ISR Theory: Part 2

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(with thanks to Phil Erickson and Josh Semeter)

Basic Structure (1)

- Scattering model
  - Density fluctuations
  - ISR as a Bragg technique
- The Radar Equation
  - Hard and "soft" targets
- Detectability of the scatter
  - Original ideas
  - What really happens
- The plasma wave spectrum
  - Relationship to plasma parameters
- Pulsed radar concepts
  - Range resolution
  - Time/distance
  - Frequency/velocity

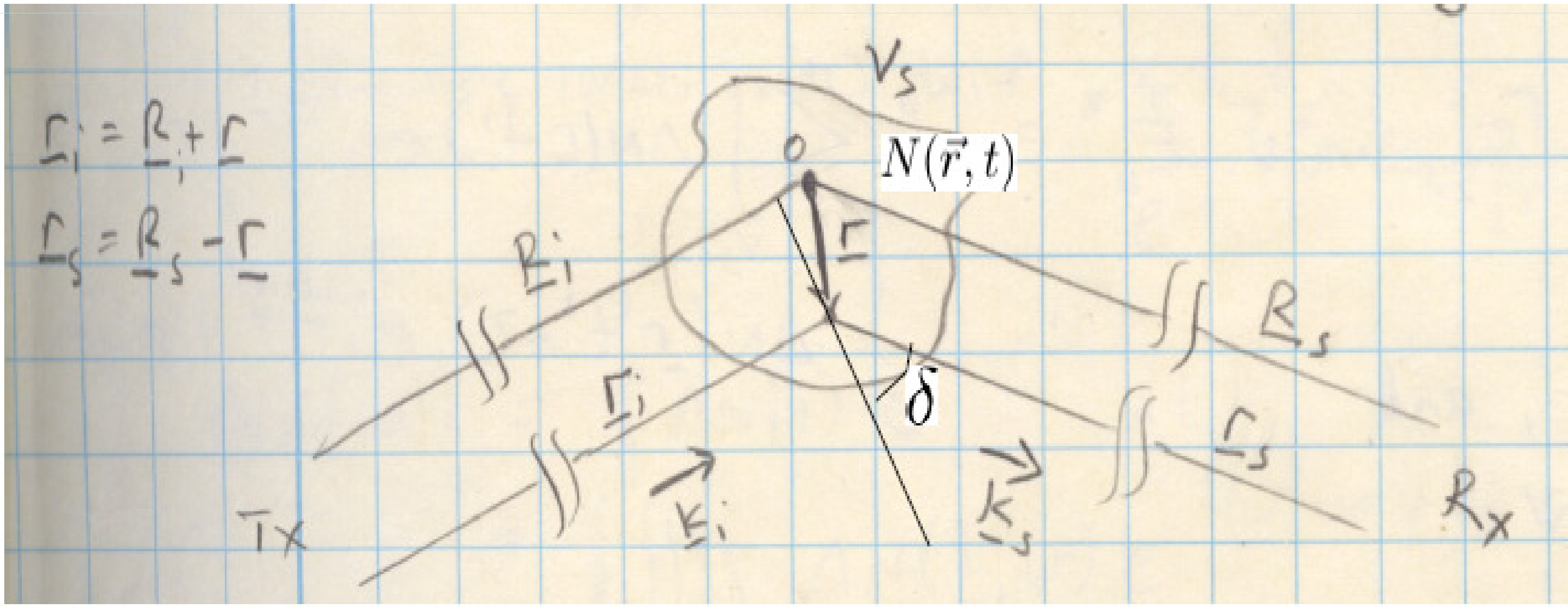
Basic structure (2)

- Doppler measurement
  - Intuitive approach
  - IQ technique
- Autocorrelation functions
  - Overspread targets
  - Why single pulses are needed
  - Why single pulses are no good
- Constraining Factors
  - How far to calculate the ACF?
  - Correlation times vs scale heights
- Clutter

Single Electron Scattering Model

Incident EM wave accelerates each charged particle it encounters.
These then re-radiate an EM wave.

For a single electron located at r = 0, we need the scattered field at a distance r_s.



Single Electron Scattering Model

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For a single electron located at r = 0, the scattered field at a distance r_s :

scattered field $\left|\vec{E}_s(\vec{r_s}, t)\right| = \frac{e^2 \mu_0 \sin\delta}{4\pi R m_e} \left|\vec{E}_i(0, t')\right|$ Incident field

$$= \frac{r_e}{R} \sin\delta \left|\vec{E}_i(0, t')\right|$$

$$r_e = \frac{e^2 \mu_0}{4\pi m_e}$$ Classical electron radius

$$t' = t - \frac{R}{c}$$ Delayed time

$\sin\delta$ Scattering angle

Scattering Model

Assume a volume filled with electron scatterers whose density is represented in space and time by

$$N(\vec{r}, t)$$

Illuminating this volume with an incident field from a transmitter location means that each electron contributes to the resulting scattered field, using *Born approximation* (each scatter is weak and does not affect others).

With geometrical considerations, scattered field at receiver location is now:

$$E_s(t) = r_e \sin\delta \; E_0 e^{j\omega_0 t} \int_{V_s} \frac{1}{r_s} N(\vec{r}, t') e^{-j(\vec{k_i} - \vec{k_s})\vec{r}} d^3\vec{r}$$

$$t' = t - \frac{r_i}{c}$$ Delayed time

Scattering Model

Assume densities have random spatial and temporal fluctuations about a background:

$$N(\vec{r},t) \rightarrow N_0 + \Delta N(\vec{r},t)$$

Further, assume backscatter (i.e. monostatic radar):

$$\vec{k} = 2\vec{k_i} \qquad\qquad r_i \equiv r_s = R$$

Then, scattered field reduces to:

$$E_s(t) \rightarrow \frac{r_e}{R}\ \sin\delta\ E_0 e^{j\omega_0 t} \underbrace{\int_{V_s} \Delta N(\vec{r},t')e^{-j\vec{k}\cdot\vec{r}}d^3\vec{r}}_{\equiv \Delta N(\vec{k},t')}$$

Scattering Model

Plasmas (ionosphere) are thermal gases and $\Delta N(\vec{r},t)$ is a Gaussian random variable, so the Central Limit Theorem applies:

statistical average → $\langle E_s(t)\rangle = \langle \Delta N(\vec{r},t)\rangle = 0$

It's much more useful to look at second order products – in other words, examine temporal correlations in the scattered field:

$$\langle E_s(t)\ E_s^*(t+\tau)\rangle \propto e^{-j\omega_0\tau}\left\langle \Delta N(\vec{k},t)\ \Delta N^*(\vec{k},t+\tau)\right\rangle$$

Useful things to measure can now be defined.

Scattering: Measurable Quantities

Defining $C_s = \frac{r_e^2 E_0^2 \sin^2 \delta}{R^2} V_s$, then

Total scattered power

$$\left\langle |E_s(t)|^2 \right\rangle = C_s \left\langle \left| \Delta N(\vec{k}) \right|^2 \right\rangle$$

and Autocorrelation function (ACF):

$$\langle E_s(t) E_s^*(t+\tau) \rangle = C_s e^{-j\omega_0 \tau} \left\langle \Delta N(\vec{k}, t) \Delta N^*(\vec{k}, t+\tau) \right\rangle$$

or Power Spectrum:

$$\left\langle |E_s(\omega_0 + \omega)|^2 \right\rangle \propto C_s \left\langle \left| \Delta N(\vec{k}, w) \right|^2 \right\rangle$$

Incoherent Scattering Model: Summary

Radar filters in k space:

$$\Delta N(\vec{r}, t) \rightarrow \Delta N(\vec{k}_r, t)$$

$$\Delta N(\vec{k}_r, t) \propto E_s(t)$$

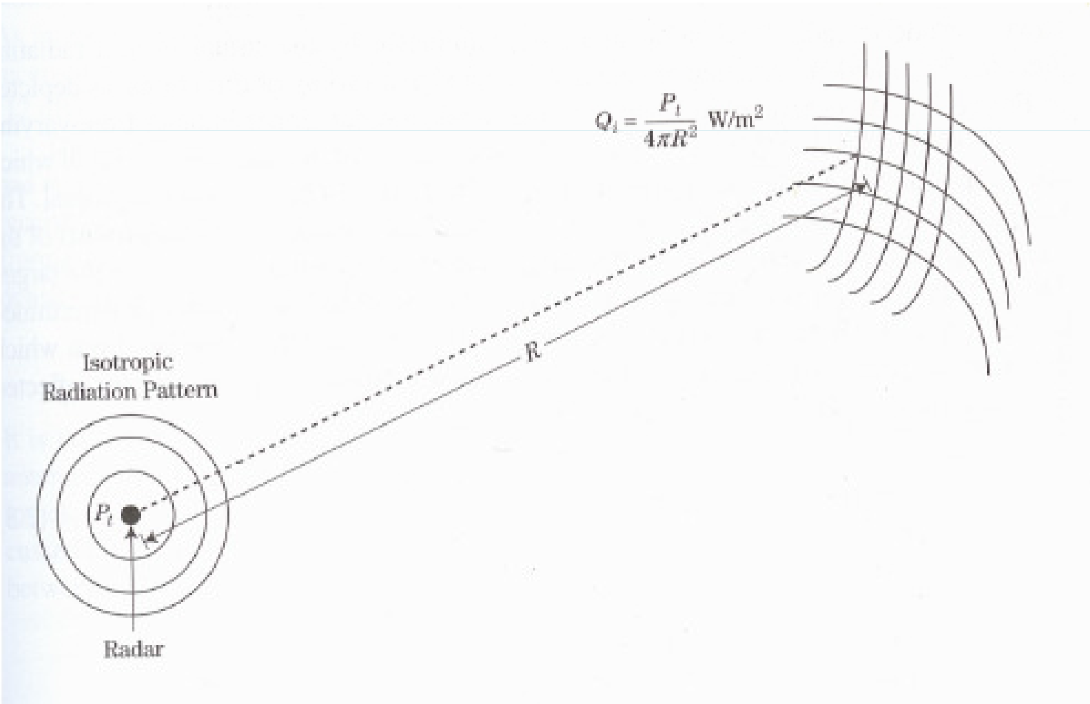
Form ACF of $E_s(t)$ for each range, average, transform:

$$\langle E_s(t) E_s^*(t+\tau) \rangle \rightarrow \left\langle \left| \Delta N(\vec{k}, w) \right|^2 \right\rangle$$

Interpret latter in terms of the medium parameters.

