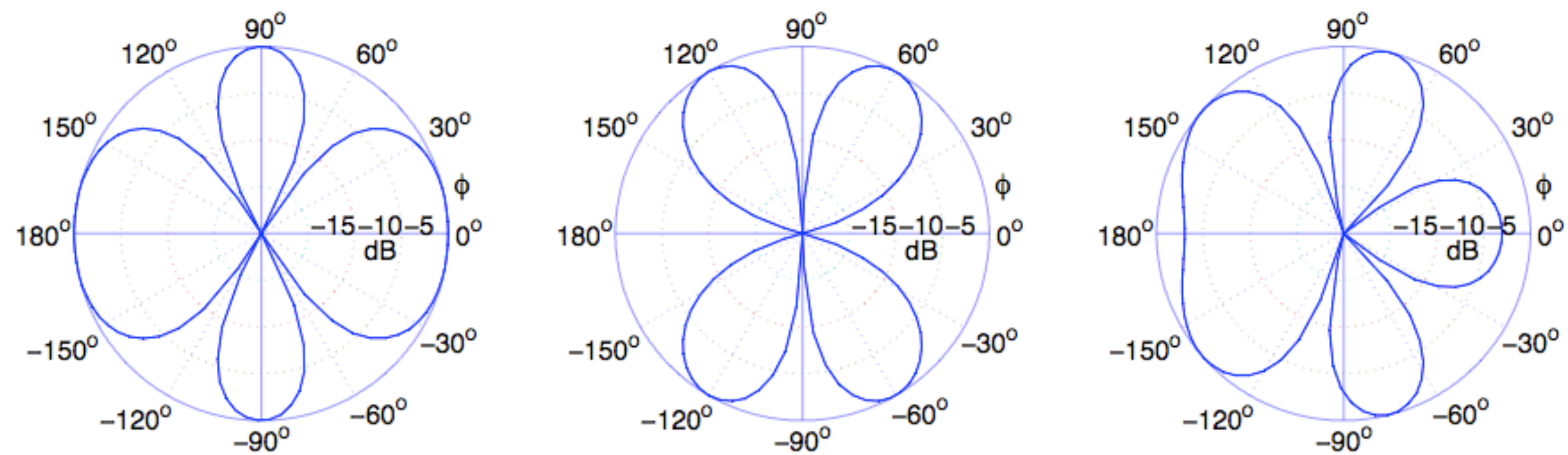


Phase and Separation effects

Two-element Array



1.00 λ, In phase, Out of phase (180°), 90°



Phase and Separation effects

Array Steering

$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)}$$

If the element constants have no phase angles, beam maximum will be in direction:

$$x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi = 0 \quad \longrightarrow \quad \theta = 0$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin\theta_0 \cos\phi_0 + y_n \sin\theta_0 \sin\phi_0 + (\psi_m + \psi_n) = 0$$

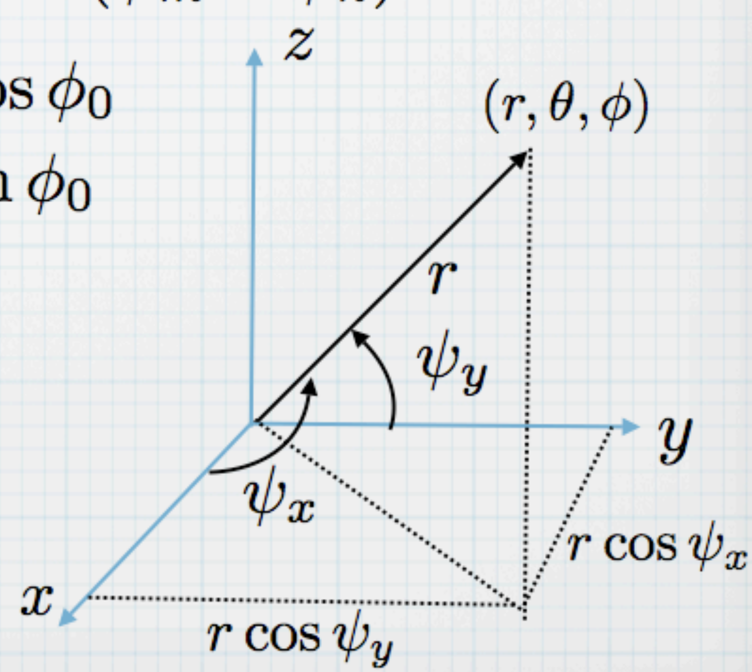
$$\psi_m = -kx_m \sin\theta_0 \cos\phi_0$$

$$\psi_n = -ky_n \sin\theta_0 \sin\phi_0$$

Direction cosines:

$$\cos\psi_{x0} = \sin\theta_0 \cos\phi_0$$

$$\cos\psi_{y0} = \sin\theta_0 \sin\phi_0$$



Array Steering

$$F_{array}(\theta,\phi)=\sum_m\sum_n I_{mn}e^{jk(x_m\sin\theta\cos\phi+y_n\sin\theta\sin\phi)}$$

$$F_{array}\quad=\quad\sum_{m,n}I_{mn}e^{jkx_m(\cos\psi_x-\cos\psi_{x0})}e^{jky_n(\cos\psi_y-\cos\psi_{y0})}$$

Say we want to point in direction (θ_0,ϕ_0)

$$x_m\sin\theta_0\cos\phi_0+y_n\sin\theta_0\sin\phi_0+(\psi_m+\psi_n)=0$$

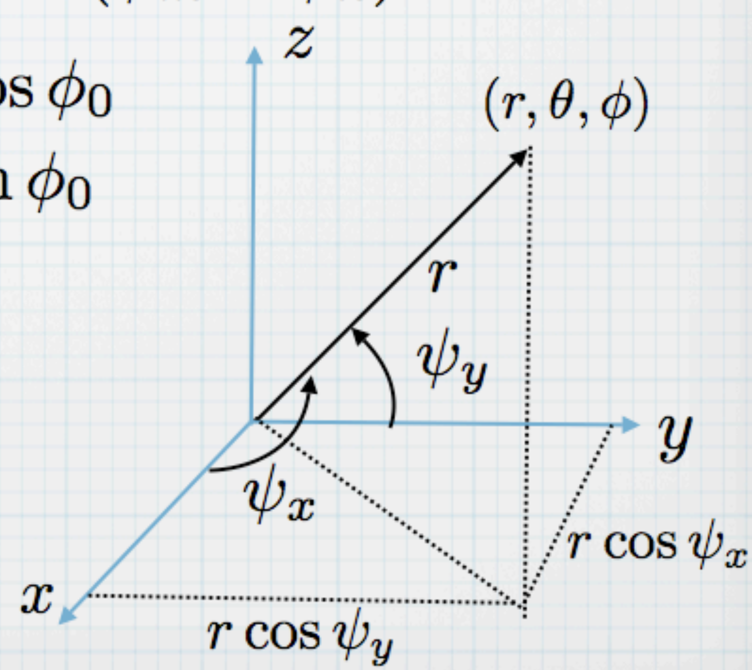
$$\psi_m=-kx_m\sin\theta_0\cos\phi_0$$

$$\psi_n=-ky_n\sin\theta_0\sin\phi_0$$

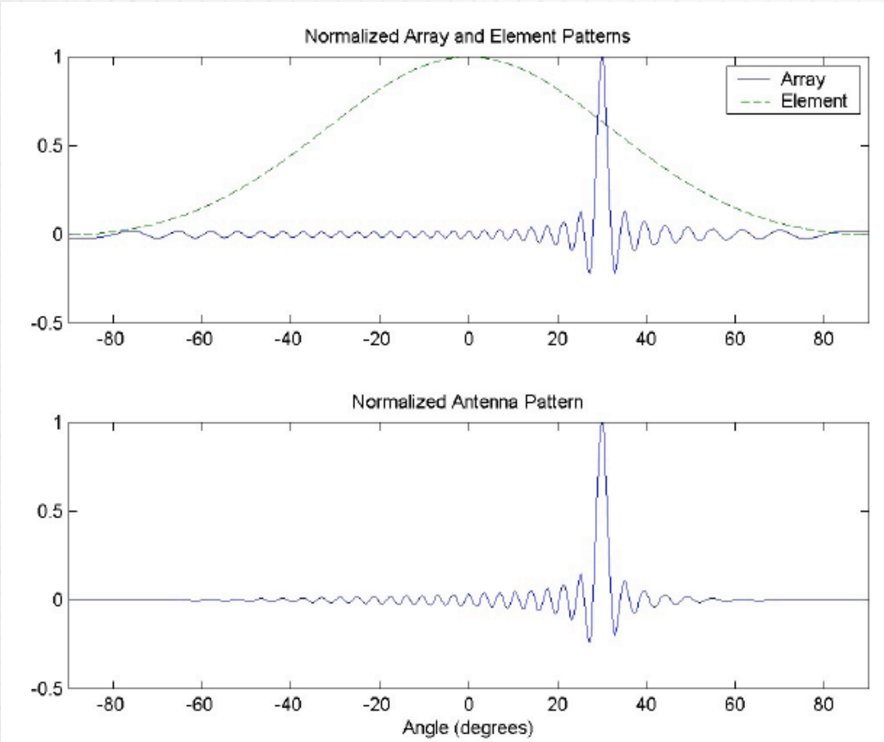
Direction cosines:

$$\cos\psi_{x0}=\sin\theta_0\cos\phi_0$$

$$\cos\psi_{y0}=\sin\theta_0\sin\phi_0$$



Directive Gain of Antenna Array

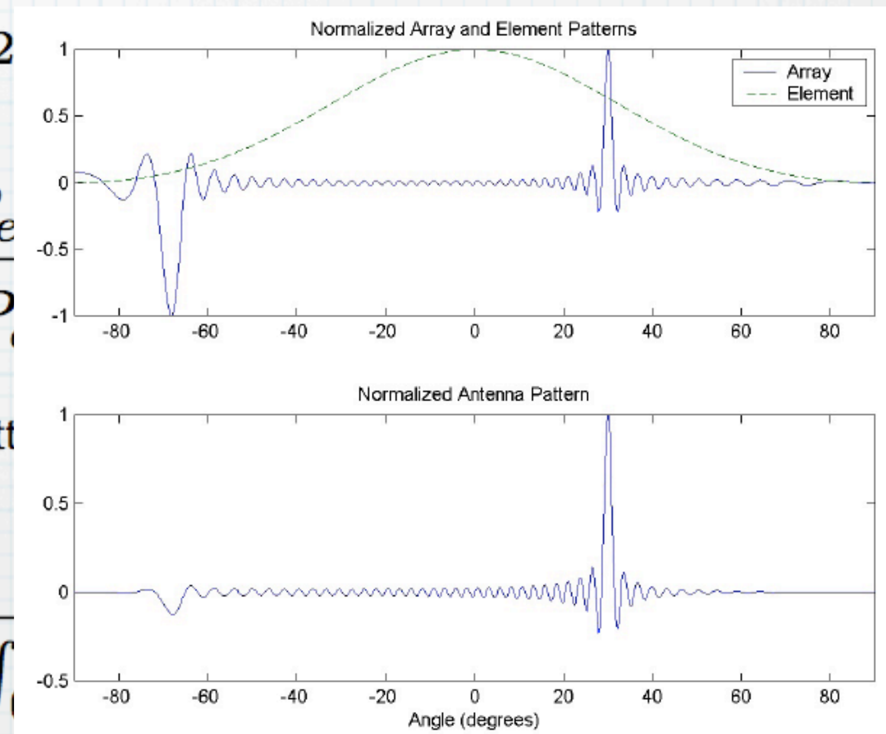


$, \phi)$ Direction / ...sity $= 4\pi R^2 \frac{\text{Power Density In } (\theta, \phi)}{\text{Total Power Radiated}}$

$|\mathbf{E}|^2 |F_{array}|^2 = P_{el} |F_{array}|^2$

$ay|^2$

P_e ... $\int_0^{2\pi} d\phi \int_0^{\pi} P$ ...