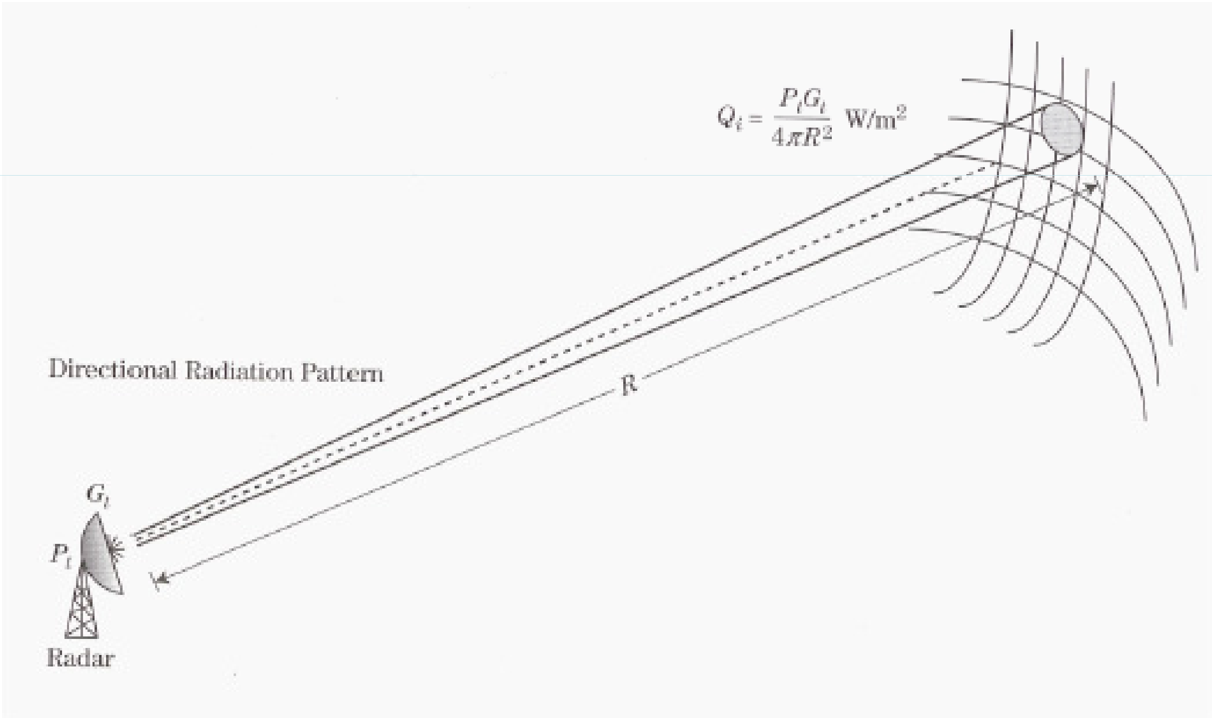
The Radar Equation: Monostatic Version

Power density at range R (isotropic): $$\frac{P_t}{4\pi R^2}$$



The Radar Equation: Monostatic Version

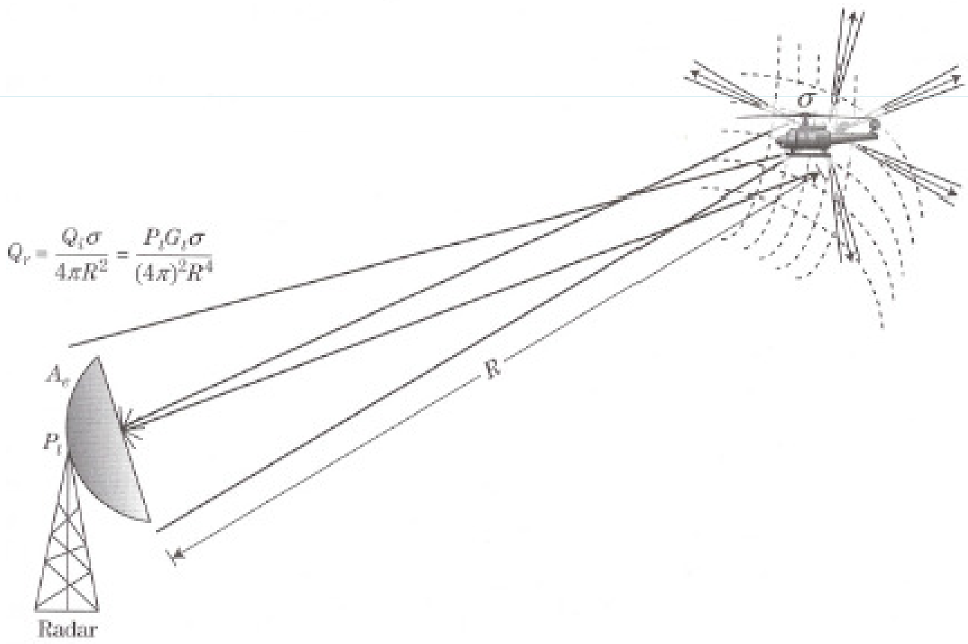
Power density at range R (directional): $\dfrac{P_t\ G}{4\pi R^2}$



The Radar Equation: Monostatic Version

Radar cross section (m^2)

Reradiated power density at Rx: $$\frac{P_t\ G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$



The Radar Equation: Monostatic Version

Total received power: $$P_r = \frac{P_t\ G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e = \frac{P_t\ G\ A_e\ \sigma}{(4\pi)^2 R^4}$$

Use gain/area relation -

The Radar Equation:

$$P_r = P_t\ \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4}\ \sigma$$

Maximum range form:

$$R_{max} = \left[\frac{P_t}{S_{min}}\ \frac{\rho_a^2 A^2}{4\pi\lambda^2}\ \sigma\right]^{\frac{1}{4}}$$

Hard vs Soft Radar Targets

Generalize radar equation for one or more scatterers, distributed over a volume:

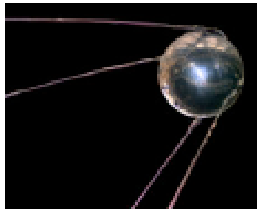
$$P_r = \int P_t \, \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4} \, \sigma(\vec{x}) \, dV_s$$

First case: single scatterer ("hard target") at single point in space:

$$\int \sigma(\vec{x}) \, dV_s = \sigma_{target} \equiv \sigma$$

Hard target radar equation:

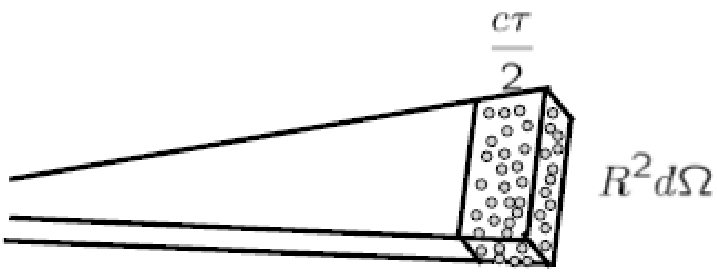
$$P_r = P_t \, \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4} \, \sigma$$


Sputnik 1 (1957-10-04)

Distributed Targets

$$\int \sigma(\vec{x})\ dV_s = \int_0^{2\pi}\int_0^{\pi} \sigma(\vec{x})\ \frac{c\tau}{2}\ R^2 d\Omega$$

$$\int \sigma(\vec{x})\ dV_s = \frac{c\tau}{2}\int_0^{2\pi}\int_0^{\pi} \sigma(\vec{x})\ R^2 \sin\theta\ d\theta\ d\phi$$



Assume volume is filled with identical, isotropic scatters

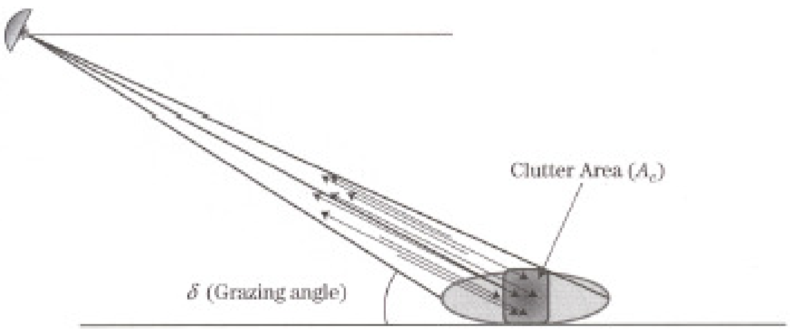
$$\int \sigma(\vec{x})\ dV_s = \frac{c\tau}{2} R^2 \sigma$$

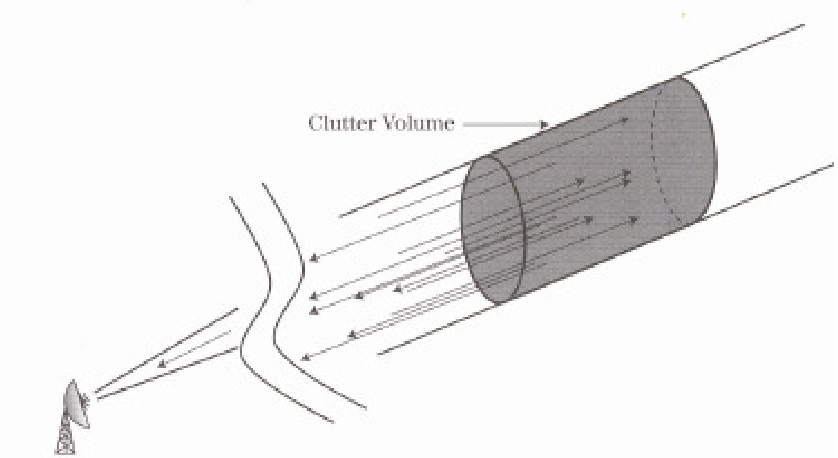
Distributed Scatterers

$$P_r = P_t \, \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4} \, \sigma \, \frac{c\tau}{2} R^2$$

The "soft target" Radar Equation

$$P_r = P_t \, \frac{c\rho_a^2 A^2 \tau}{8\pi\lambda^2 R^2} \, \sigma$$





Remote sensing a plasma: The experimental (radar) view

Suppose we transmit a wave towards a plasma and measure the scattered wave:

$$P_{rec} = (P_{inc}) A_{scat} (\frac{A_{rec}}{4\pi R^2})$$

$$A_{scat} = \sigma_{radar} V_s$$ (ionosphere is a beam filling target)

$$\sigma_{radar} = 4\pi \sigma_{total}$$ (Solid angle)

$$\left(\frac{P_{rec}}{P_{inc}}\right)\left(\frac{4\pi R^2}{A_{rec}}\right)\left(\frac{1}{V_s}\right) = 4\pi r_e^2 \sin^2\delta \left\langle |\Delta N(k)|^2 \right\rangle$$

Measurable experimentally

Detectability of scatter from ionospheric plasma

Assume a beam filling plasma at F region altitudes (300 km) with very high electron density (1E12 electrons per m3):

Classical electron scattering cross-section $\sigma_e = 10^{-28} m^2/e^-$

Assume a pulse length of 10 km.
Assume a cross-beam width of 1 km (~ Arecibo).

$$\sigma_{tot} \sim 10^{-6} m^2$$

NB: Born approximation is very valid, since total amount of scattered power in the volume ~ 1E-12

Detectability of scatter from ionospheric plasma

For fraction of scattered power actually received, assume isotropic scatter and a BIG 100 m class antenna:

$$f_{rec} = \frac{A_{rec}}{4\pi R^2} \sim \frac{10^4 m}{4(300 \times 10^3 m)^2}$$

About -80 dB (1E-8): not much. So:

$$\frac{P_{rec}}{P_{tx}} \sim 10^{-20}$$

So a radar with 1 MW transmitted signal receives 10 femtowatts of incoherently scattered power from free electrons in the ionosphere.

REALLY not very much.

Detectability of scatter from ionospheric plasma

What matters, though, is the signal to noise ratio:

$$P_{noise} = (k_B T_{eff})\,(BW)$$

Typical effective noise temperatures ~100 to 200 K at UHF frequencies (430 MHz, say).

Assume the bandwidth is set by thermal electron motions in a Boltzmann sense:

$$3k_B T_e \sim m_e v_{e,th}^2$$

$$v_{e,th} \sim \sqrt{\frac{3k_B T_e}{m_e}} \sim 2 \times 10^5 m/s$$

$$BW \sim (v_{e,th})\,(2)(2)(\frac{f_{tx}}{c}) \sim 10^6 Hz$$

Detectability of scatter from ionospheric plasma

Finally,

$$P_{noise} \sim 2 \times 10^{-15} W$$

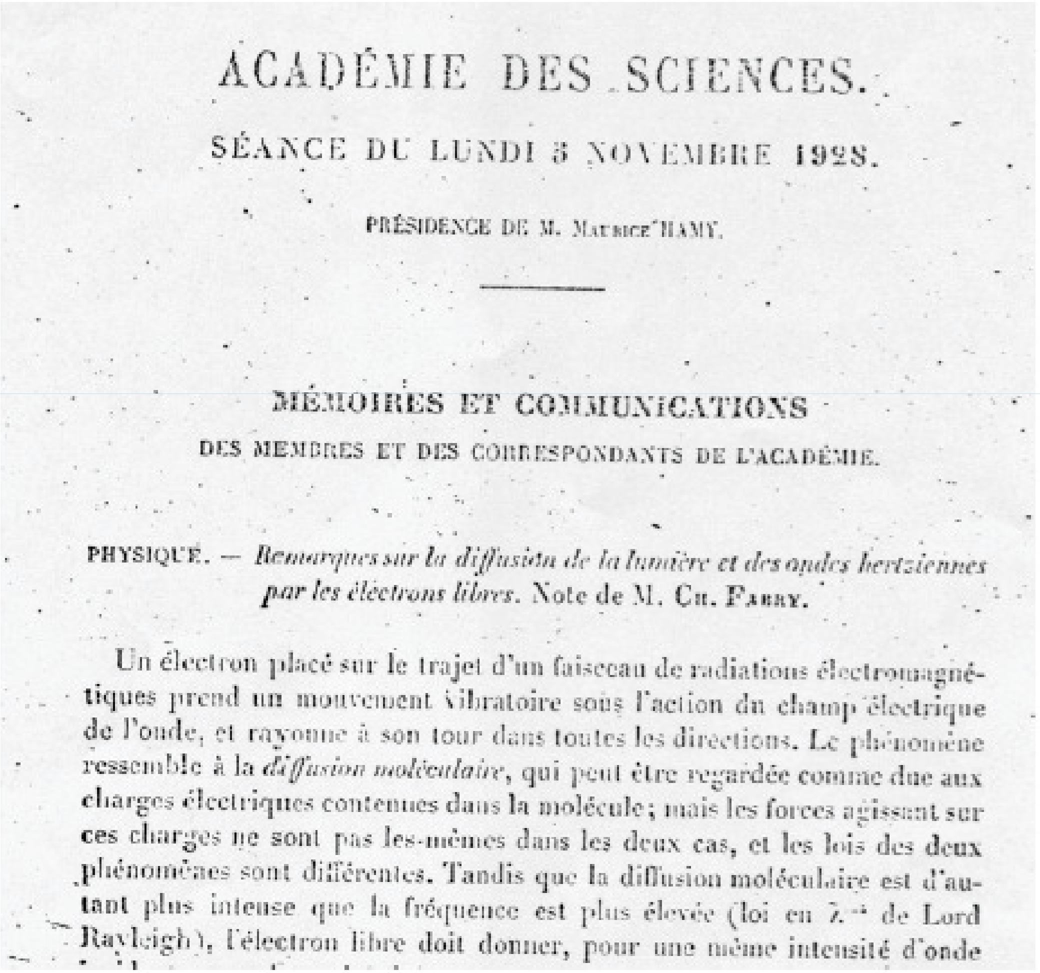
$$S/N \sim 5$$

Workable!

But you need a megawatt class transmitter and a huge antenna.

1950s: technology makes this possible (radio astronomy + construction = large antennas, military needs = high power transmitters)

Incoherent Scatter Concepts Are Older Than You Think



ACADÉMIE DES SCIENCES.

SÉANCE DU LUNDI 5 NOVEMBRE 1928.

PRÉSIDENCE DE M. MAURICE HAMY.

MÉMOIRES ET COMMUNICATIONS
DES MEMBRES ET DES CORRESPONDANTS DE L'ACADÉMIE.

PHYSIQUE. — *Remarques sur la diffusion de la lumière et des ondes hertziennes par les électrons libres.* Note de M. CH. FABRY.

Un électron placé sur le trajet d'un faisceau de radiations électromagnétiques prend un mouvement vibratoire sous l'action du champ électrique de l'onde, et rayonne à son tour dans toutes les directions. Le phénomène ressemble à la *diffusion moléculaire*, qui peut être regardée comme due aux charges électriques contenues dans la molécule; mais les forces agissant sur ces charges ne sont pas les mêmes dans les deux cas, et les lois des deux phénomènes sont différentes. Tandis que la diffusion moléculaire est d'autant plus intense que la fréquence est plus élevée (loi en λ^{-4} de Lord Rayleigh), l'électron libre doit donner, pour une même intensité d'onde

Remarques sur la diffusion de la lumière et des ondes hertziennes par les electrons libres

C. Fabry
1928

Charles Fabry
1867-1945

$$\sigma = \frac{8\pi}{3}\frac{e^4}{m^2}$$

Electron scattering cross section (fundamental)

Incoherent Scatter Concepts Are Older Than You Think

Without worrying about noise:
- Rayleigh scattering $\propto \lambda^4$ [why is the sky blue?]
- Incoherent scatter independent of wavelength
  [but it's weak]

Remarques sur la diffusion de la lumière et des ondes hertziennes par les electrons libres

C. Fabry
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Incoherent Scatter concept!

For luminous radiations whose wavelength is very small, there is no phase relation between the elementary waves sent out by the different electrons of even a small volume and it is the intensities which add up. Thus, if a certain volume contains a total number of electrons n, then the power that it scatters is that transmitted by an area S = nσ. With the degrees of ionization that can actually exist, the scattering of light by electrons is always very slight. That is why it plays no appreciable role in the production of light in the diurnal sky *.

First Incoherent Scatter Radar

- W. E. Gordon of Cornell is credited with the idea for ISR.
- *"Gordon (1958) has recently pointed out that scattering of radio waves from an ionized gas in thermal equilibrium may be detected by a powerful radar."* (Fejer, 1960)
- Gordon proposed the construction of the Arecibo Ionospheric Observatory for this very purpose (NOT for radio astronomy as the primary application)

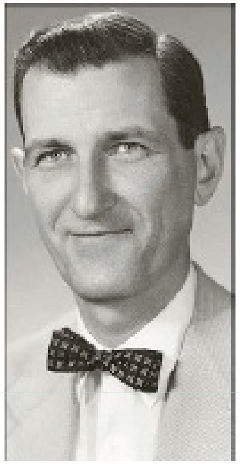
~40 megawatt-acres

- 1000' Diameter Spherical Reflector
  - 62 dB Gain
- 430 MHz line feed 500' above dish
- Gregorian feed
- Steerable by moving feed.

Proceedings of the IRE, November 1958

Incoherent Scattering of Radio Waves by Free Electrons with Applications to Space Exploration by Radar*

W. E. GORDON†, MEMBER, IRE



Introduction

FREE electrons in an ionized medium scatter radio waves incoherently so weakly that the power scattered has previously not been seriously considered. The calculations that follow show that this incoherent scattering, while weak, is detectable with a powerful radar. The radar, with components each representing the best of the present state of the art, is capable of:

1) measuring electron density and electron temperature as a function of height and time at all levels in the earth's ionosphere and to heights of one or more earth's radii;
2) measuring auroral ionization;
3) detecting transient streams of charged particles coming from outer space; and
4) exploring the existence of a ring current.

* Original manuscript received by the IRE, June 11, 1958; revised manuscript received, August 25, 1958. The research reported in this paper was sponsored by Wright Air Dev. Ctr., Wright-Patterson Air Force Base, O., under Contract No. AF 33(616)-5547 with Cornell Univ.

† School of Elec. Eng., Cornell Univ., Ithaca, N. Y.

First Incoherent-Scatter Radar

- **K.L. Bowles [Cornell PhD 1955]**, Observations of vertical incidence scatter from the ionosphere at 41 Mc/sec. *Physical Review Letters* 1958:

"The possibility that incoherent scattering from electrons in the ionosphere, vibrating independently, might be observed by radar techniques has apparently been considered by many workers although seldom seriously because of the enormous sensitivity required…"

First Incoherent-Scatter Radar

…*Gordon (*W.E. Gordon from Cornell*) recalled this possibility to the writer* [spring 1958; D. T. Farley] *while remarking that he hoped soon to have a radar sensitive enough to observe electron scatter in addition to various astronomical objects…"*

Bowles executed the idea - hooked up a large transmitter to a dipole antenna array in Long Branch Ill., took a few measurements.

Gordon presenting on same day at October 21, 1958 Penn State URSI meeting:

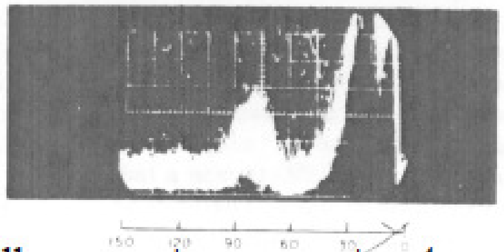
"…And then I want to tell you about a telephone call that I just had."

VOLUME 1, NUMBER 12 PHYSICAL REVIEW LETTERS DECEMBER 15, 1958

Table I. Parameters of radar equipment used.

Operating frequency	40.92 Mc/sec
Peak pulse power	(4 to 6) × 10^5 watts
Pulse duration	(50 to 150) × 10^{-6} sec
Average power	4 × 10^4 watts maximum
Receiver bandwidth	10, 15, or 30 kc/sec
Antenna cross section	116 × 140 meters (1024 half-wave elements in phase above ground)
Antenna polarization	north-south
Calculated antenna gain	~ 35 decibels/isotropic

~6 week setup time



Oscilloscope + camera + ~4 sec exposure (10 dB integration)

FIG. 2. Pulse [illegible] andwidth 30 kc.

Incoherent Scattering Detectability

Bowles' results found approximately the expected amount of power scattered from the electrons (scattering is proportional to charge to mass ratio - electrons scatter the energy).

BUT: his detection with a 20 megawatt-acre system at 41 MHz (high cosmic noise background; should be marginal) implies a spectral width 100x narrower than expected – almost as if the much heavier (and slower) ions were controlling the scattering spectral width.

In fact, they do.

Calculating the fluctuation spectrum

$$\left\langle \left| \Delta N(\vec{k},\omega) \right|^2 \right\rangle = \frac{1}{2} N_0 \frac{k_B T}{m} \left(\frac{k^2}{\omega^2} \right)$$

Insert plasma dispersion relation here

We need the full dispersion relation expression.

This is not a plasma waves course so we won't derive it, but the two most important modes are:

1) *Ion-acoustic fluctuations* [sound waves in plasma]

$$\frac{\omega}{k} = \sqrt{\frac{k_B T_e + \gamma_i k_B T_i}{m_i}} = V_s$$

NB: ordinary acoustic waves: adiabatic compression / decompression of fluid particles.

Ion-acoustic fluctuations: restoring force = electromagnetic

Calculating the fluctuation spectrum

$$\left\langle \left| \Delta N(\vec{k},\omega) \right|^2 \right\rangle = \frac{1}{2} N_0 \frac{k_B T}{m} \left(\frac{k^2}{\omega^2} \right)$$

Insert plasma dispersion relation here

We need the full dispersion relation expression.

This is not a plasma waves course so we won't derive it, but the two most important modes are:

2) *Langmuir oscillations* (Plasma oscillations):

$$\omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{th}^2 \quad v_{th}^2 = 2k_B T_e / m_e$$

Akin to Brunt-Våisålå oscillations in fluid (parcel in presence of density gradient) - here, electrostatic field is restoring force, and electron pressure gradient transmits information

Irving Langmuir (1881 - 1957)

When Langmuir arrived at the Laboratory, the director, Willis R. Whitney, told him to look around and see if there was anything he would like to "play with." Whitney would often ask him, "Are you having any fun today?" One day, after three years of apparently unproductive research, Langmuir answered, "I'm having a lot of fun, but I really don't know what good this is to the General Electric Company." Whitney replied. "That's not your worry. That's mine."

- Thermal fluctuations in an ordinary collision dominated gas can be considered to be made up of sound waves.
- In a plasma, the fluctuations are ion-acoustic waves and electrostatic plasma (Langmuir) waves.

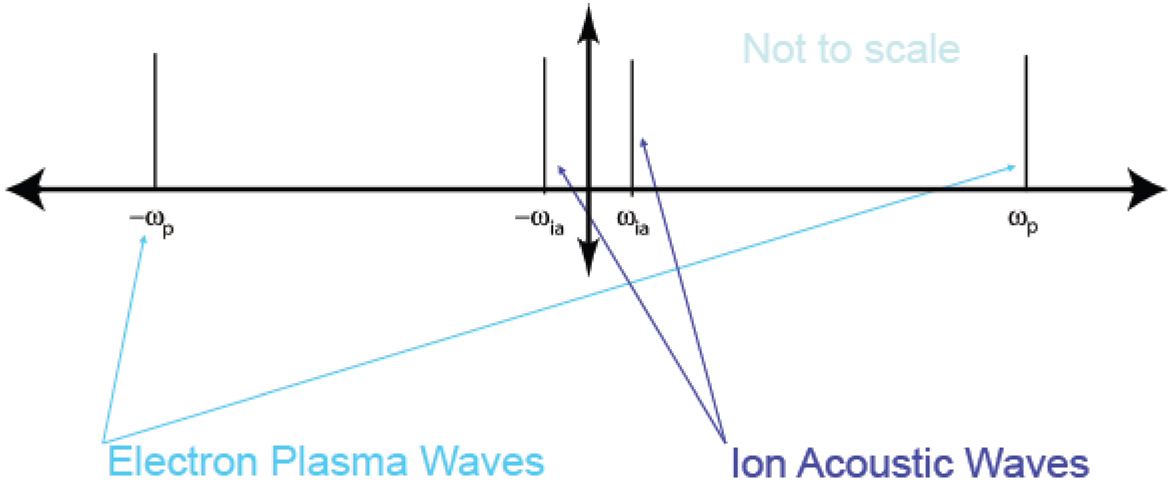
Ions

Thermal velocity

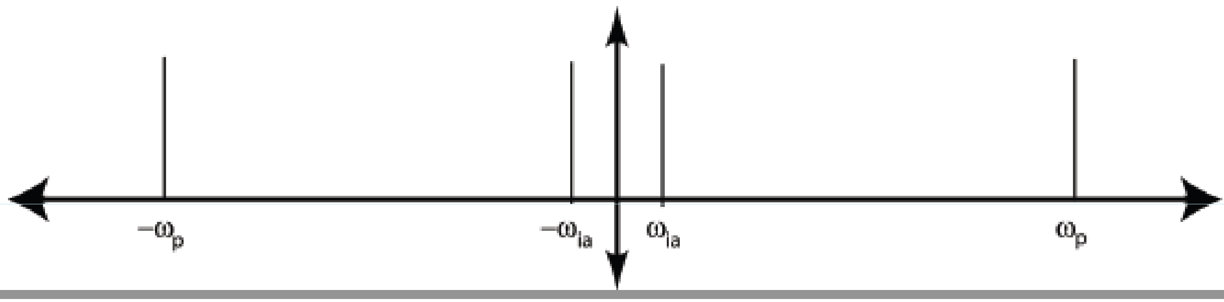
$$v_{TH} = \sqrt{\frac{2kT}{m}}$$

$$v_{THe} = \sqrt{\frac{m_i}{m_e}} v_{THi}$$

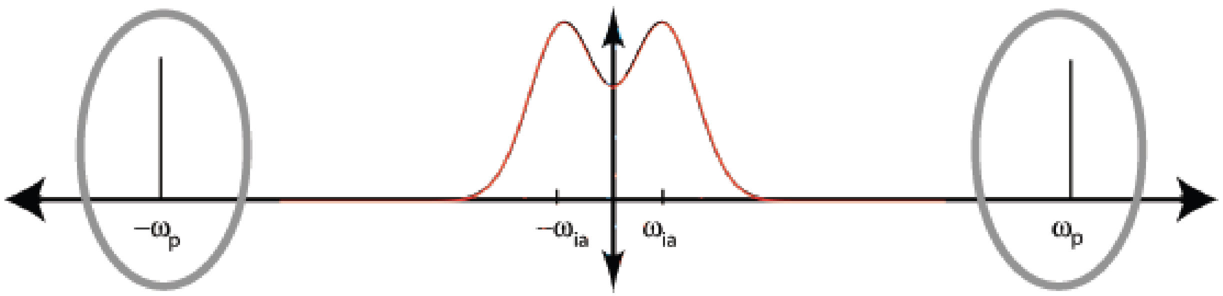
Electron Gas



Wave Spectrum (ISR Spectrum)



Why aren't the Langmuir (plasma) waves damped?
Electron thermal velocity ~ 125 km/s but plasma wave frequency ~ several MHz – Not much interaction and not much damping.