If element pattern is much broader than array patt

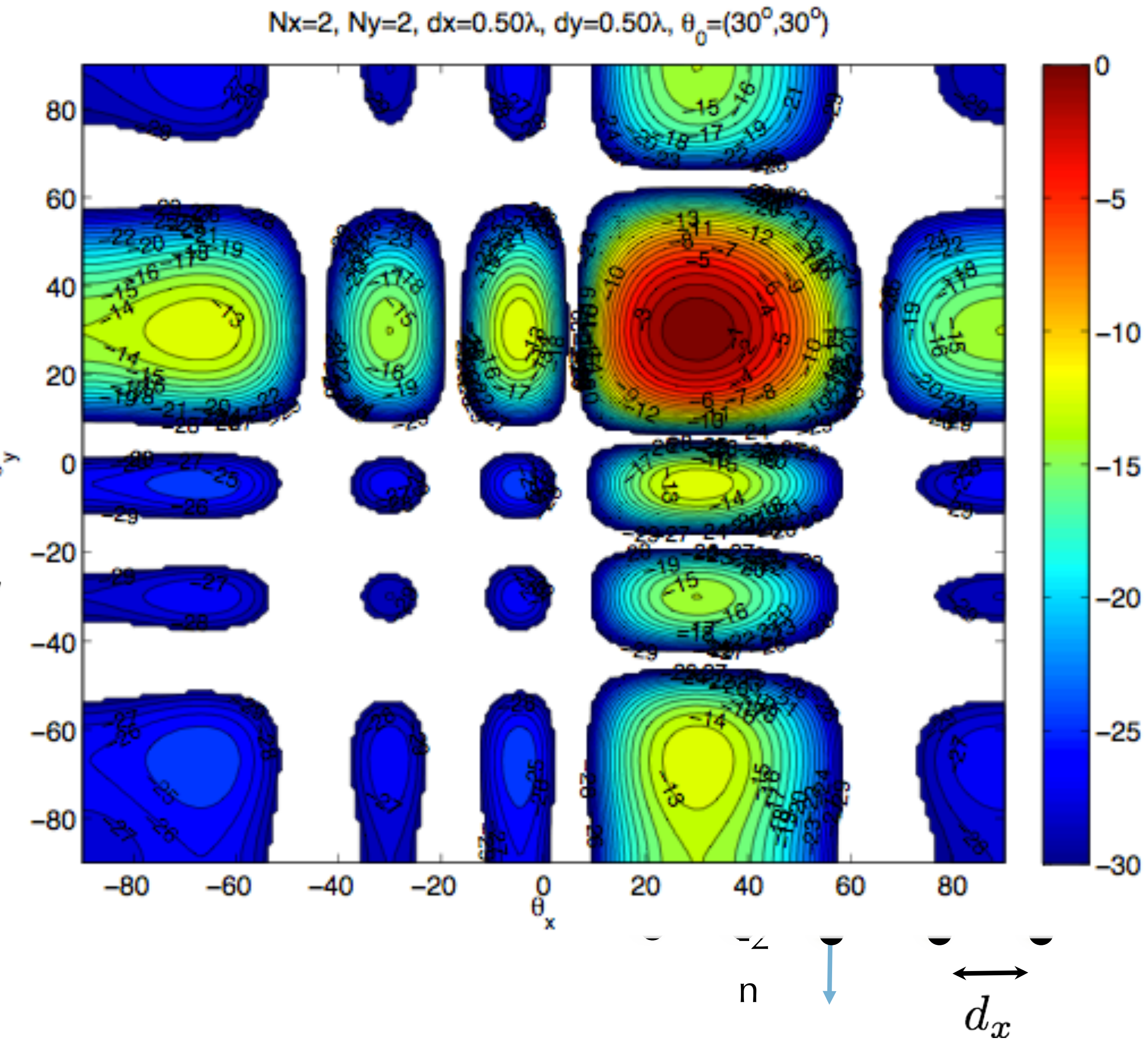
$$D(\theta, \phi) = 4\pi r^2 \frac{\quad}{\int_0^{2\pi} d\phi \int \ldots}$$

Rectangular Planar Array

$$F_{array} = \sum_{m,n} I$$

$$= I_{00} \sum$$

$$\theta_x = \pi/2 - \psi_x$$
$$\theta_y = \pi/2 - \psi_y$$



Note: No "-z" computed!

The Fourier Analogy

$$F_{array} = \sum_m I_m e^{jkdm(\cos\psi_x - \cos\psi_{x0})}$$

$$= \sum_m I_m e^{jm\gamma}$$

$$= \sum_m I_m z^m$$

Array factor can be interpreted as DFT of weighting factors

Array factor in spatial z domain

$$I_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} F_{array}(\gamma)e^{-j\gamma m}d\gamma$$

Inverse DFT - principle of many array design methods (analogous to FIR filter design)

Visible Region and Grating Lobes

Recall
(1d array pointed broadside):

$$F_{array} = \sum_m I_m e^{jkdm\cos\psi_x} = \sum_m I_m e^{jm\gamma}$$

Can see that:

$$-kd \leq \gamma \leq kd$$

Visible Region

$$d \leq \lambda/2 \rightarrow \gamma \leq 2\pi$$

$$d > \lambda/2 \rightarrow \gamma > 2\pi$$

Values of F_{array} repeat - Grating Lobes

Grating lobes are analogous to classical undersampling (spectral aliasing).

Uniform, Linear Array

Back to linear x array:

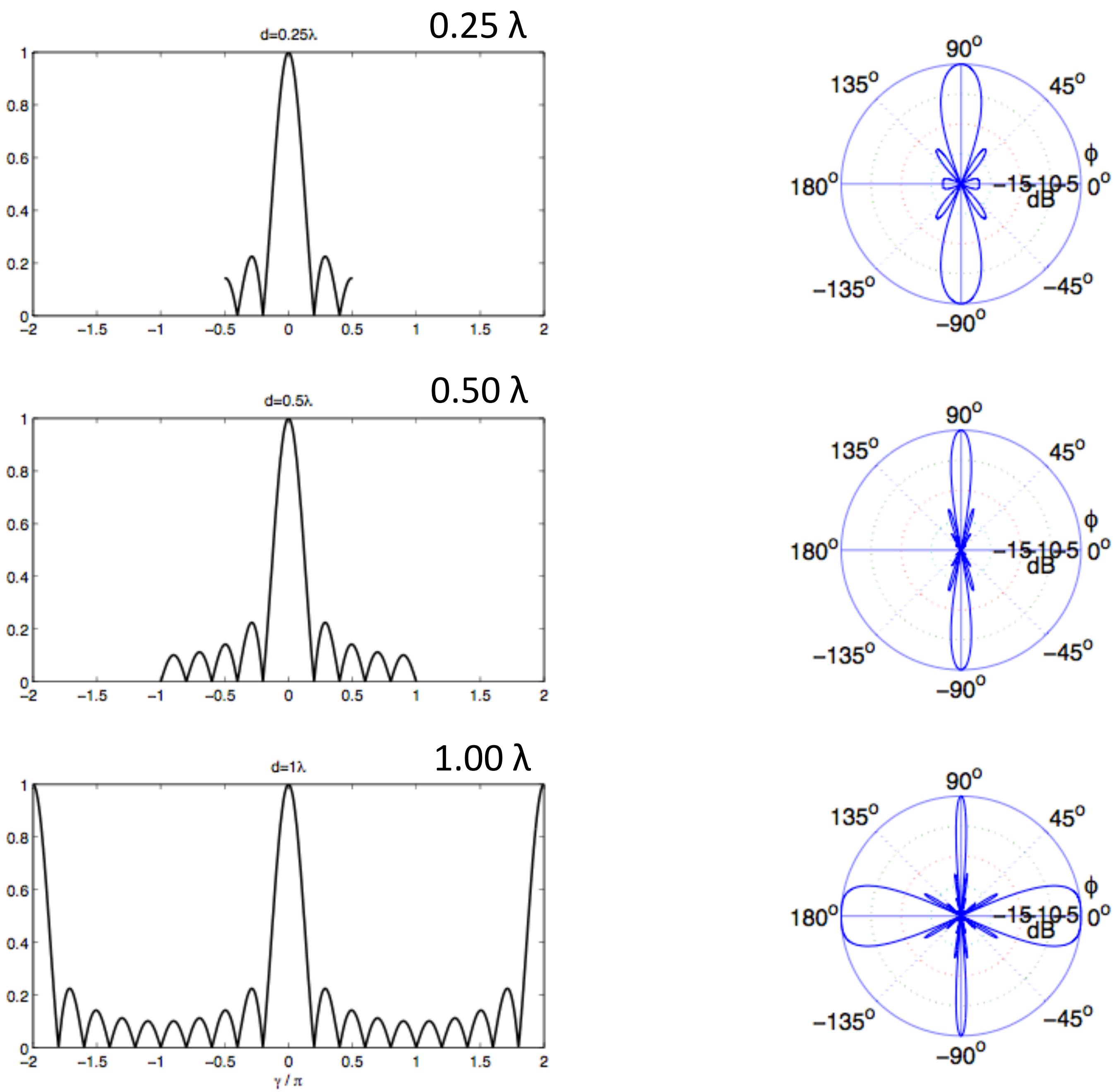
$$F_{array} = \sum_m I_m e^{jkdm(\cos\psi_x - \cos\psi_{x0})} = \sum_m I_m e^{jm\gamma}$$

If weights are uniform:

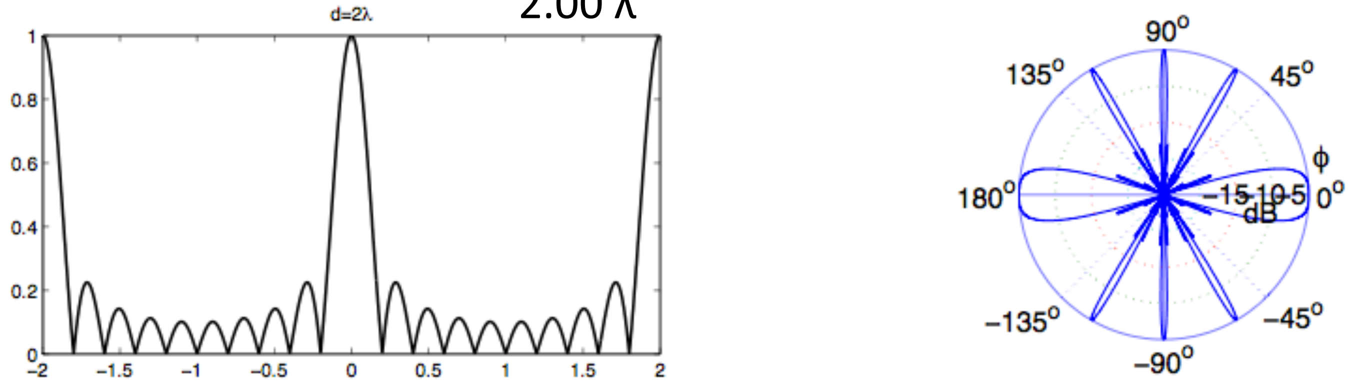
$$\begin{aligned} F_{array} &= I_0 \left[1 + e^{j\gamma} + e^{j2\gamma} + ... + e^{j(N-1)\gamma}\right] \\ &= I_0 \frac{e^{jN\gamma} - 1}{e^{j\gamma} - 1} \\ &= I_0 \frac{\sin\frac{N\gamma}{2}}{\sin\frac{\gamma}{2}} e^{j(N-1)\gamma/2} \end{aligned}$$

Sinc function, comes from DFT of rectangular window

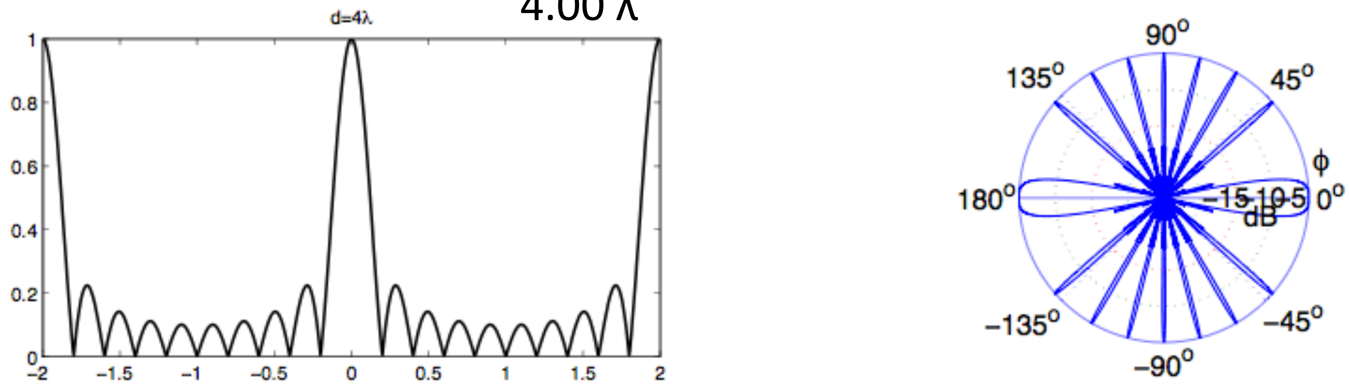
Note that the larger the array, the narrower the beam **HPBW ≈ λ/D**


0.25 λ
d=0.25λ
0.50 λ
d=0.5λ
1.00 λ
d=1λ
γ / π
90°
45°
135°
180°
0°
−45°
−135°
−90°
ϕ
−15 −10 −5
dB

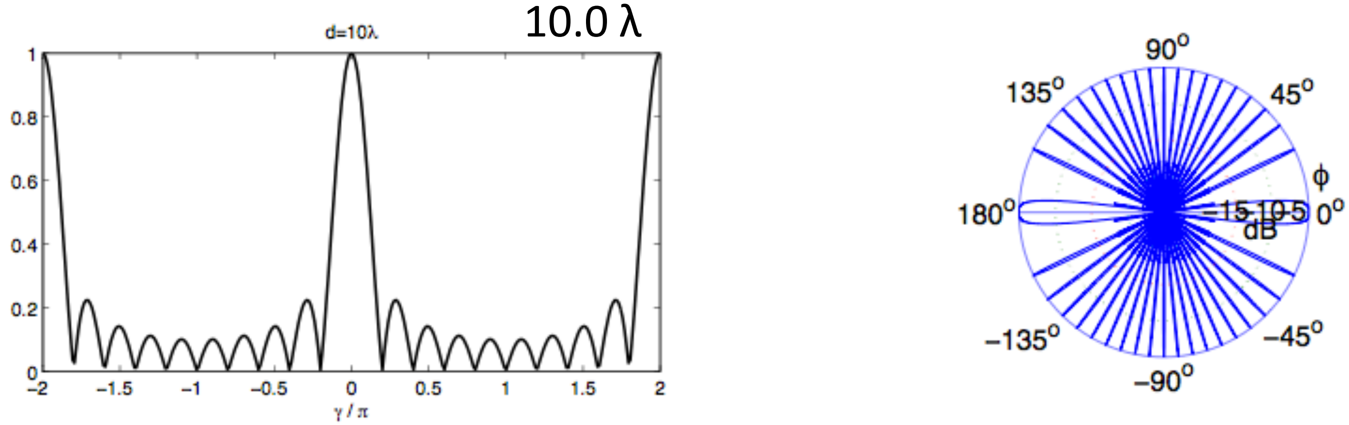
2.00 λ



4.00 λ



10.0 λ



Steering and Grating Lobes

For arbitrary steering direction:

$$F_{array} = \sum_m I_m e^{jkdm(\cos\psi_x - \cos\psi_{x0})} = \sum_m I_m e^{jm(\gamma-\gamma_0)}$$

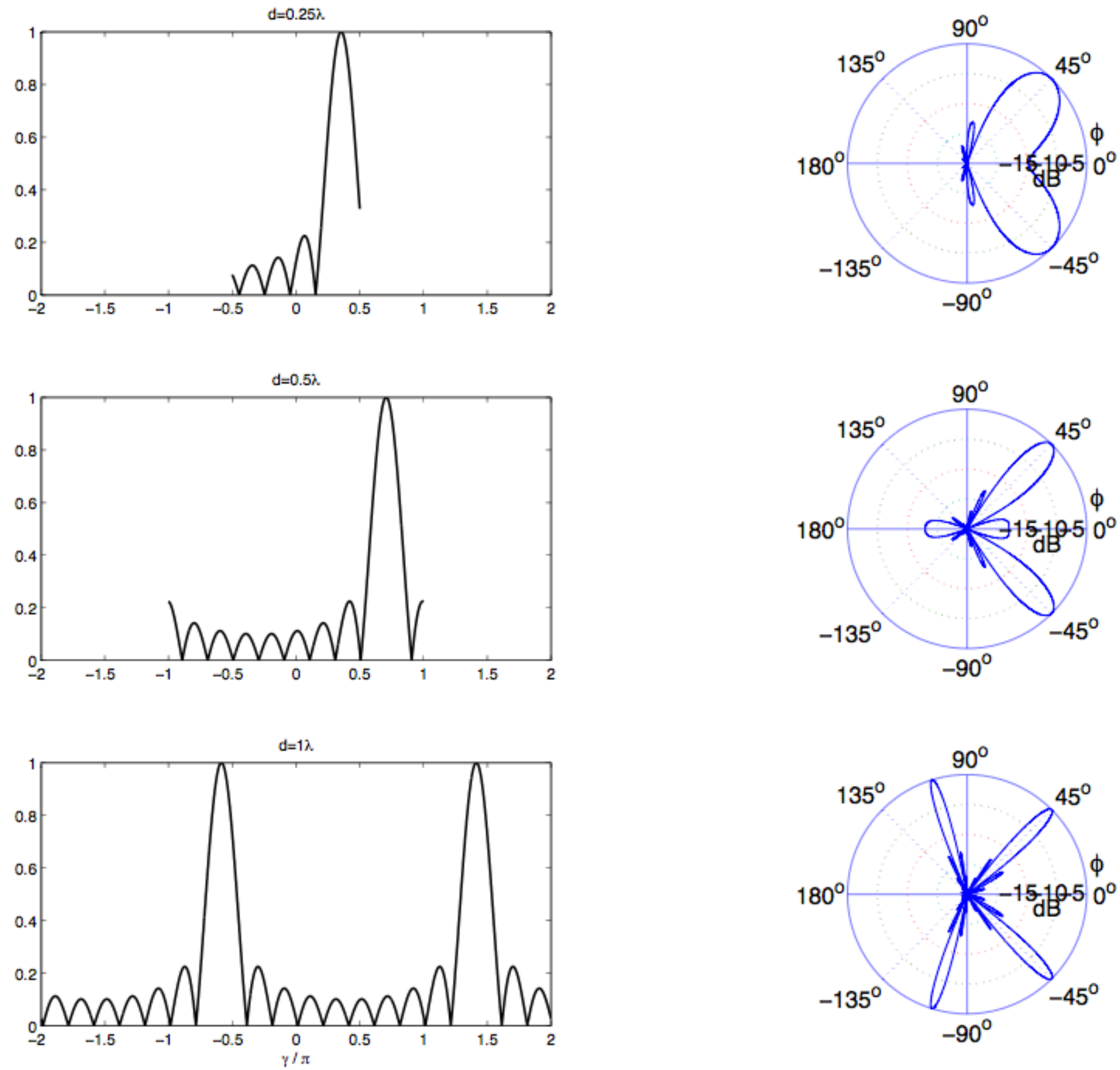
$$-kd(1+\cos\psi_{x0}) \leq \gamma \leq kd(1-\cos\psi_{x0})$$

Modified Visible Region

For no grating lobes,

$$\gamma \leq 2\pi \qquad d \leq \frac{\lambda}{1+|\cos\psi_{x0}|}$$

Also note that beam broadens as $\sin\psi_0$ as beam is steered



d=0.25λ
d=0.5λ
d=1λ
γ / π
90°
135°
45°
180°
0°
-135°
-45°
-90°
ϕ
dB

Method of Moments (mutual coupling)

RADIO SCIENCE, VOL. 46, RS2012, doi:10.1029/2010RS004518, 2011

A review on array mutual coupling analysis

C. Craeye[1] and D. González-Ovejero[1]

Received 8 September 2010; revised 14 December 2010; accepted 6 January 2011; published 8 April 2011.

[1] An overview about mutual coupling analysis in antenna arrays is given. The relationships between array impedance matrix and embedded element patterns, including beam coupling factors, are reviewed while considering general-type antennas; approximations resulting from single-mode assumptions are pointed out. For regular arrays, a common Fourier-based formalism is employed, with the array scanning method as a key tool, to explain various phenomena and analysis methods. Relationships between finite and infinite arrays are described at the physical level, as well as from the point of view of numerical analysis, considering mainly the method of moments. Noise coupling is also briefly reviewed.

Citation: Craeye, C., and D. González-Ovejero (2011), A review on array mutual coupling analysis, *Radio Sci.*, *46*, RS2012, doi:10.1029/2010RS004518.