ISR in a nutshell

Here's what we measure:

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

⇐ The "radar equation"

P_r = Received power

P_n = Received noise power

P_t = Transmitted power

σ = Radar cross section

G = Antenna gain

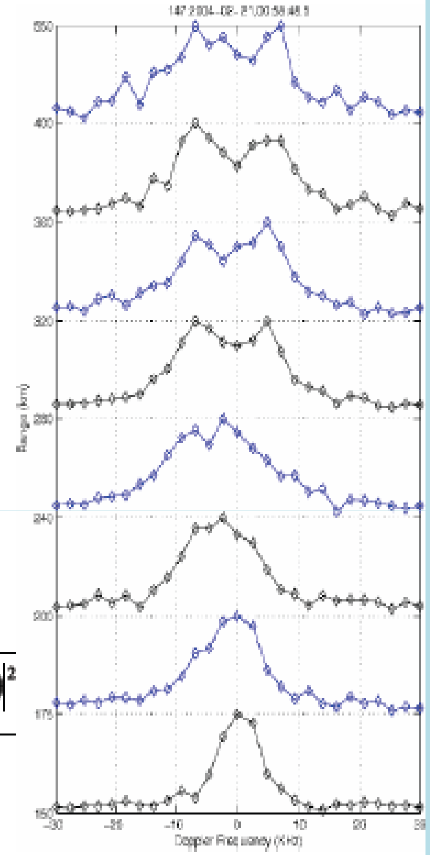
A = Antenna area

k_B = Boltzman's constant

T = Temperature

B = Bandwidth

N_{sys} = System noise temperature



Here's the theory:

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \times F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

$$F_e(\omega) = 1 - \omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$

The ISR model

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \cdot F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

where:

$$F_e(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)d\tau$$

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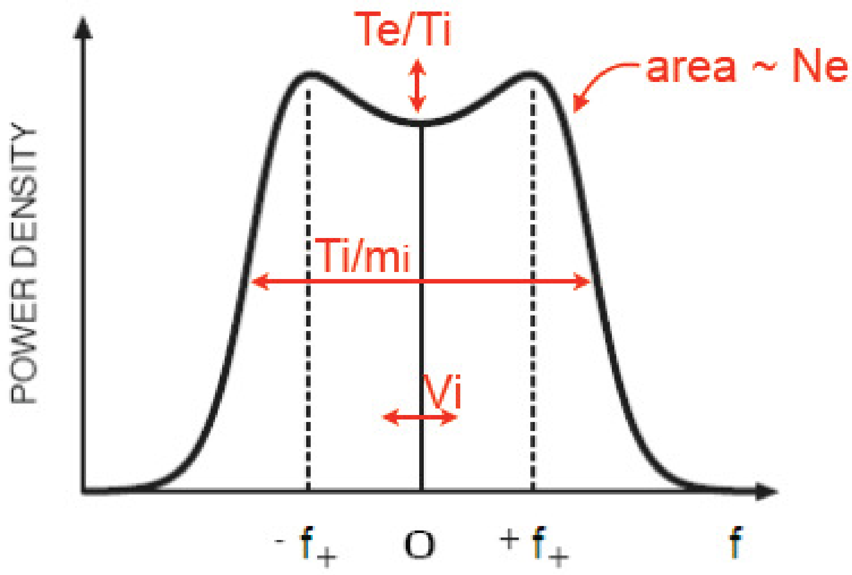
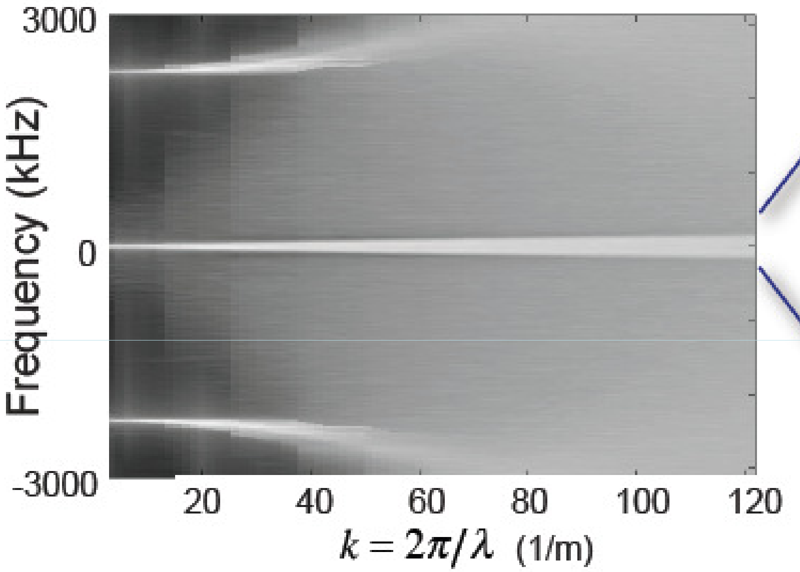
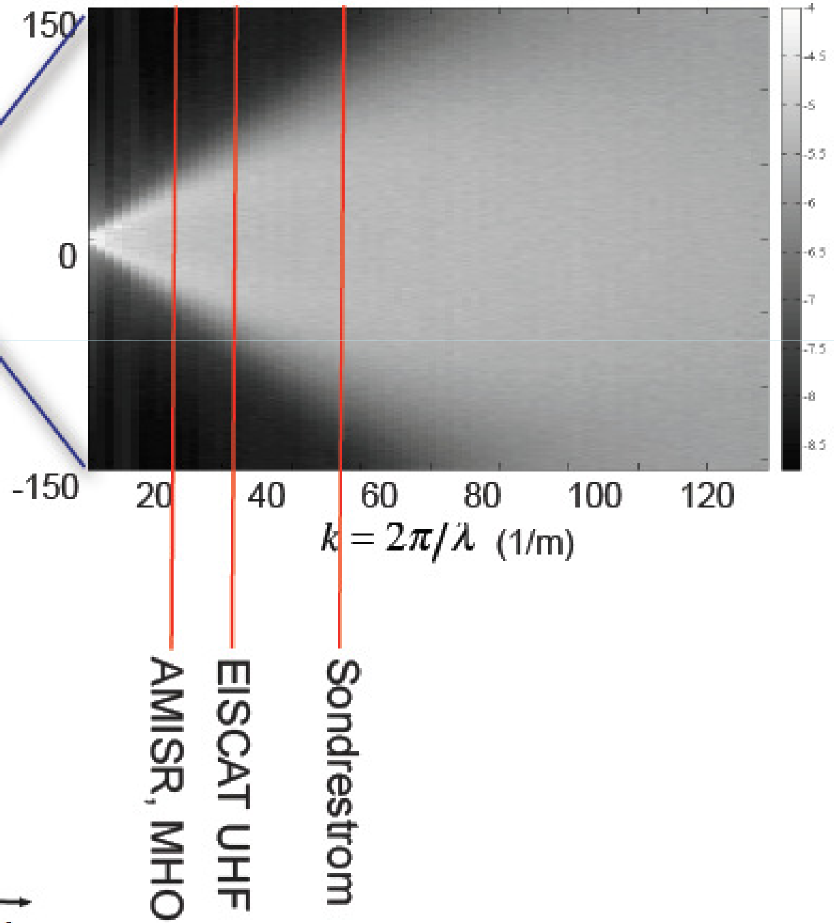
From Evans, IEEE Transactions, 1969

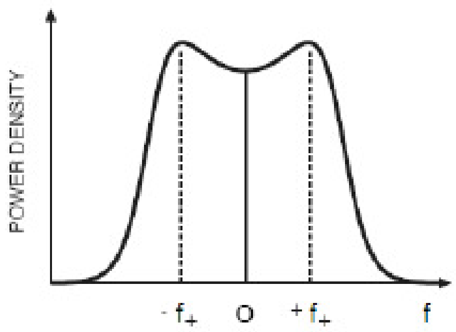


ISR Measures a Cut Through This Surface

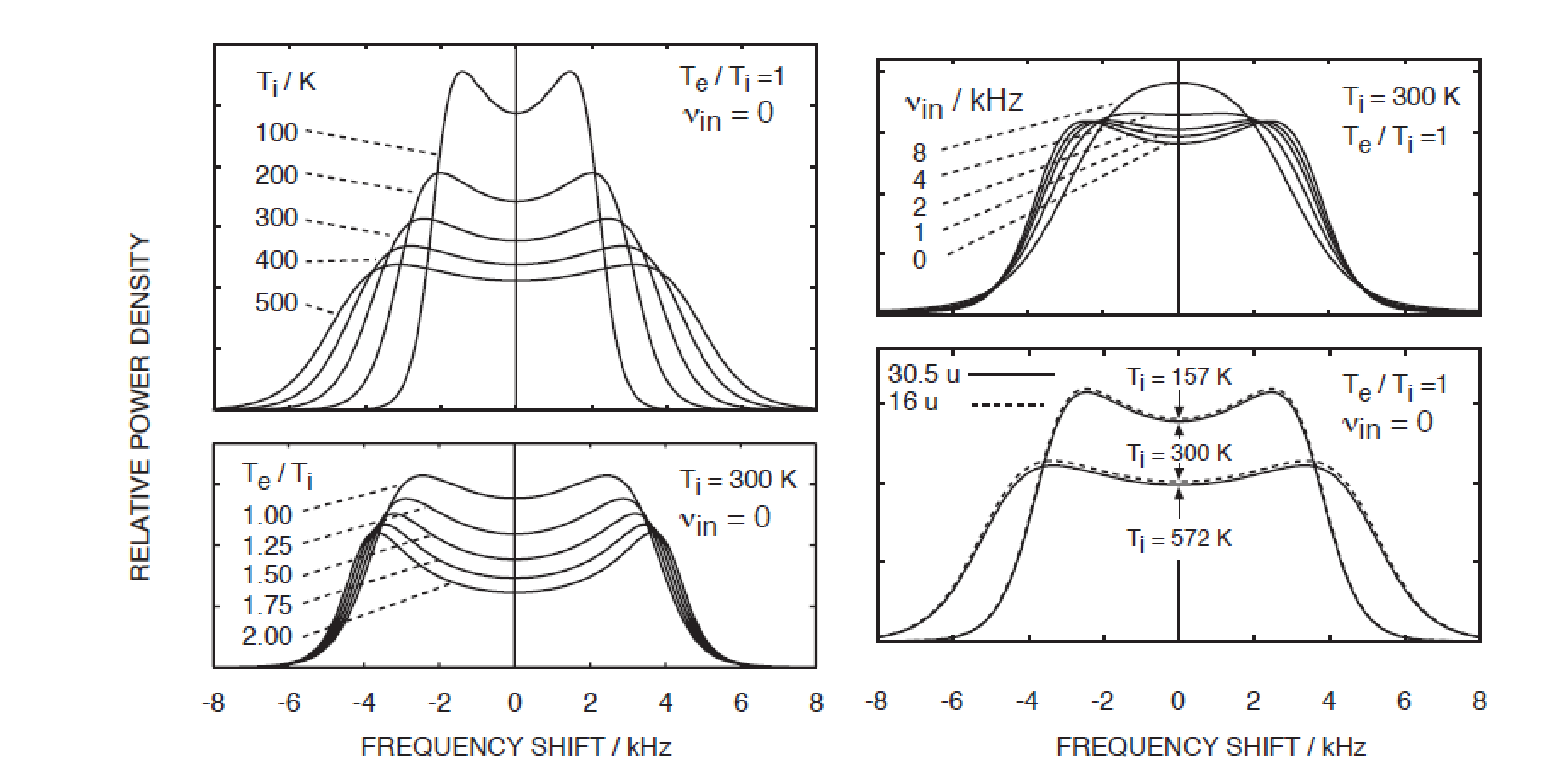




Ion-acoustic "lines" are broadened by Landau damping



Dependence on Plasma Parameters



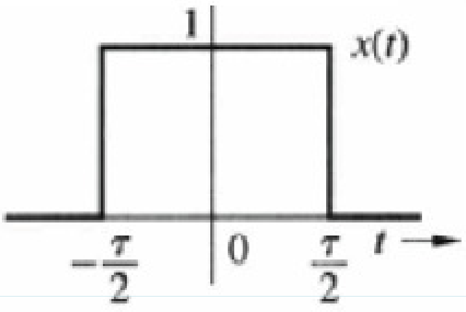
To see how incoherent scatter spectrum depends on the plasma parameters, play with the widget at:

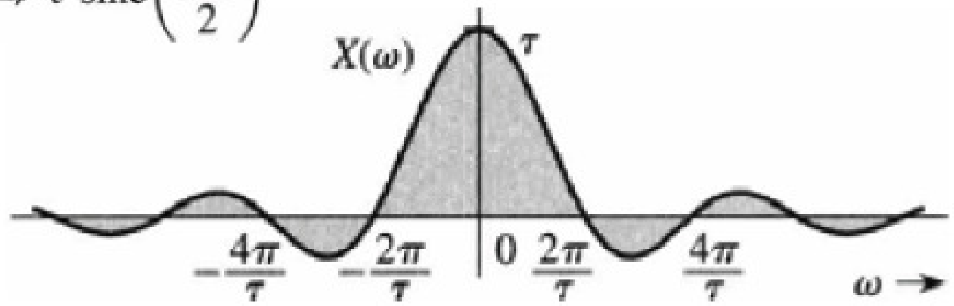
http://madrigal.haystack.mit.edu/madrigal/ISR/spectrum/