Mutual Coupling / Impedance

- Array gain - related to gain of individual element.
- Gain of isolated element very different from element gain within array.
- Element pattern will also vary across array.
- **Actual element gain usually not known - must be simulated/measured.**

For an N element array:

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

Mutual impedance

Self impedance

- Solve for I
- Compute Poynting vector
- Use this to compute radiation pattern

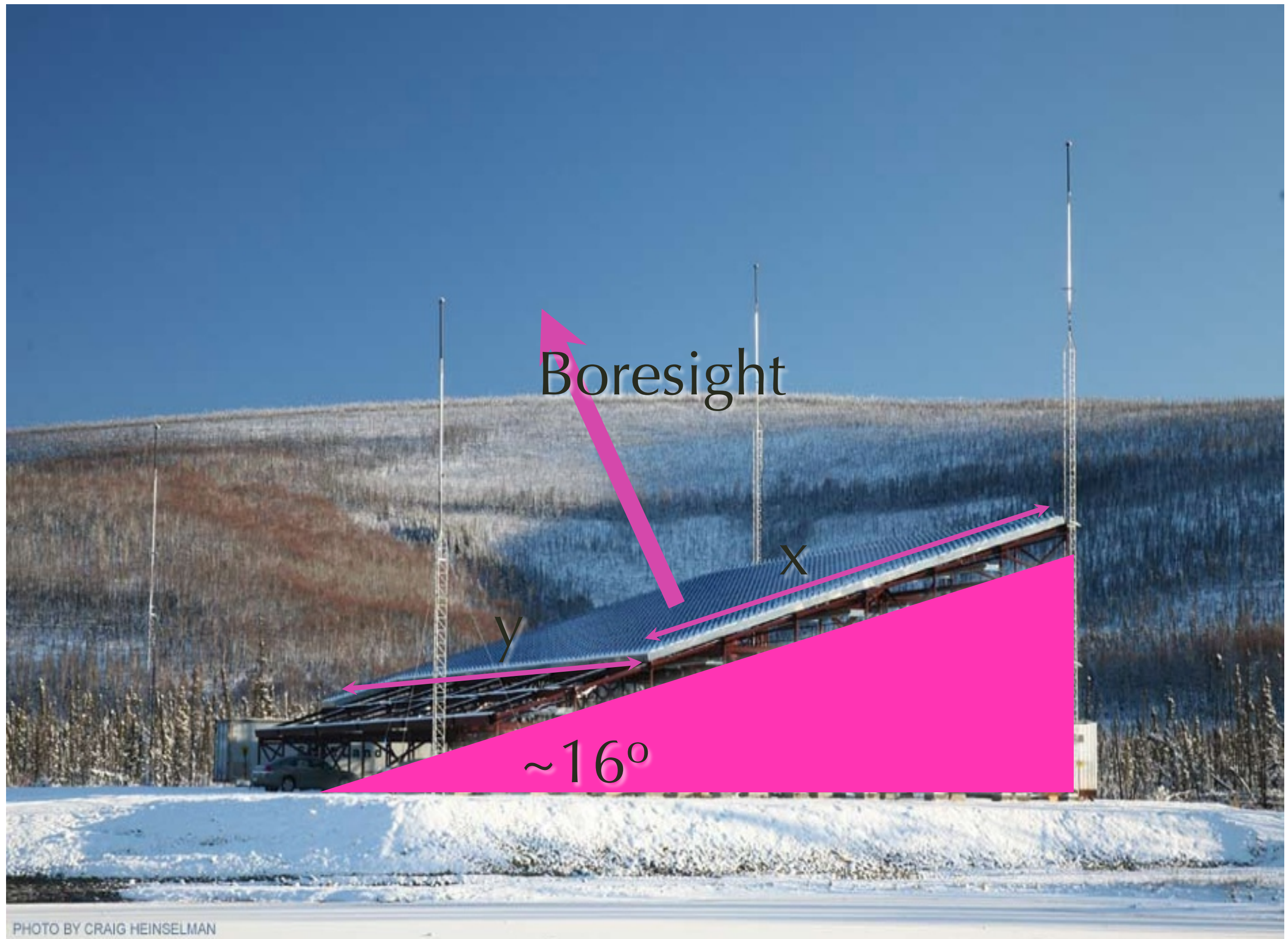
- Important to minimize mutual coupling -> Can cause problems (standing waves “hot spots”, etc

AMISR (PFISR, RISR-N, RISR-C)

- Jicamarca – Phased array with very large collecting area, but:
  - (a) “Passive”, (b) Modular but not portable, (c) Fixed pointing
- MU Radar – Active phased array, but not good for IS
- AMISR – “Modern” Incoherent Scatter Radar constructed by the NSF

AMISR (2)

- wavelength ~67 cm
- elements separated by less than a wavelength



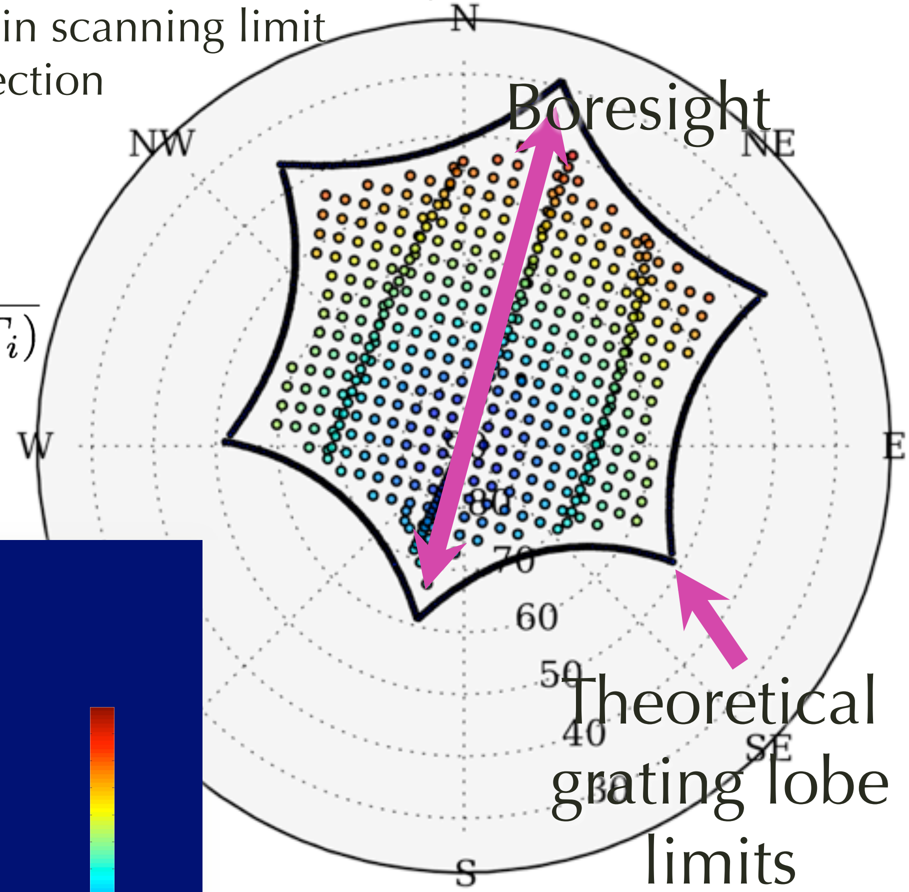
AMISR (3)

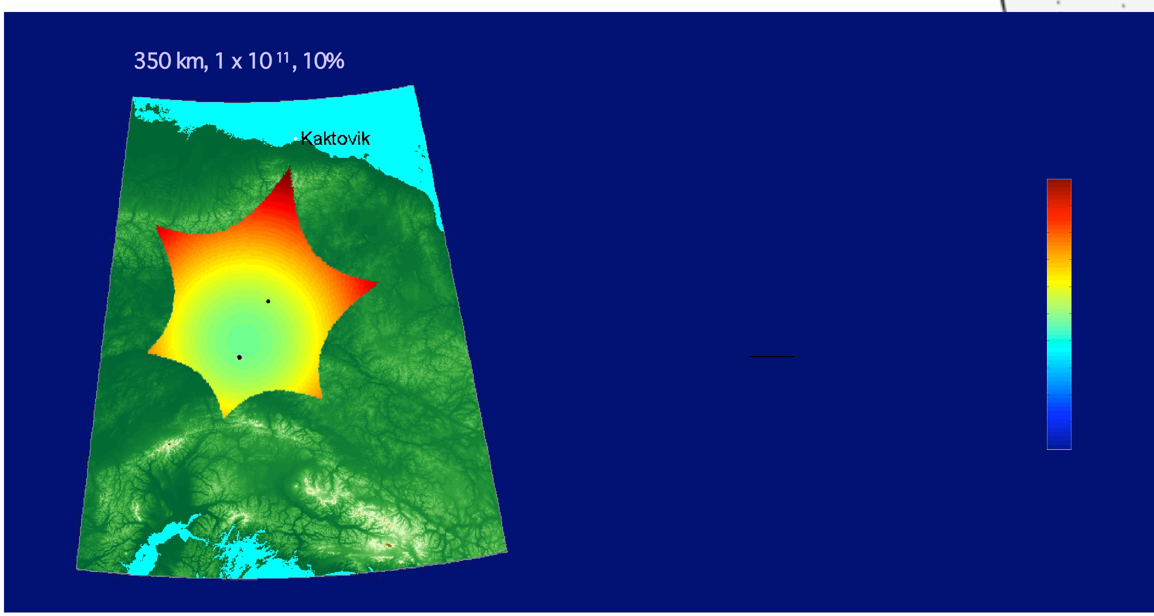
- **Recall:** Grating lobes will appear when beam is scanned far enough - makes it impossible to do incoherent scatter science beyond certain scanning limit
- **Recall:** Gain pattern will vary with scan direction

Should have seen an equation like:

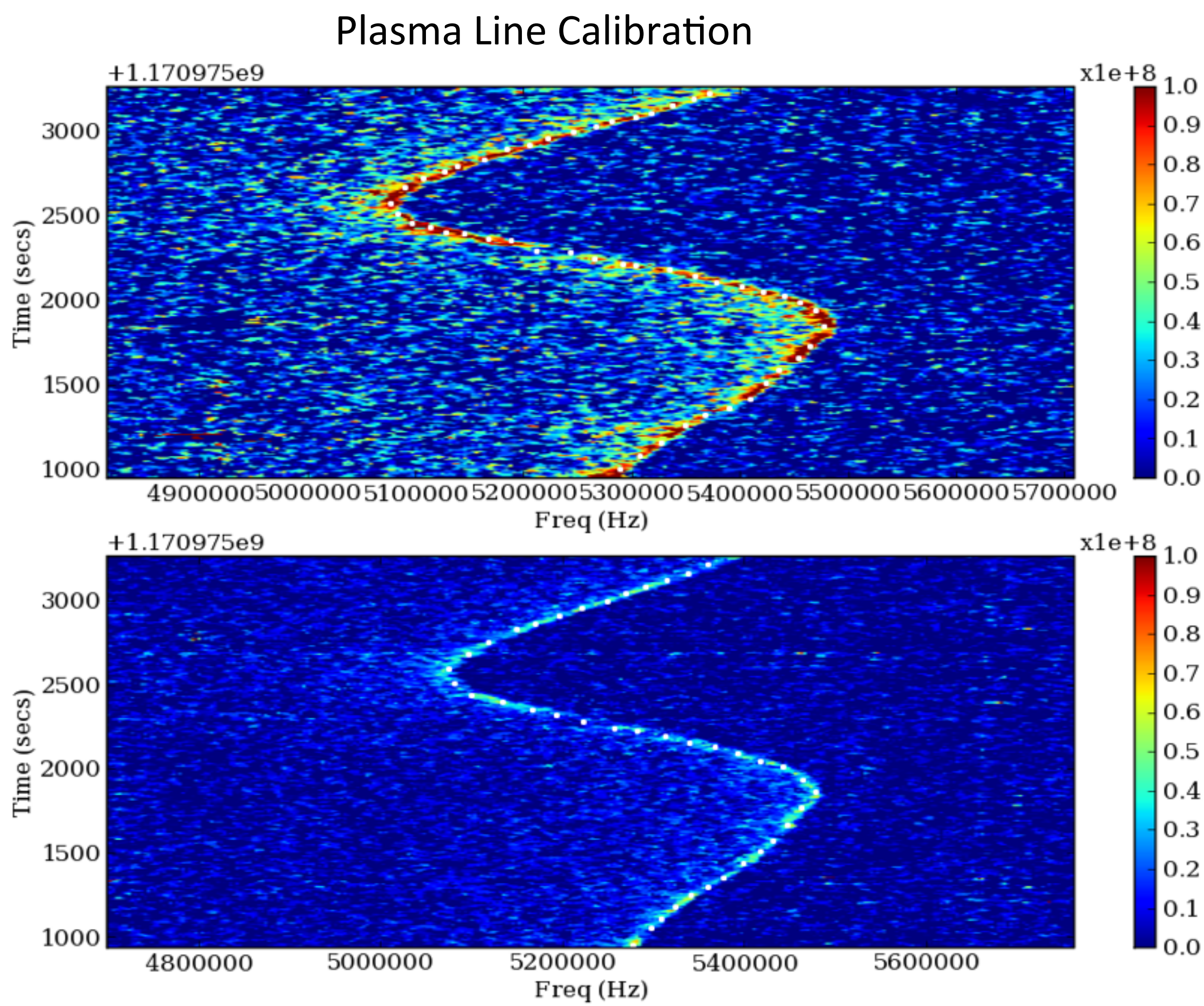
$$P_r = \frac{P_t \tau_p K_{sys}}{r^2} \frac{N_e}{(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_e/T_i)}$$