Gate function

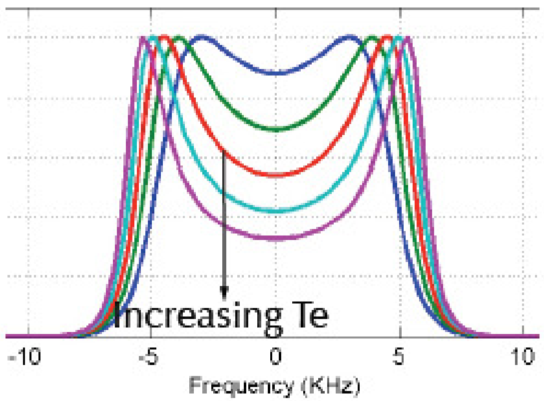
$$\text{rect}(t/\tau) = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

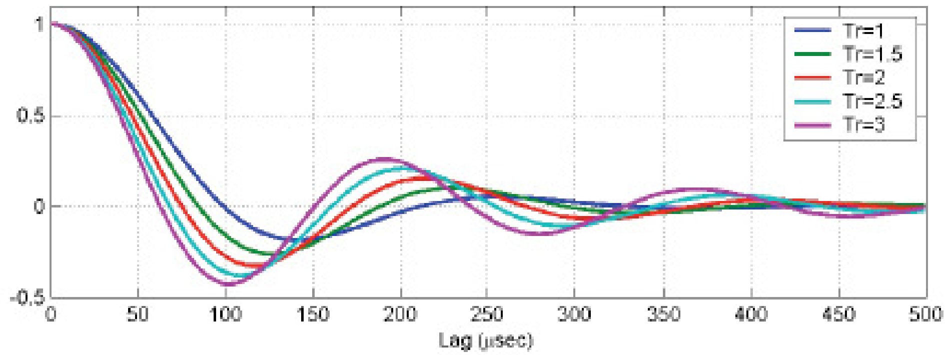
$$\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \,\text{sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\Longleftrightarrow$ Autocorrelation function (ACF)

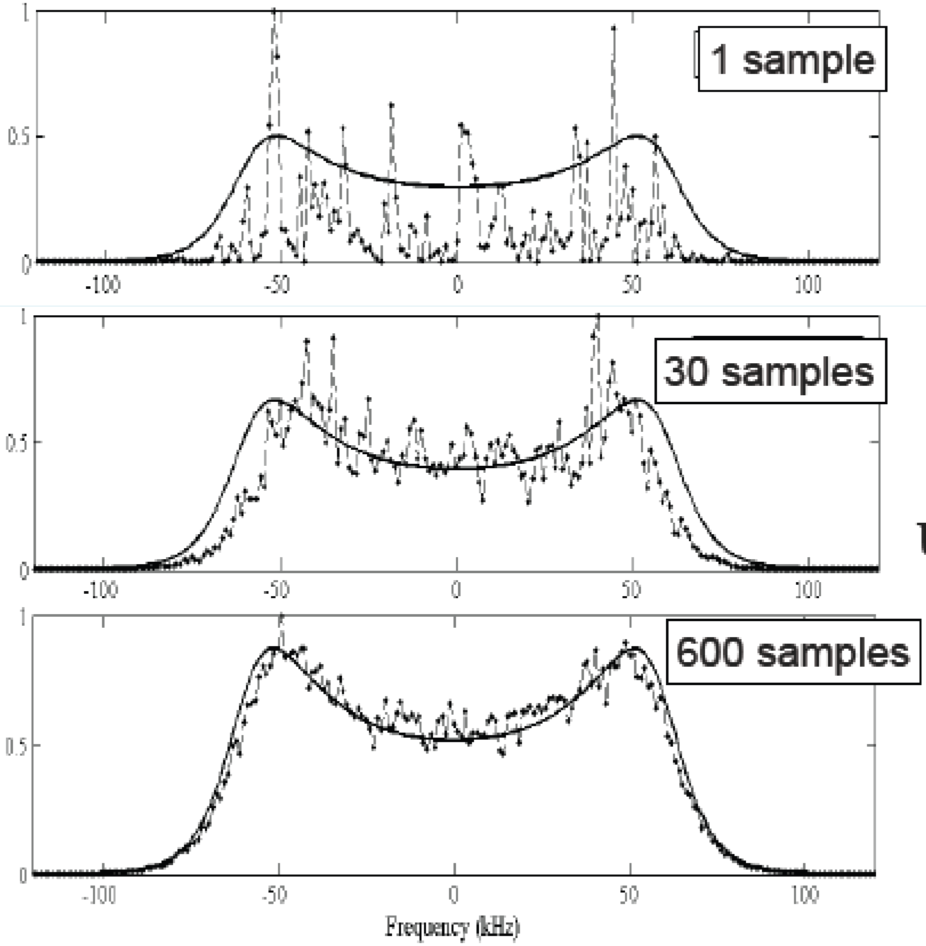




Not surprisingly, the ISR ACF looks like a sinc function…

Incoherent Averaging

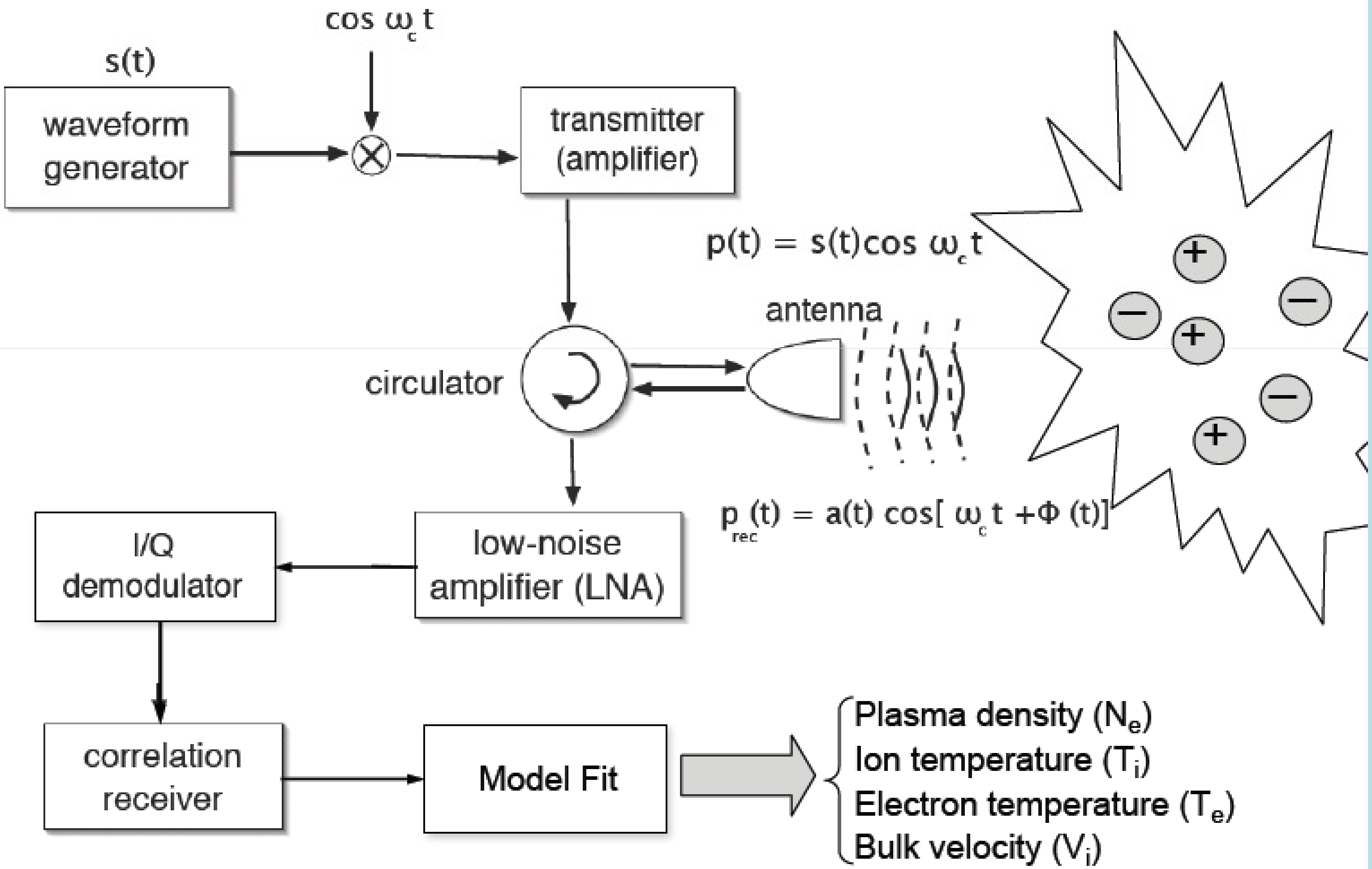
Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

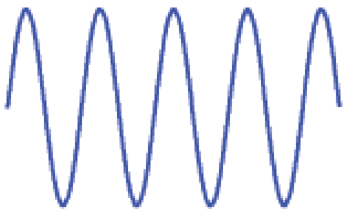
$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

Components of a Pulsed Doppler Radar



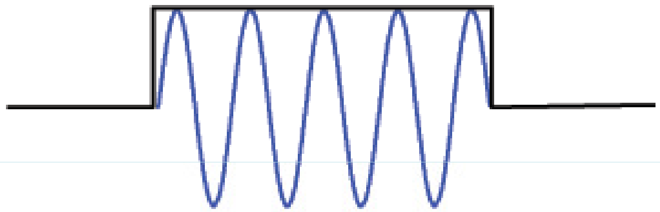
Waves versus Pulses

What do radars transmit?

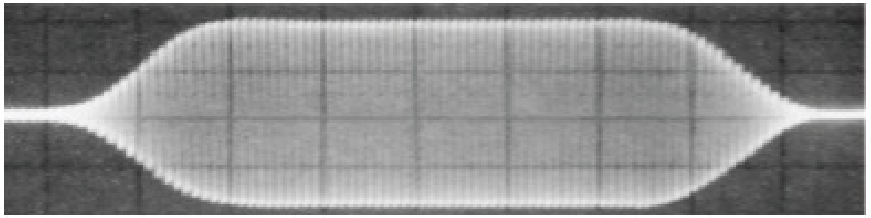


Waves?

or Pulses?



Waves, modulated by "on-off" action of pulse envelope



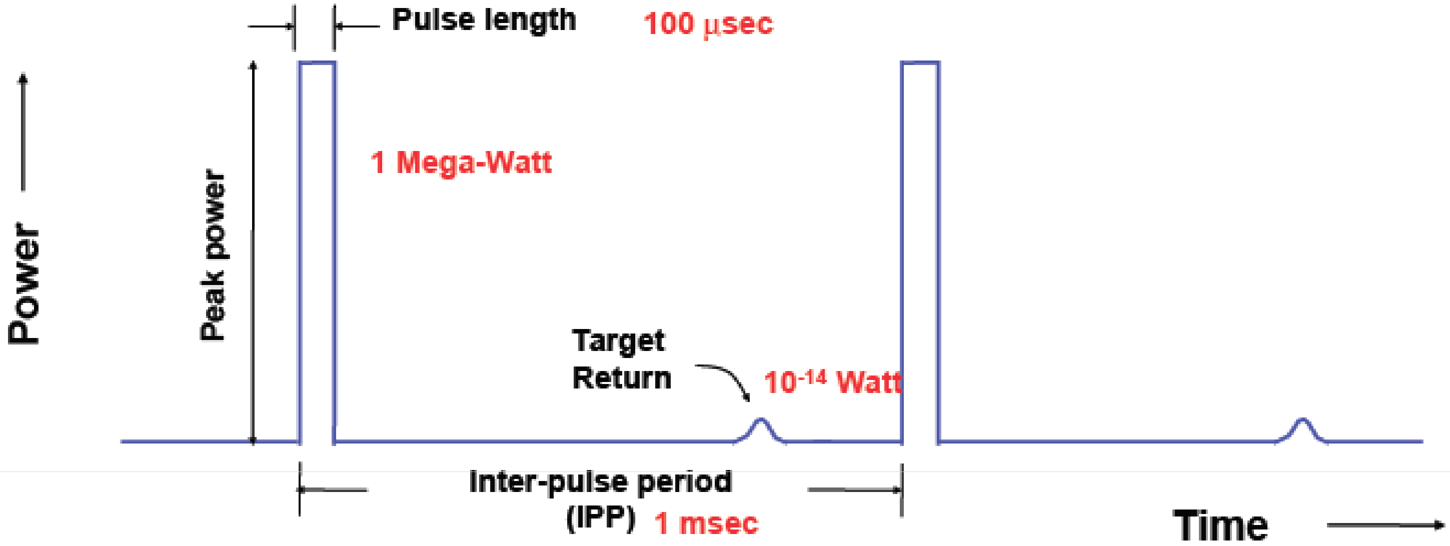
How many cycles are in a typical pulse?

PFISR frequency: 449 MHz

Typical long-pulse length: 480 μs

215,520 cycles!