System Constant becomes dependent on look direction

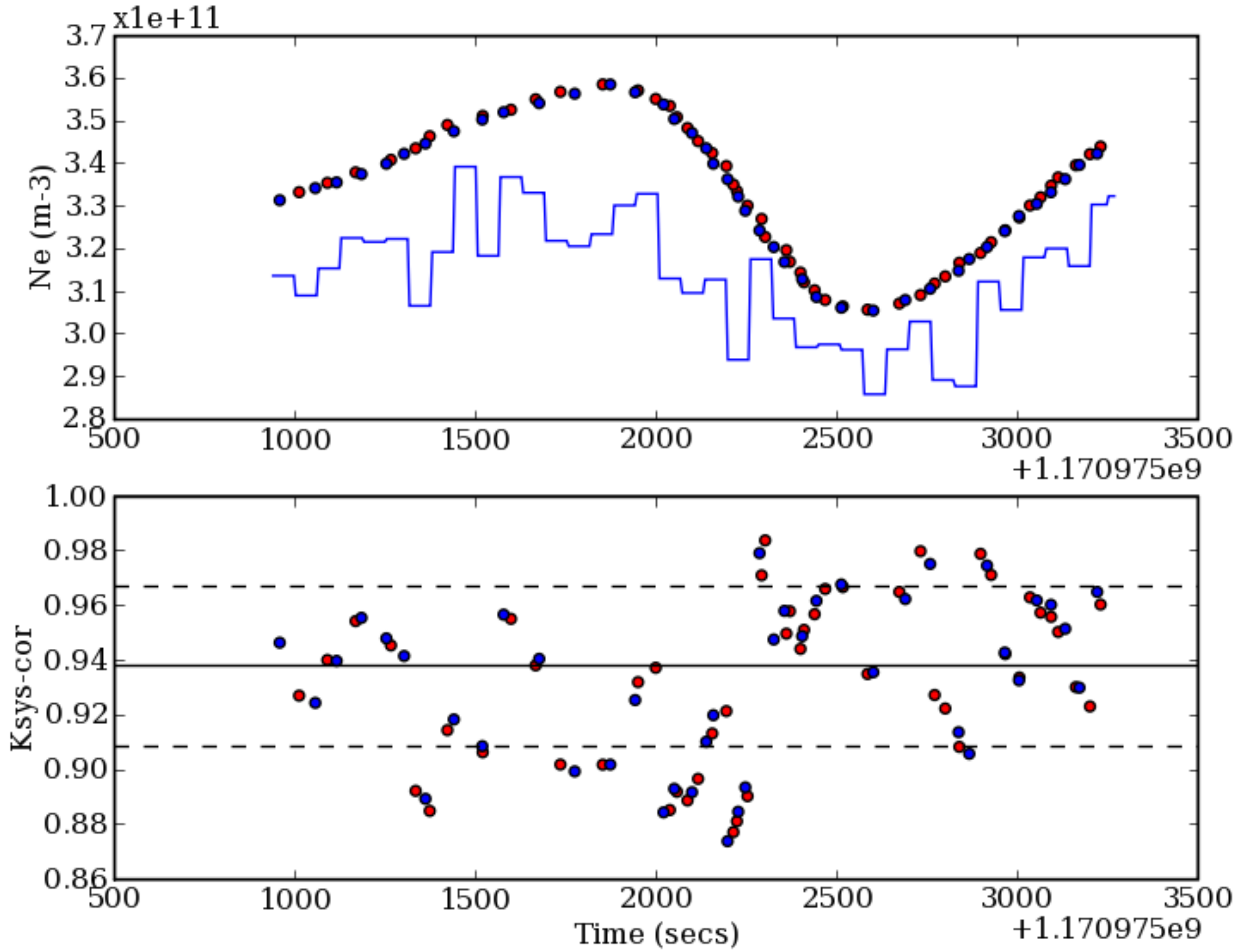




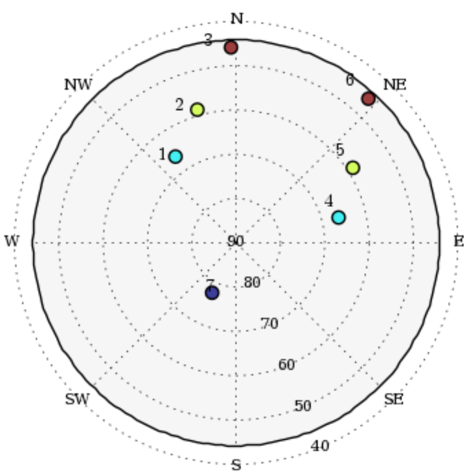
AMISR (4) Plasma Line calibration

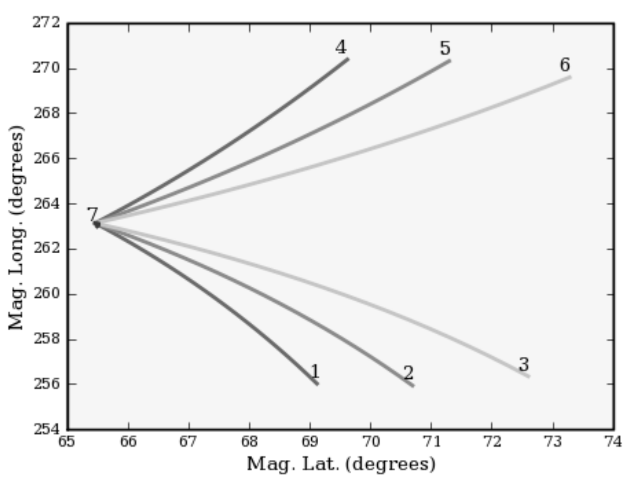


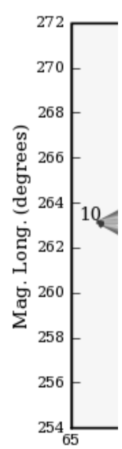
AMISR (5) Plasma Line Calibration

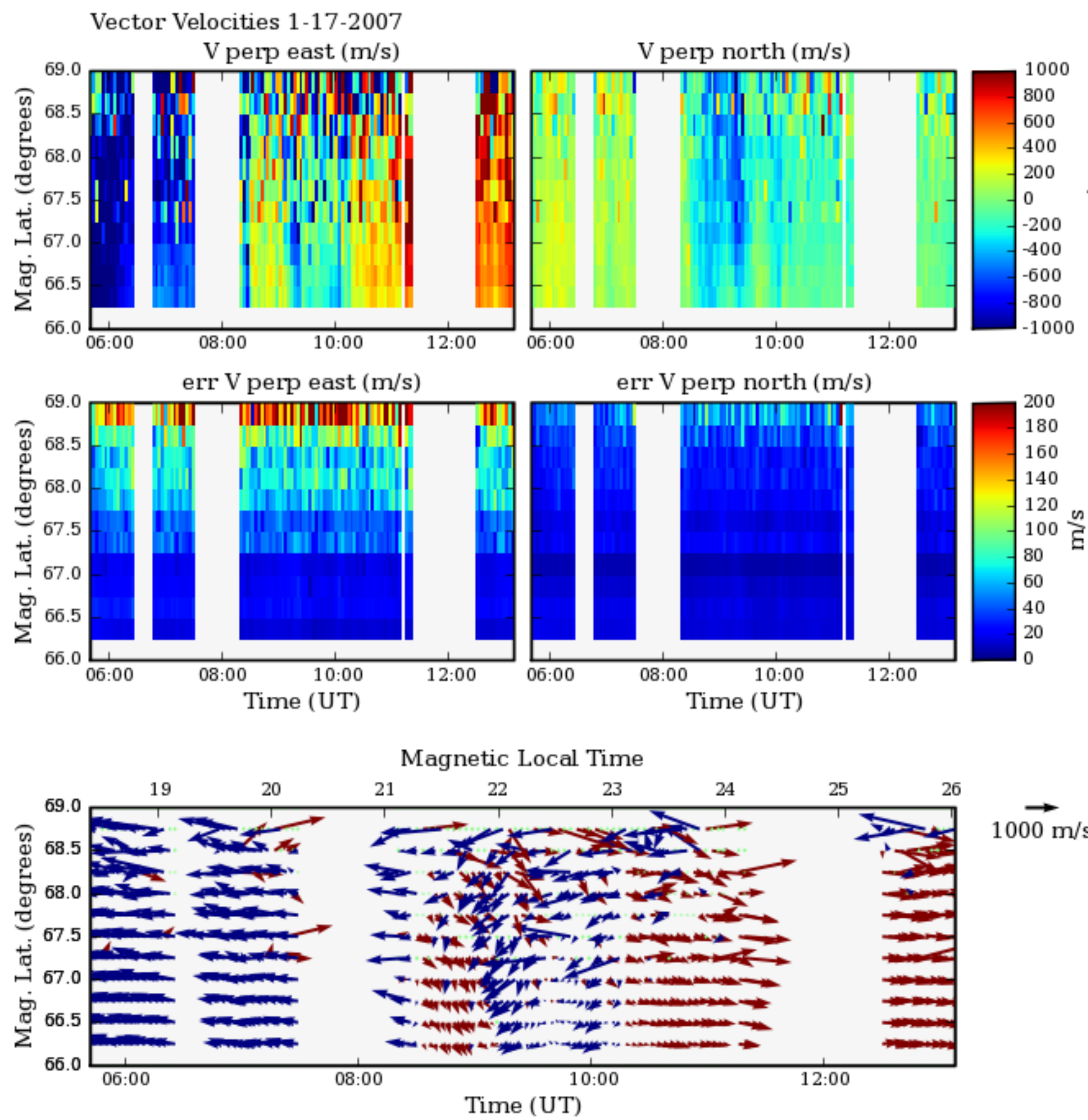


AMISR (6) Electric field Estimation



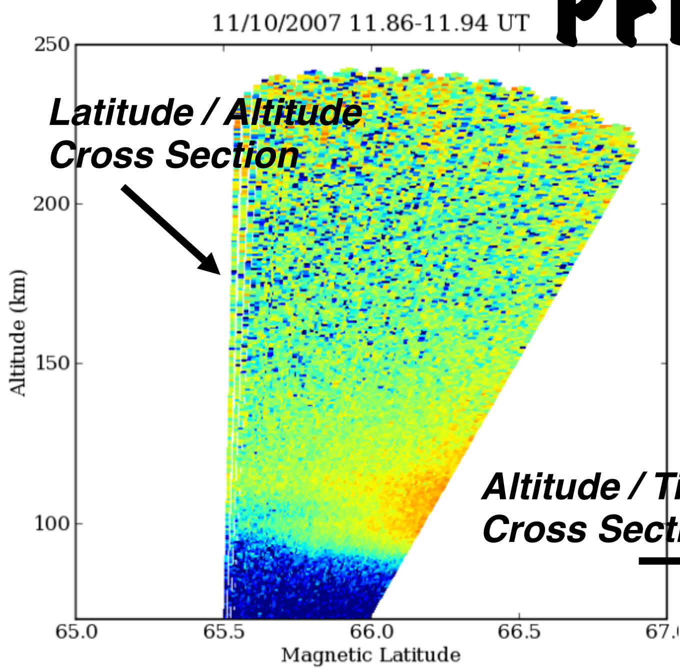




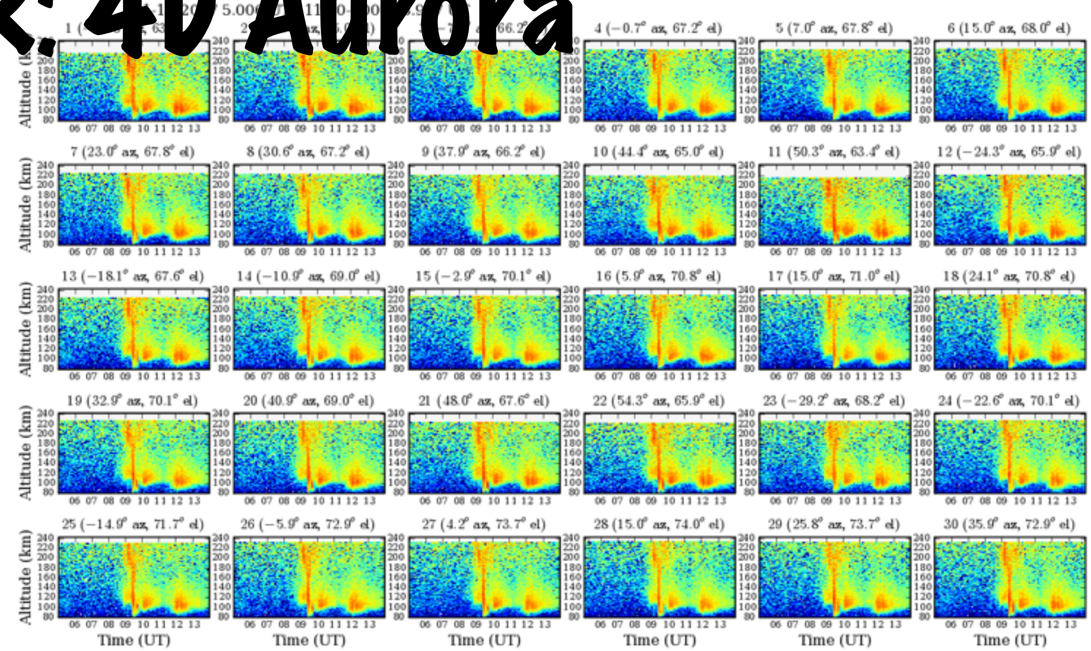


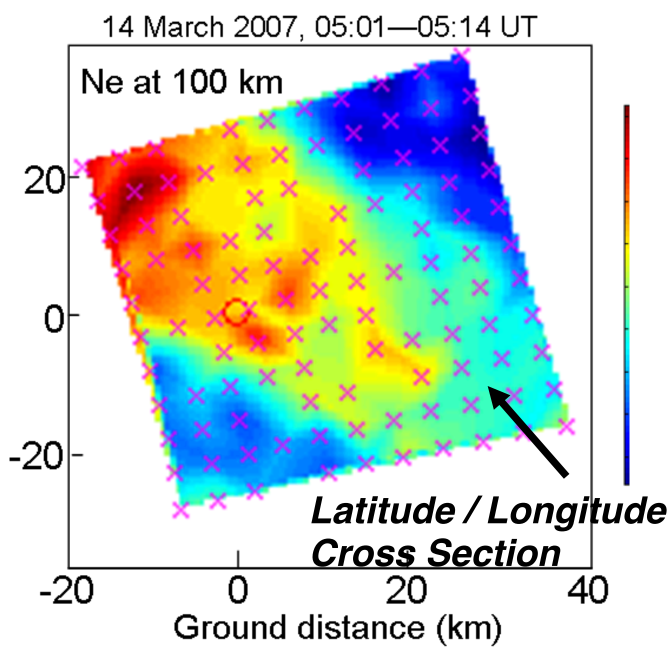
PFISR: 4D Aurora

Latitude / Altitude Cross Section



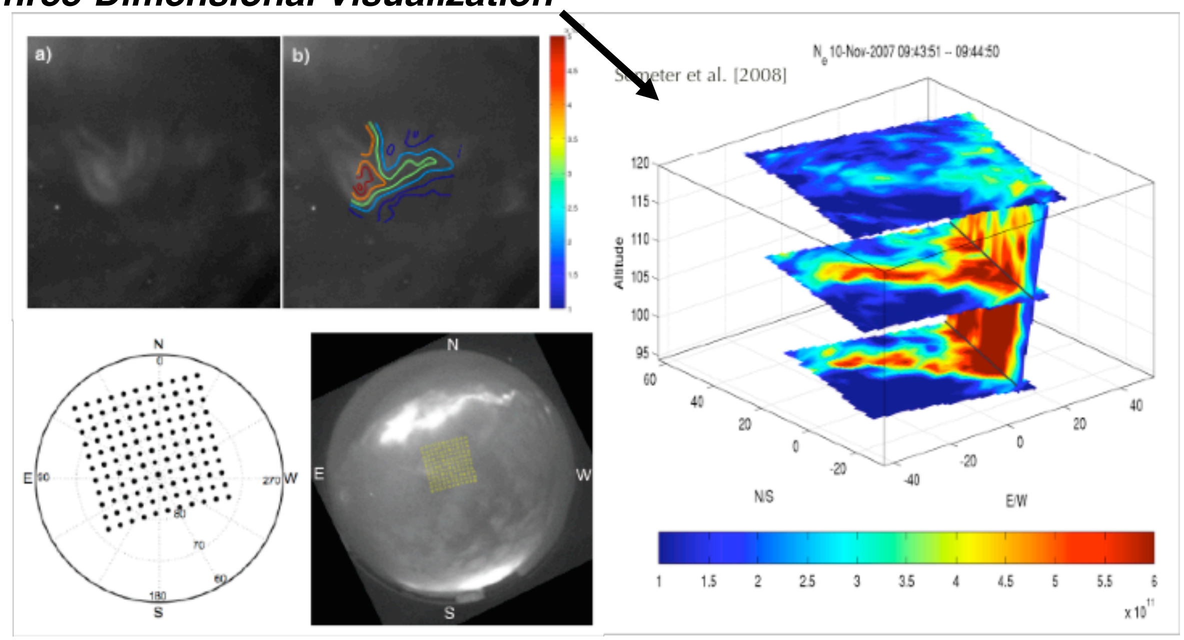
Altitude / Time Cross Section