Pulsed Radar



Duty cycle = $\frac{\text{Pulse length}}{\text{Pulse repetition interval}}$ 10%

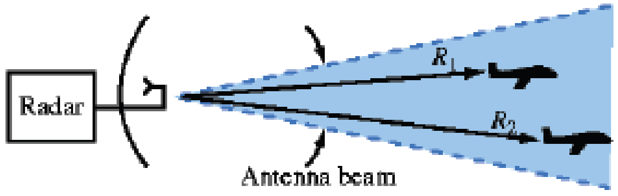
Average power = Peak power * Duty cycle 100 kWatt

Pulse repetition frequency (PRF) = 1/(IPP) 1 kHz

Continuous wave (CW) radar: Duty cycle = 100% (always on)

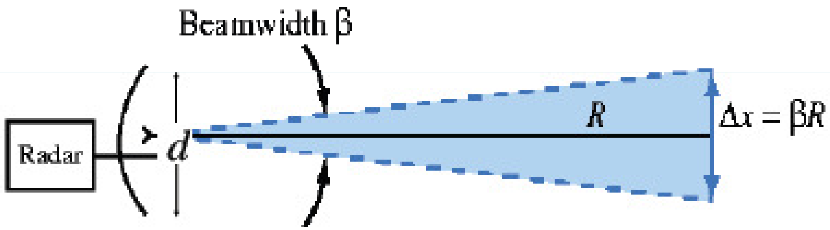
Range

Range resolution: Set by pulse length, given in units of time, τ_p, or length, c τ_p



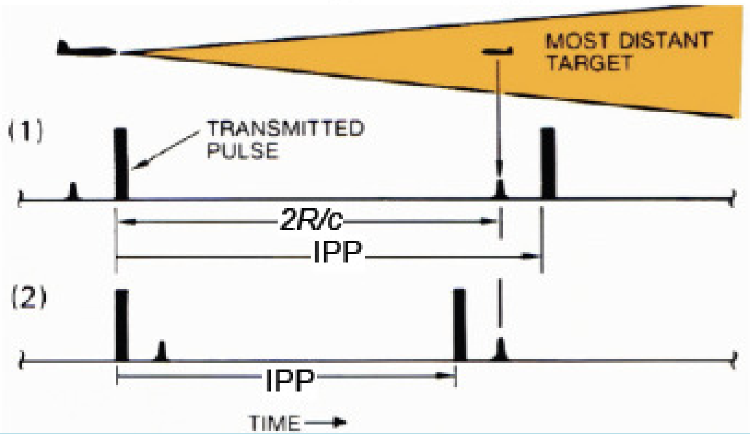
$$\Delta R = R_2 - R_1 = \frac{c\tau_p}{2}$$

Cross-range resolution: Set by "beam width" (in degrees) and target range



$$\beta \approx \frac{\lambda}{d} \text{ radians}$$

Maximum unambiguous range: Set by Inter-pulse Period (IPP)



IPP = Interpulse period (s)
PRF = pulse repetition frequence
= 1/IPP (Hz)

$$R_u = \frac{c \text{ IPP}}{2}$$

Doppler

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o\left(t+\frac{2R}{c}\right)\right]$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + v_o t$$

Substituting:

$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v_o}{c}}_{-f_D}\right)t + \underbrace{\frac{2\pi f_o R_o}{c}}_{\text{constant}}\right]$$

$$\boxed{f_D = \frac{-2f_o v_o}{c} = \frac{-2v_o}{\lambda_o}}$$

By convention, positive Doppler frequency shift $\Longleftrightarrow$ Target and radar closing

Two key concepts

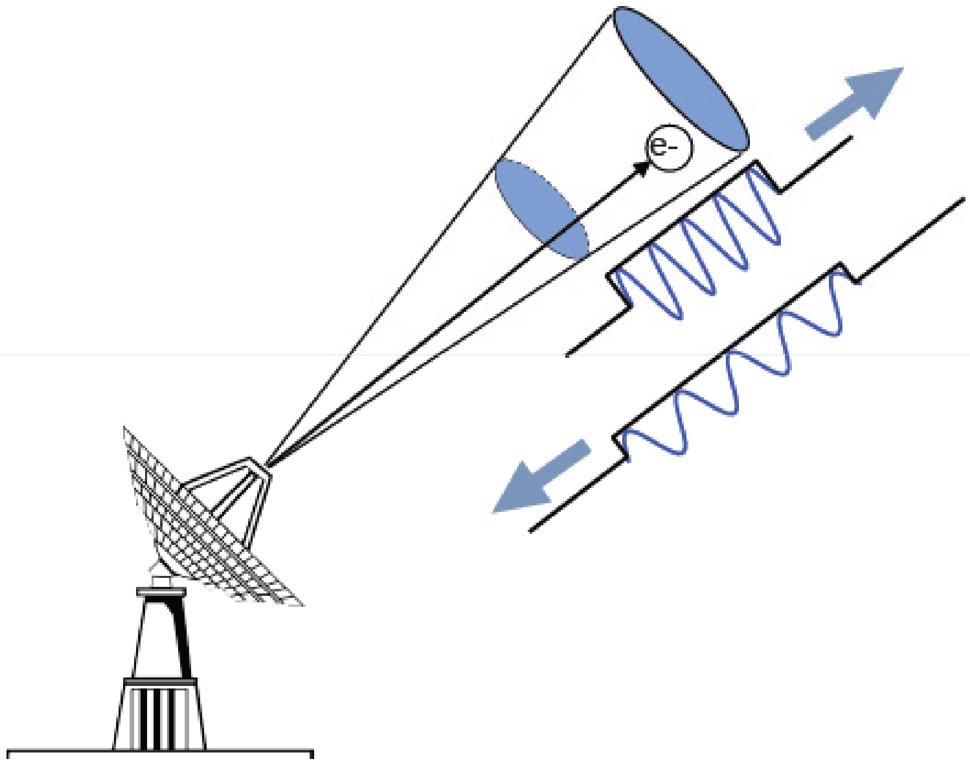
Two key concepts:

Distant ⇔ Time

$$R = c\Delta t/2$$

Velocity ⇔ Frequency

$$v = -f_D \lambda_0 /2$$



A Doppler radar measures backscattered power as a function range and velocity.
Velocity is manifested as a Doppler frequency shift in the received signal.

Two key concepts

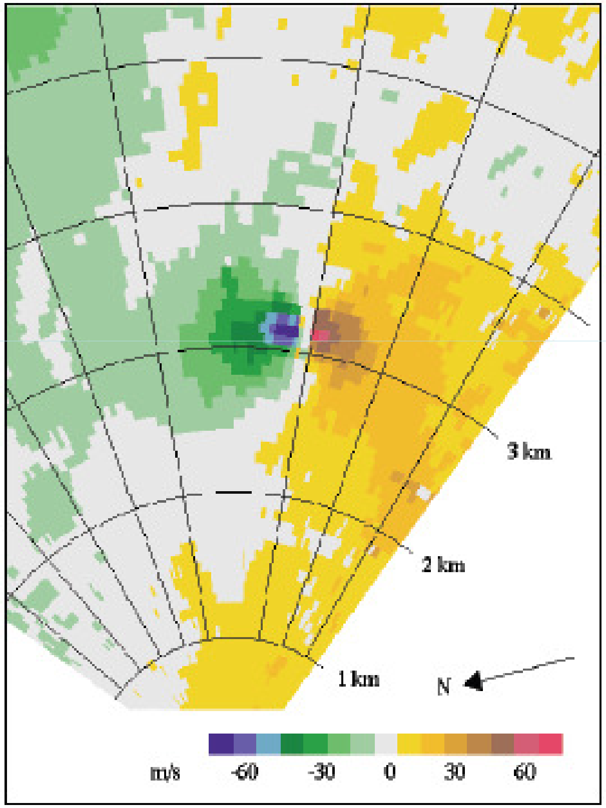
Two key concepts:

Distant ⟺ Time

$$R = c\Delta t/2$$

Velocity ⟺ Frequency

$$v = -f_D \lambda_0 /2$$



A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

Concept of a "Doppler Spectrum"

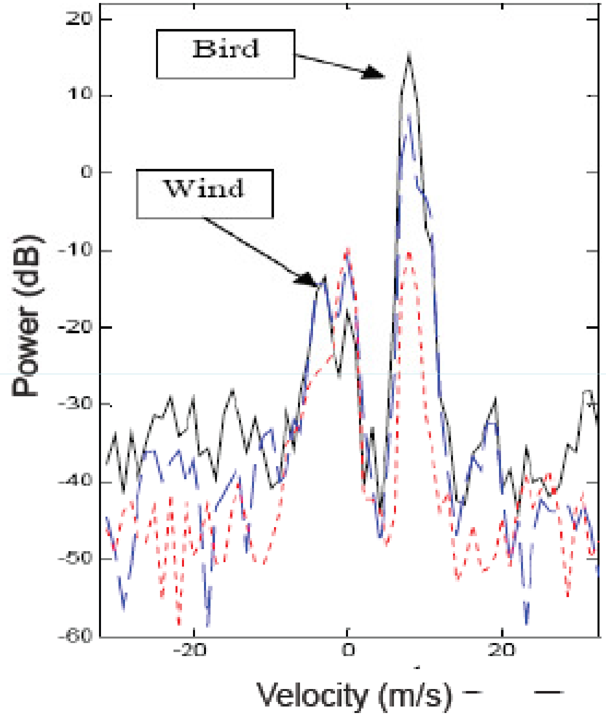
Two key concepts:

Distant ⟺ Time

$$R = c\Delta t/2$$

Velocity ⟺ Frequency

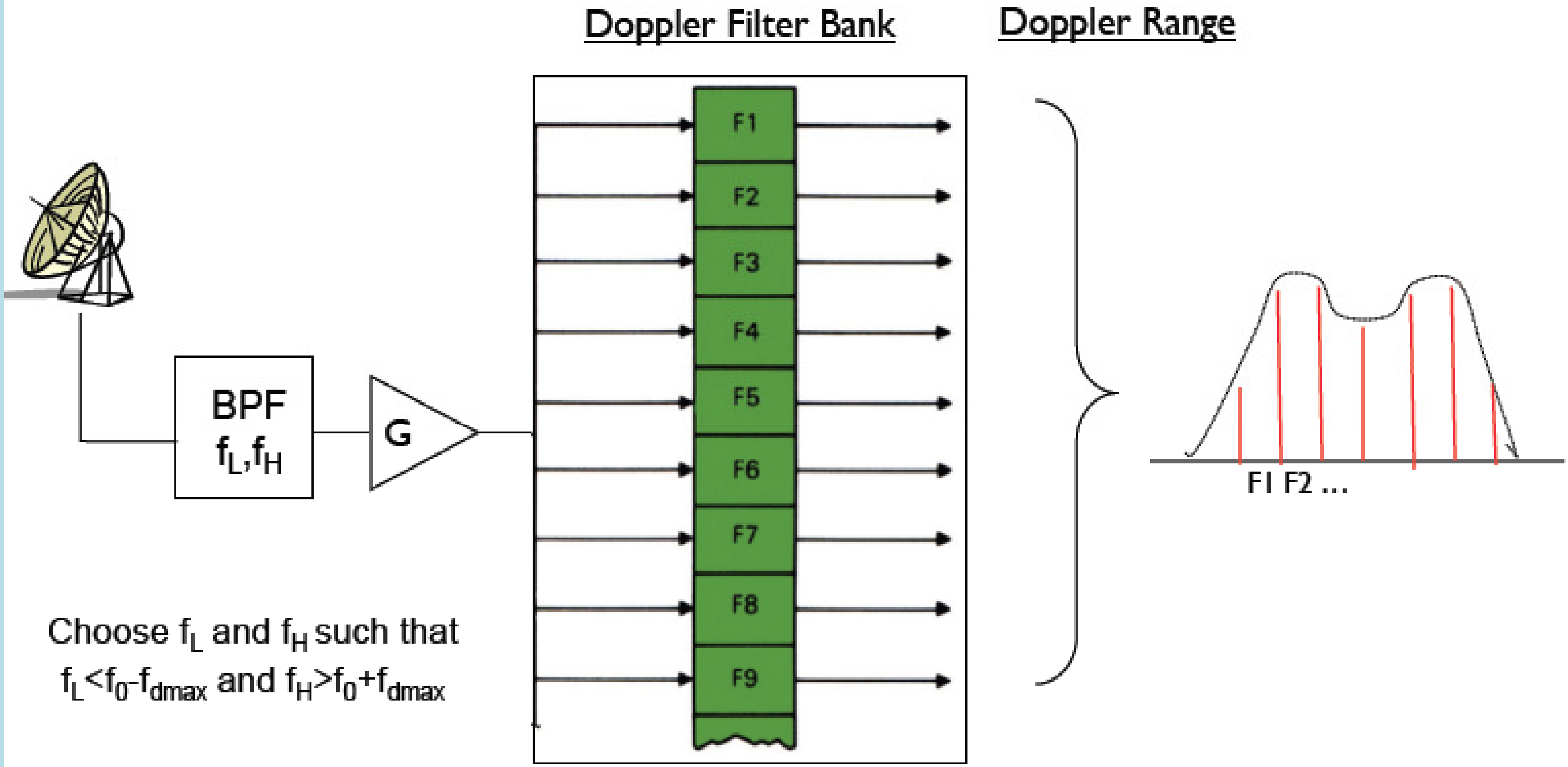
$$v = -f_D \lambda_0 / 2$$



If there is a distribution of targets moving at different velocities (e.g., electrons in the ionosphere) then there is no single Doppler shift but, rather, a Doppler spectrum.

What is the Doppler spectrum of the ionosphere at UHF (λ of 10 to 30 cm)?

ISR Receiver: Doppler filter bank approach

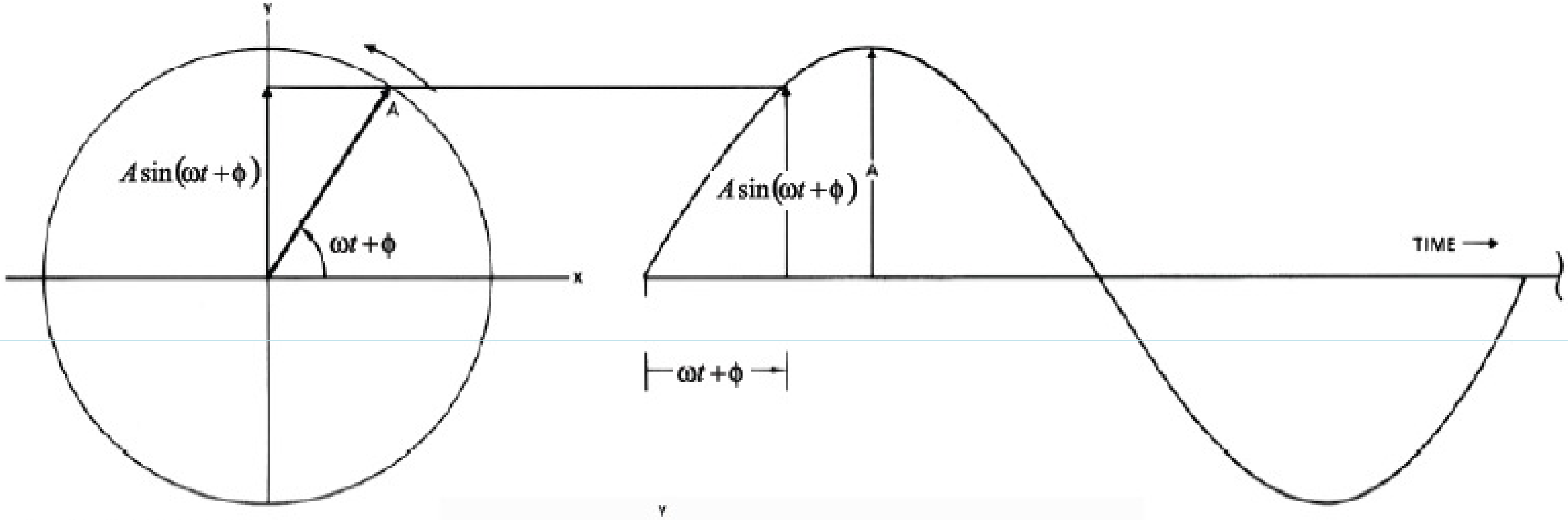


Practical Problem: It is hard to make narrow band (High Q) RF filters:

$$Q = \frac{f_0}{f_H - f_L}$$

Doppler Detection: Intuitive Approach

Phasor diagram is a graphical representation of a sine wave



<u>I & Q components*</u>

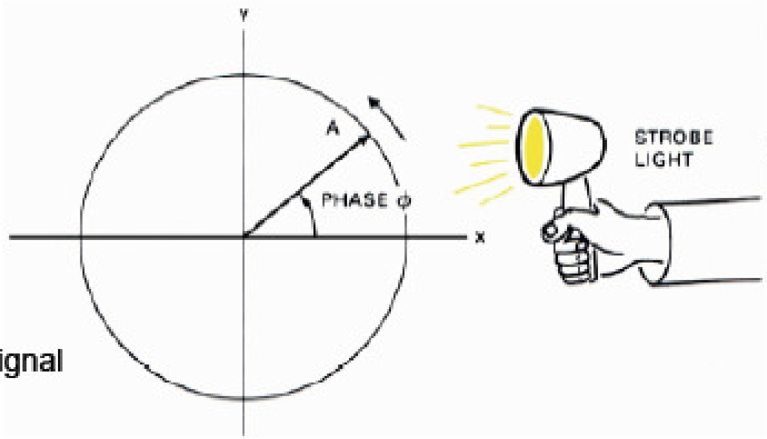
I => in-phase component

Acos(ϕ)

Q => in-quadrature component

Asin(ϕ)

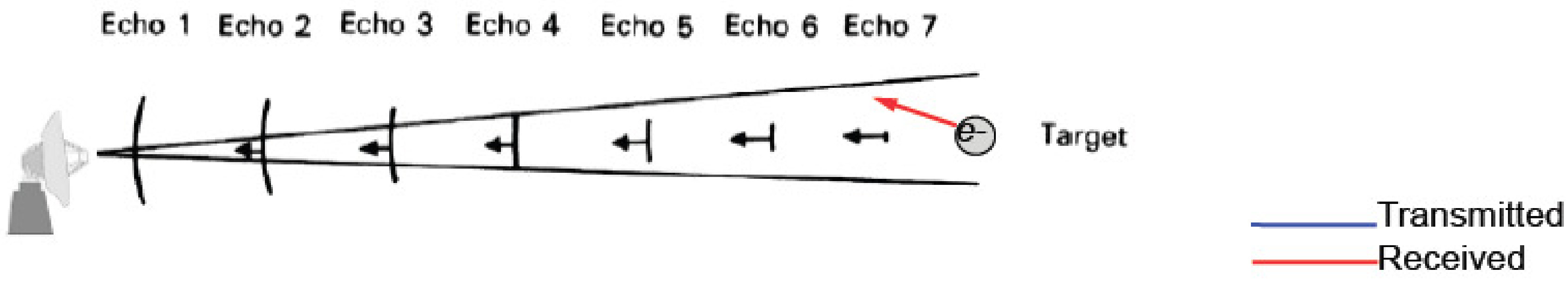
*relative to reference signal

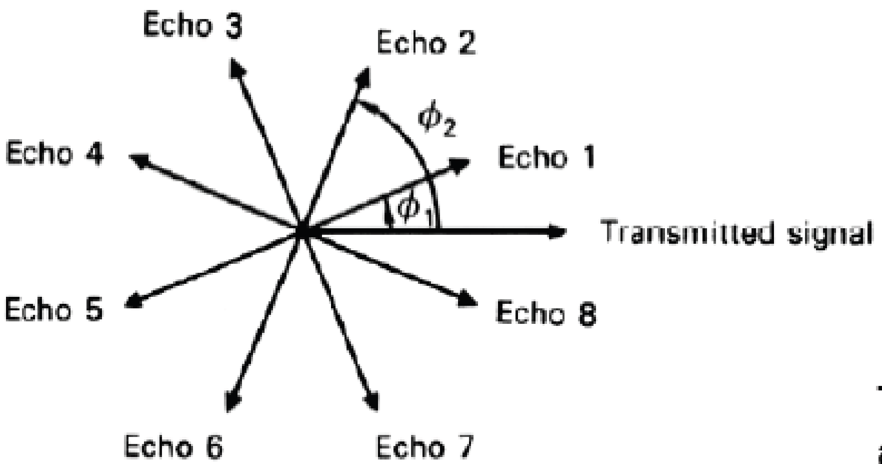


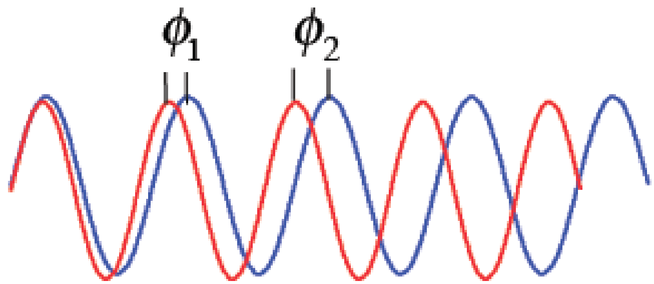
Consider strobe light as cosine reference wave at same frequency but with initial phase = 0

Doppler Detection: Intuitive Approach

Closing on target – positive Doppler shift

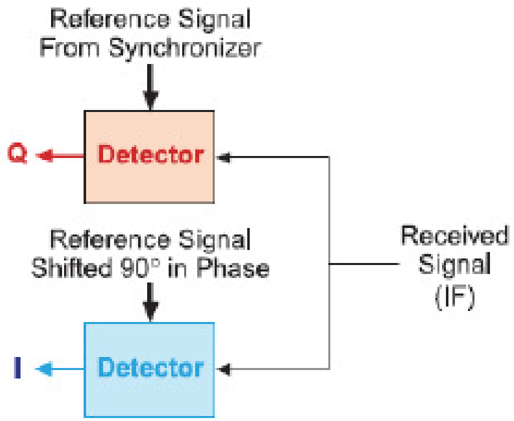






Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

I and Q Demodulation



in-phase (I) channel:

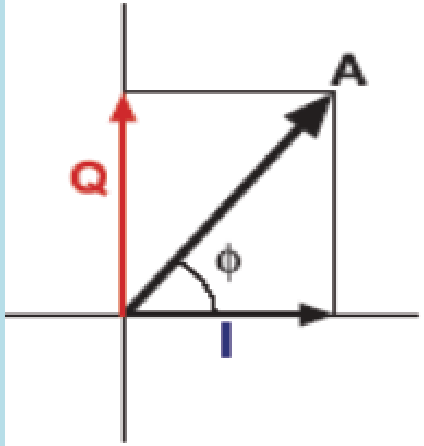
$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\cos(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{\cos(\phi(t)+2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\sin(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{-\sin(\phi(t)+2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$

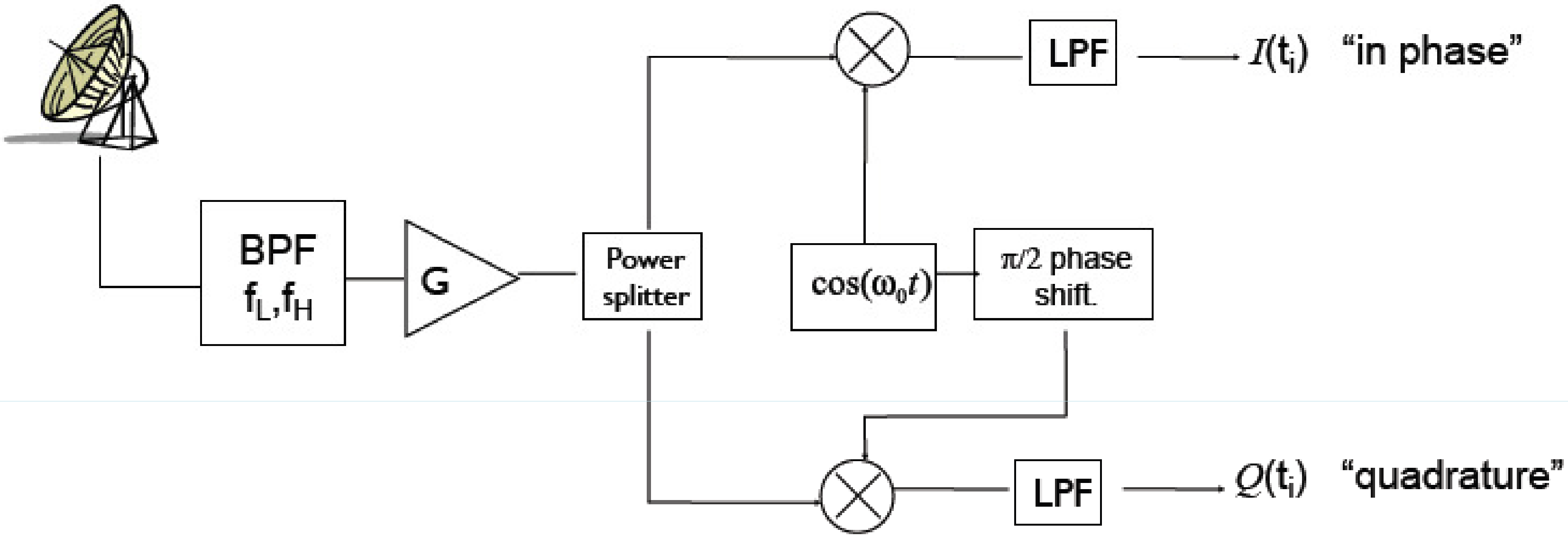


I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

The fundamental output of a pulsed Doppler radar is a time series of complex numbers.

ISR Receiver: I and Q plus correlation



We have time series of *V(t)* =*I(t)* + *jQ(t)*, how do I compute the Doppler spectrum?