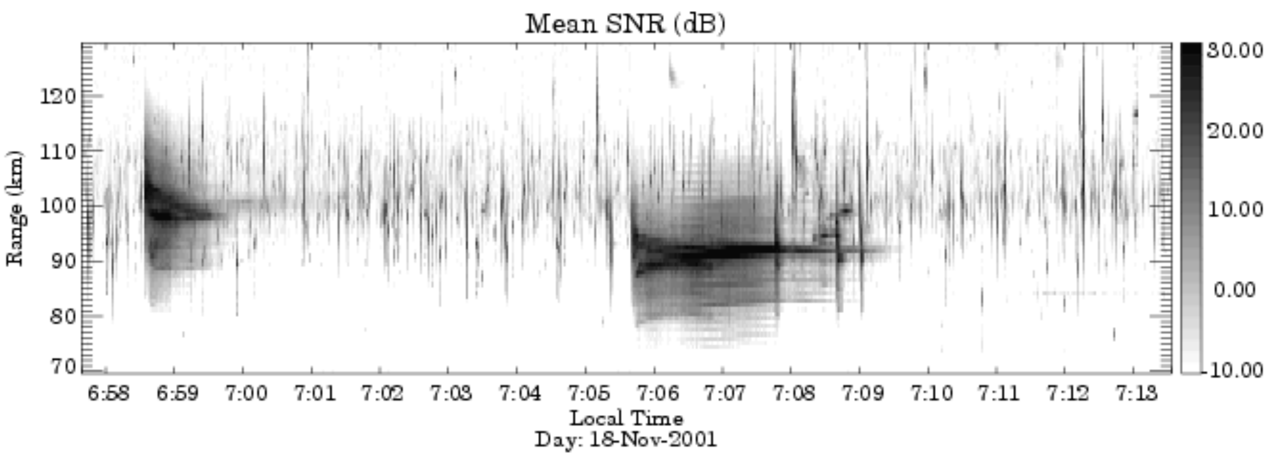


Latitude / Longitude Cross Section

Three-Dimensional Visualization

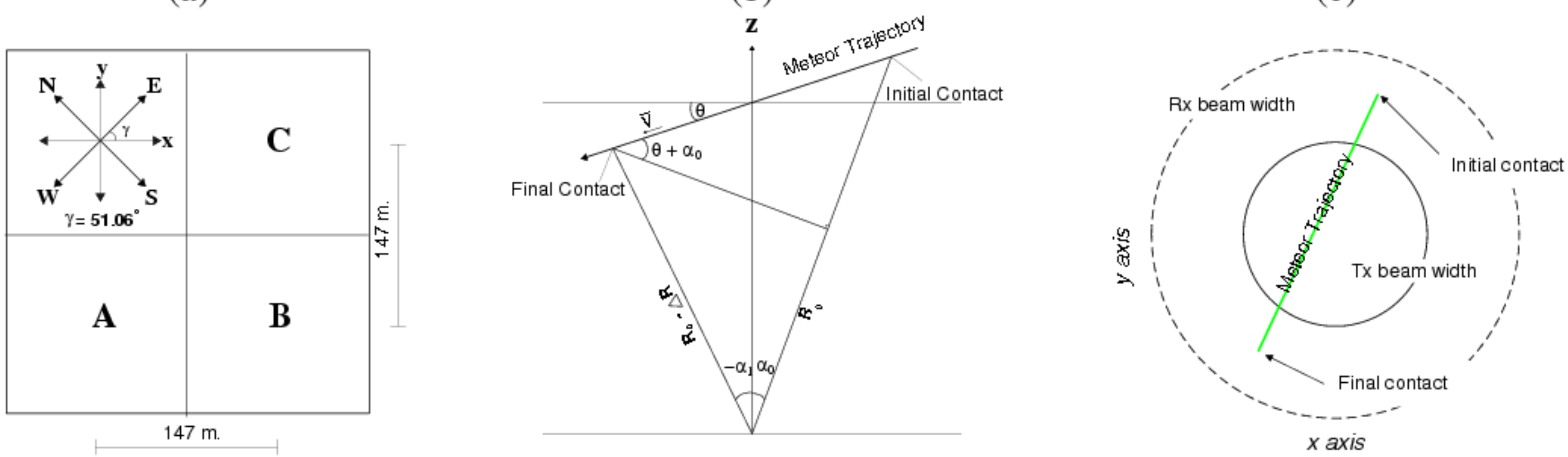


Interferometry at Jicamarca
Meteor-heads: SNR and Configuration



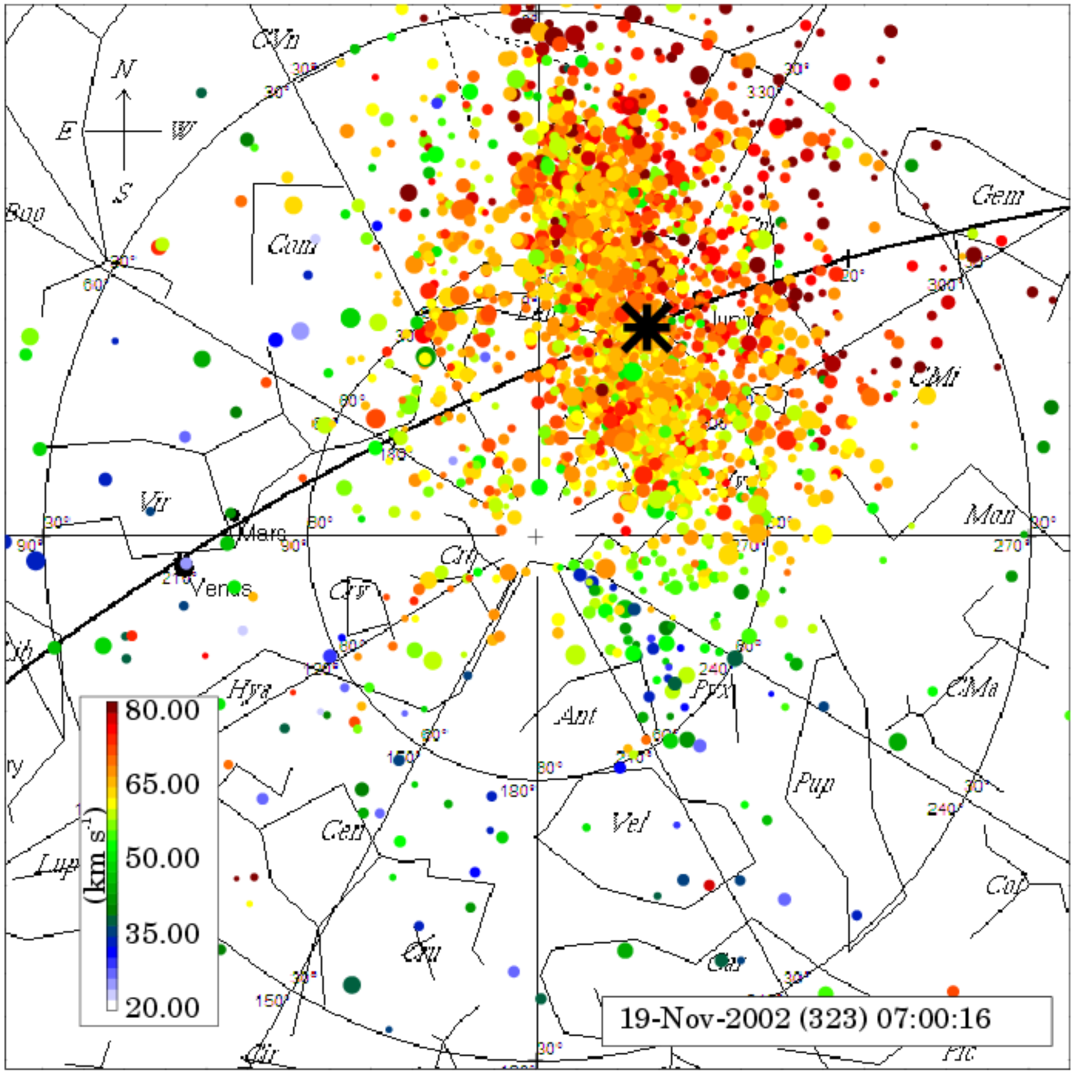
- Jicamarca detects ~1 meteor head/sec around sunrise.
- Using interferometry and special signal processing, we can determine directly: absolute velocities and decelerations, where they are coming from, range and time of occurrence, SNR.

(a) (b) (c)



[from *Chau and Woodman*, 2004]

Meteor-heads: Where do they come from?



- Most meteors come from the Apex direction. The dispersion around the Apex is ~18^{o} transverse to Ecliptic plane, and ~8.5^{o} in heliocentric longitude. Both in the Earth initial frame of reference.

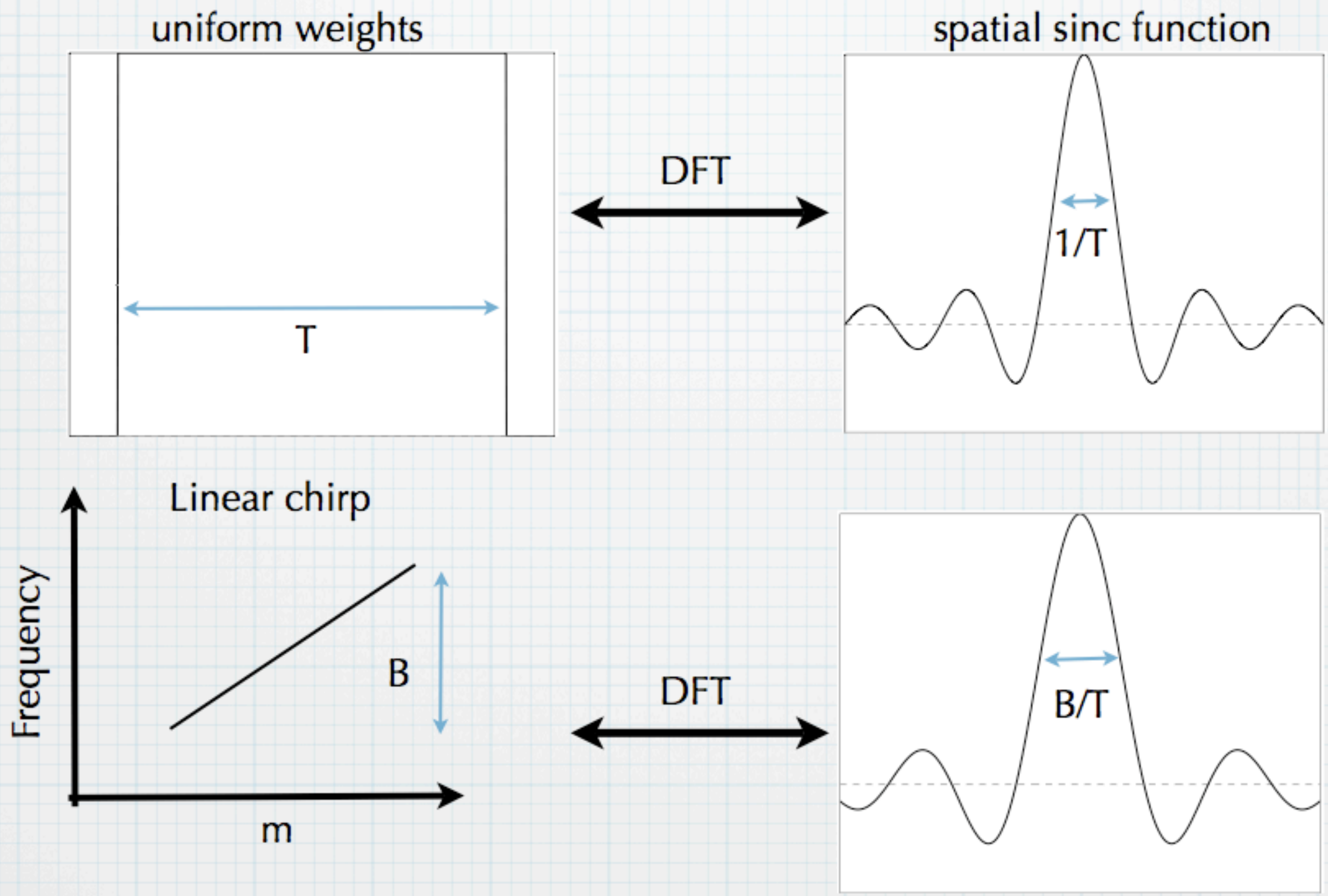
[from *Chau and Woodman, 2004*]

Antenna Compression: Motivation

- Use high power with wide beams (imaging work, spaced antenna, aspect sensitivity measurements, etc.)
- Some systems have the high power transmitters, but single antenna modules do not support such a high power (e.g., Jicamarca). Other systems have distributed power (e.g., MU, MAARSY, AMISR)
- Approaches:
  - Parabolic phase front (like Chirp)
  - Binary phase coding

The Fourier Analogy (2)

One application - beam broadening



Parabolic phase front: Details

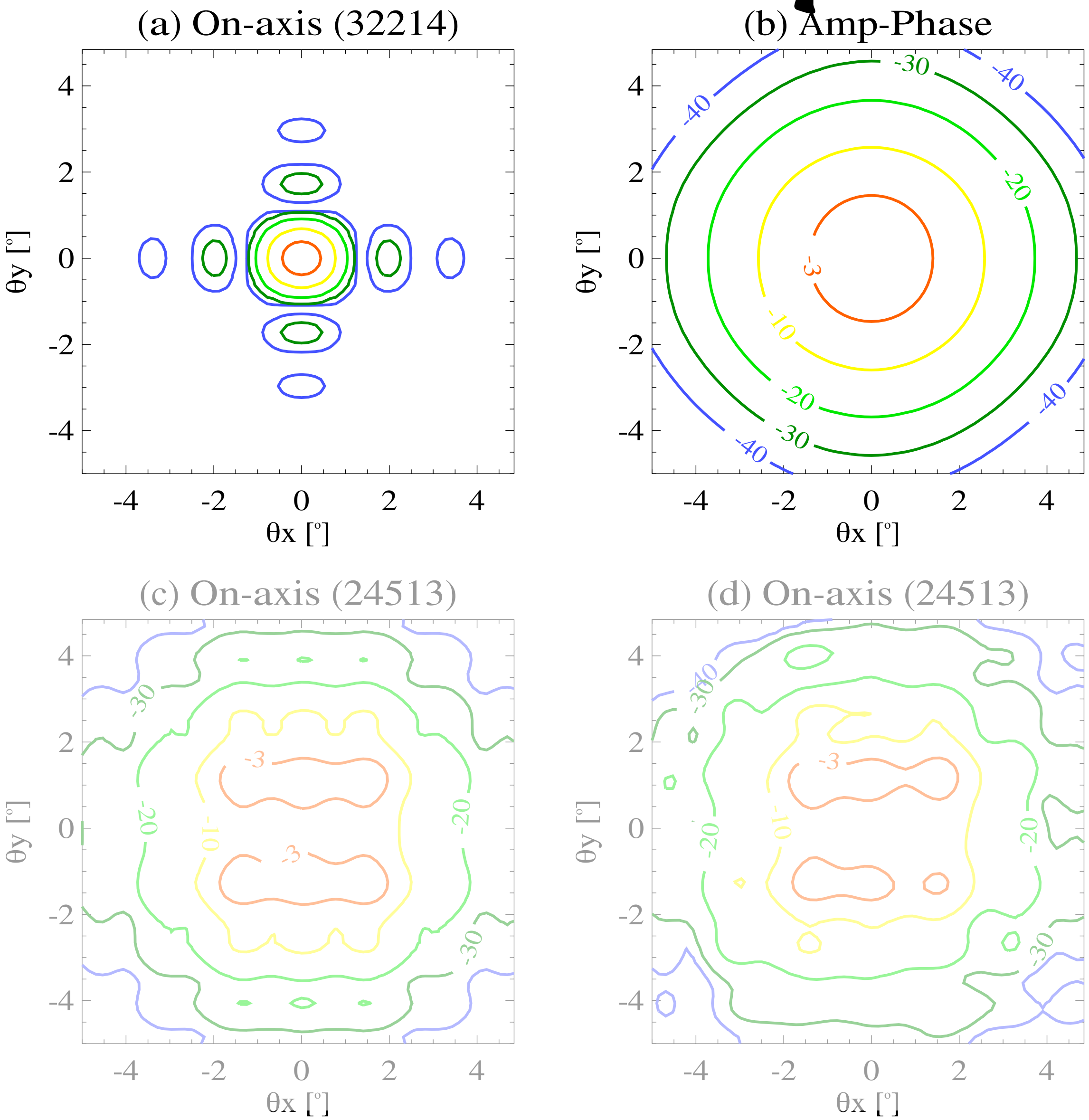
- Recall

$$F_{array}(\theta_x, \theta_y) = \sum_{i=1}^{M} g_i e^{jk(x_i\theta_x + y_i\theta_x) + j\phi_i}$$

- Wider beams can be obtained by using parabolic phase fronts.

$$\phi_i = \phi_{ox} \times (x_i - \bar{x})^2 + \phi_{oy} \times (y_i - \bar{y})^2$$

Antenna compression at PFISR



- Wide beam ~3 times wider!
- Determine which meteors in the narrow beam are coming from sidelobes (~15 %)
- Increase number of large cross-section meteor detections

[from *Chau et al.*, 2009]

Antenna Compression: Complementary 2D Binary Coding

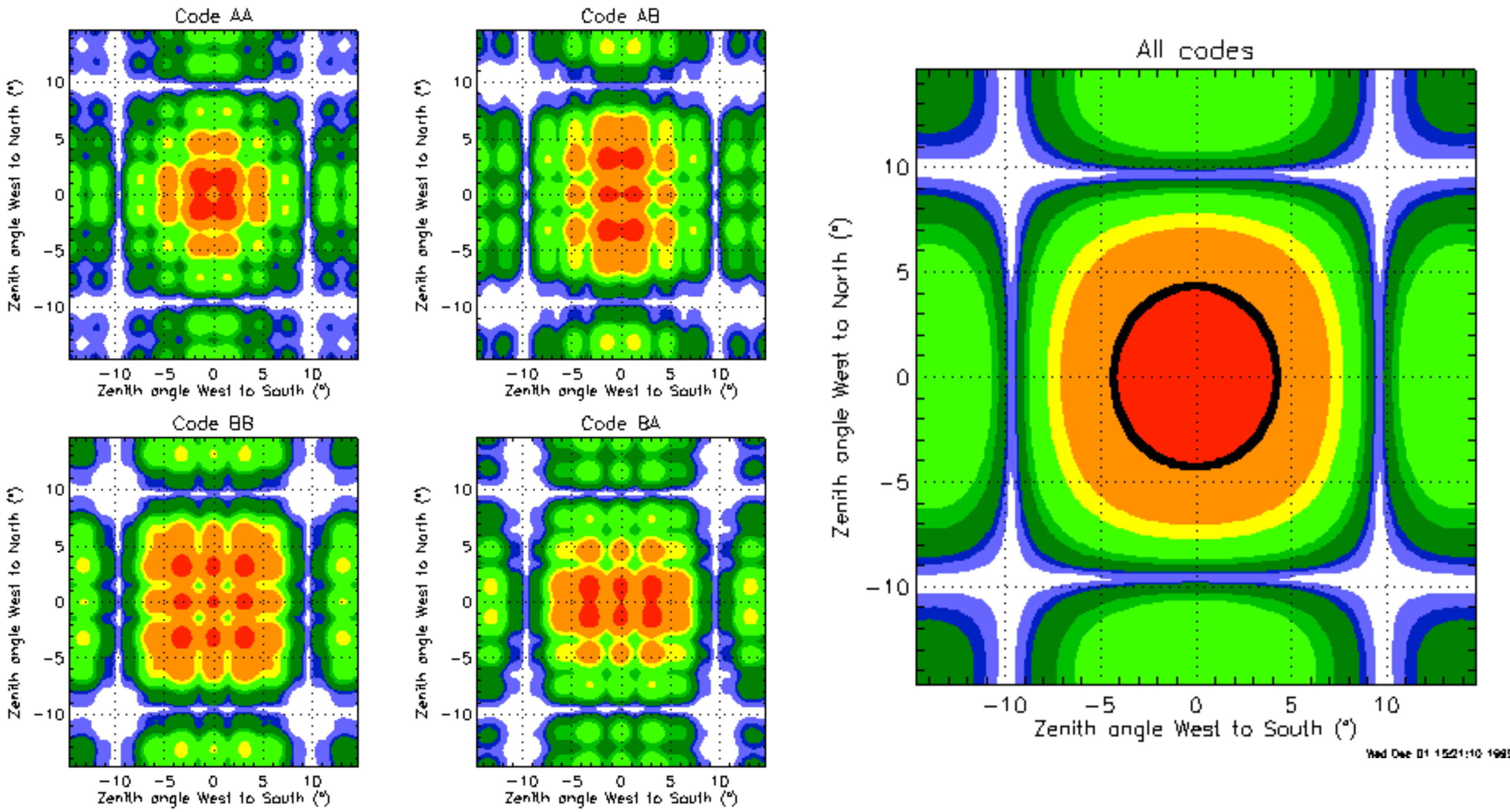
- Evolution from 1D complementary codes (A and B).
- Different sets are obtained by finding all combinations of A and B (i.e., AA, BB, AB, BA).
- Transmission is performed with each 2D code.
- Decoding is performed by adding the second order statistics of each code, the results is equivalent to using one module for transmission.

Binary coding : Antenna Codes

A\A	1	1	1	-1	1	1	1	-1	A/B
1	1	1	1	-1	1	1	1	-1	1
1	1	1	1	-1	1	1	1	-1	1
1	1	1	1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	1	1	1	1	-1	1
1	1	1	-1	1	1	1	-1	1	1
1	1	1	-1	1	1	1	-1	1	1
-1	-1	-1	1	-1	1	1	-1	1	1
1	1	1	-1	1	-1	-1	1	-1	-1
B/B	1	1	-1	1	1	1	-1	1	B\A

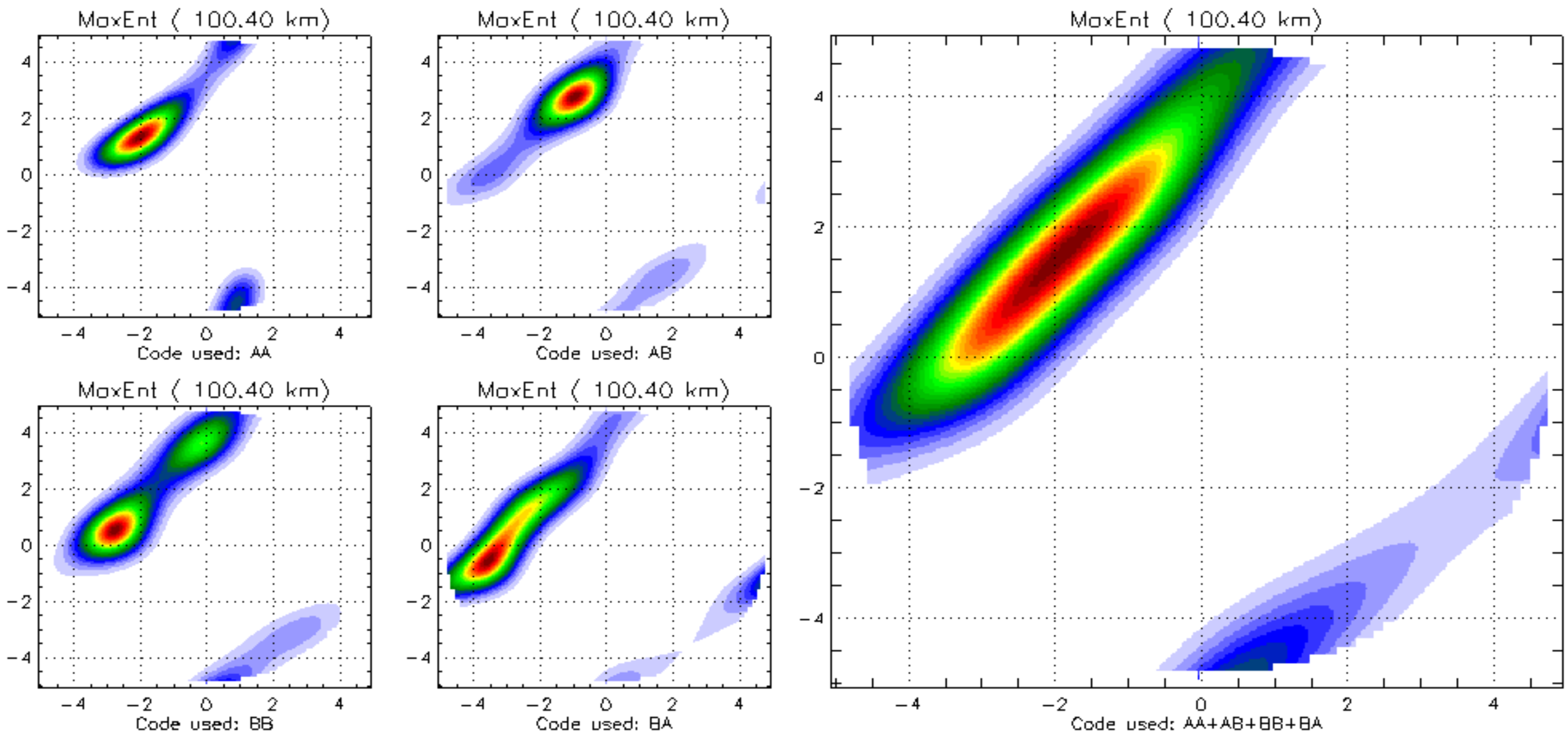
[from *Woodman and Chau*, 2001]

Binary coding: Antenna Patterns



Binary coding: EEJ Results at Jicamarca

(before and after adding statistics)



[from *Chau et al.,* 2009]

What are the Measurement Improvements

- Inertia-less antenna pointing
  - Pulse-to-pulse beam positioning
  - Supports great flexibility in spatial sampling
  - Helps remove spatial/temporal ambiguities
  - Eliminates need for predetermined integration (dish antenna dwell time)
  - Opens possibilities for in-beam imaging through, e.g., interferometry