Estimate the autocorrelation function (ACF) by computing products of complex voltages ("lag products")

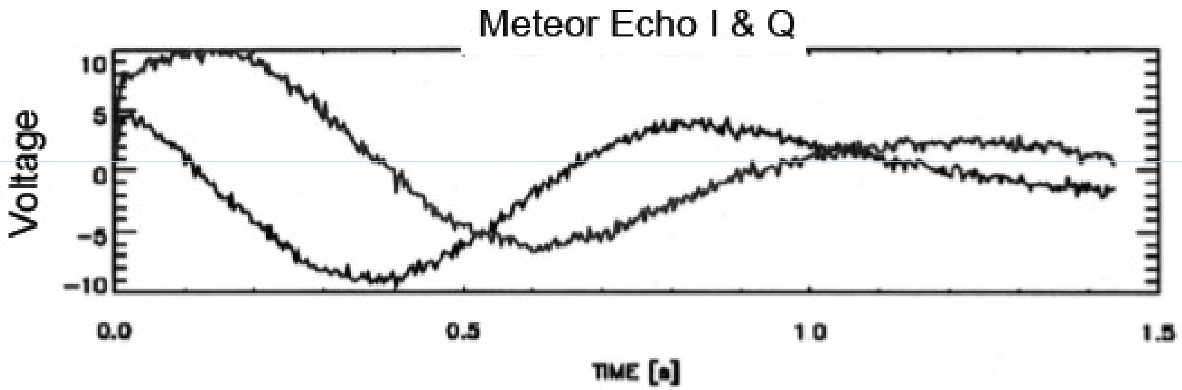
$$\rho(\tau) = \frac{\langle V(t) V^*(t+\tau) \rangle}{S}$$

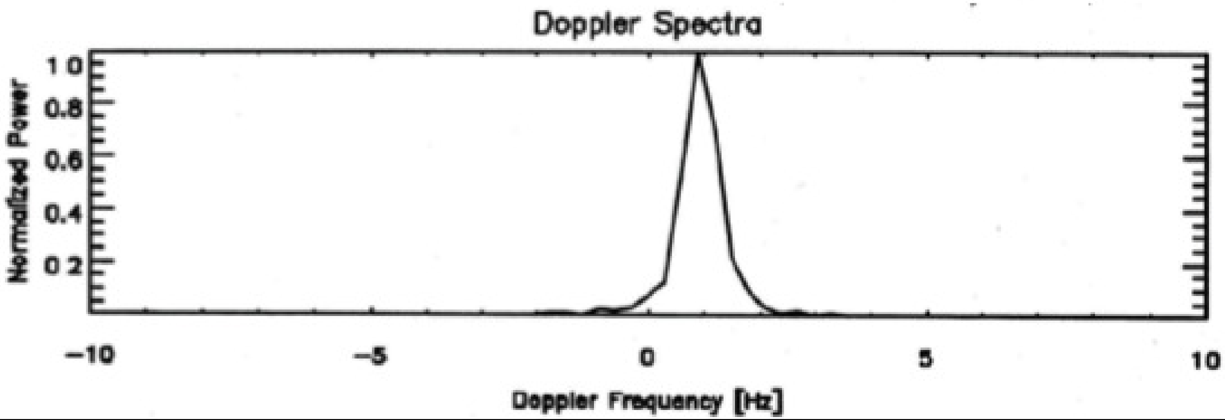
FFT

Power spectrum is Fourier Transform of the ACF

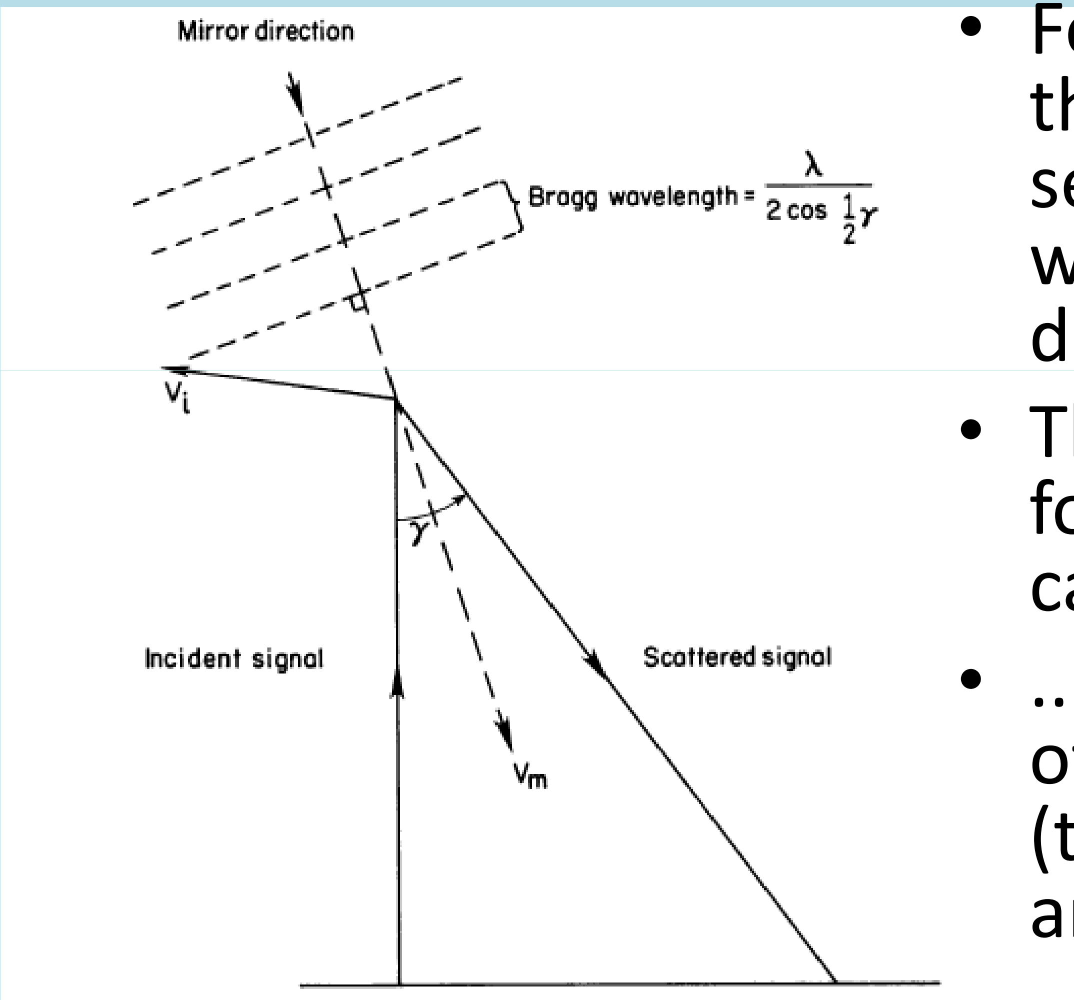
Example: Doppler Shift of a Meteor Trail

- Collect N samples of $I(t_k)$ and $Q(t_k)$ from a target
- Compute the complex FFT of $I(t_k)+jQ(t_k)$, and find the maximum in the frequency domain
- Or compute "phase slope" in time domain.





A note about bistatic radars



- For a bistatic radar, the receiver is sensitive to plasma waves in the mirror direction
- This is most important for velocity (vector) calculations…
- ..but actually it affects other parameters too (think temperature anisotropy)

Overspread Target

Typical ion-acoustic velocity: 3 km/s
Doppler shift at 450 MHz: 10kHz
Correlation time: 1/10kHz = 0.1 ms
Required PRF to probe ionosphere (500km range): 300 Hz

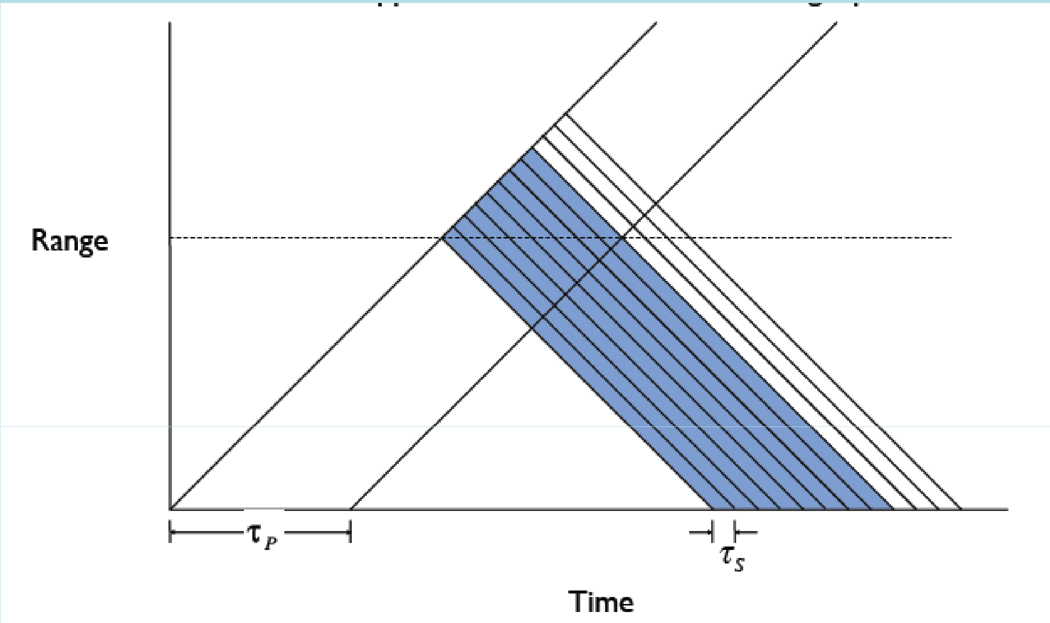
Plasma has completely decorrelated by the time we send the next pulse.

$f_d >> 1/\tau$ (Doppler changes significantly during one pulse)

- Must sample multiple times per pulse
- Result: Doppler can be determined from single pulse.

Computing the ACF

In the correlator:



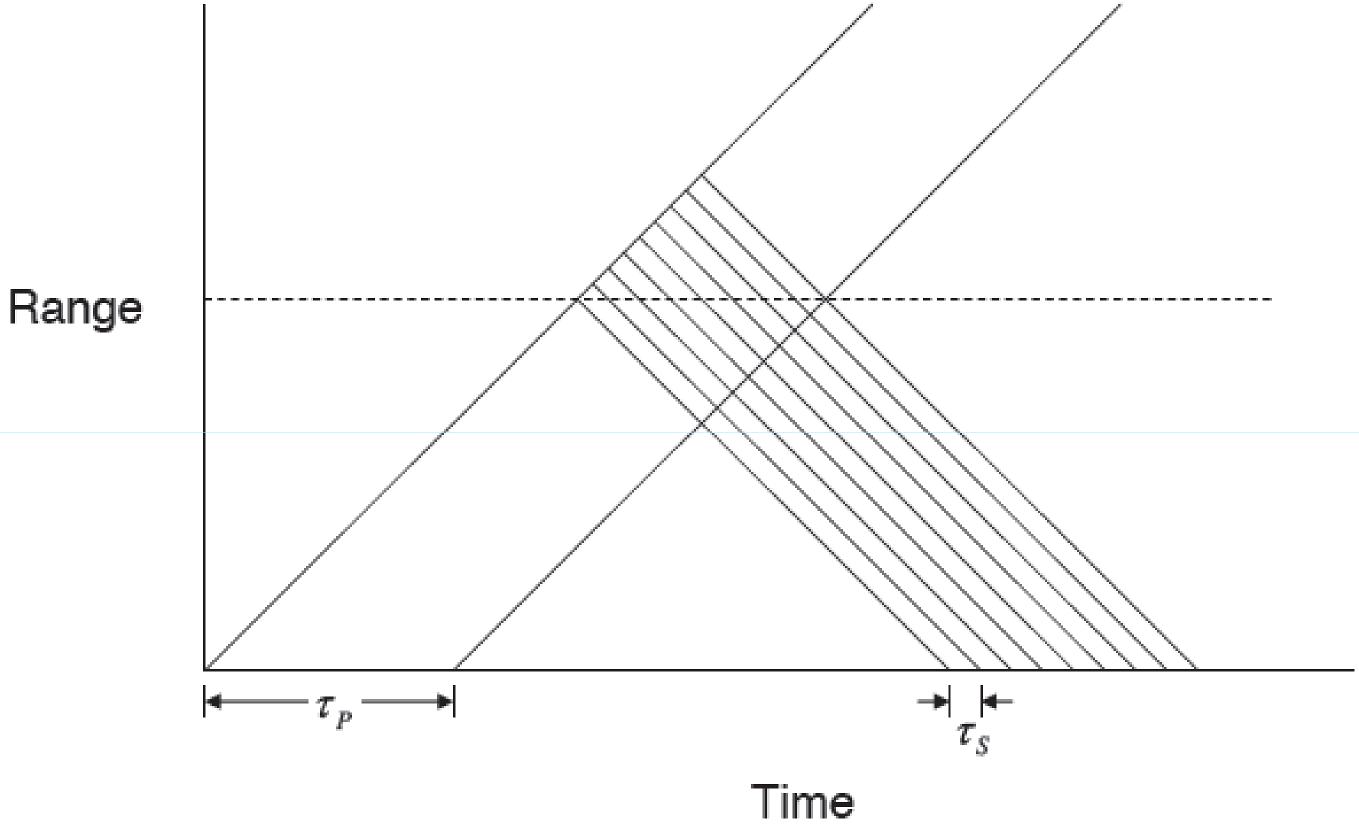
$R_{i,j} = \sum(X_i X_j + Y_i Y_j)$

$I_{i,j} = \sum(X_i Y_j - Y_i X_j)$

For lag 0: i = j (imaginary part identically zero)

For lag 1: j-i=1, and so on…

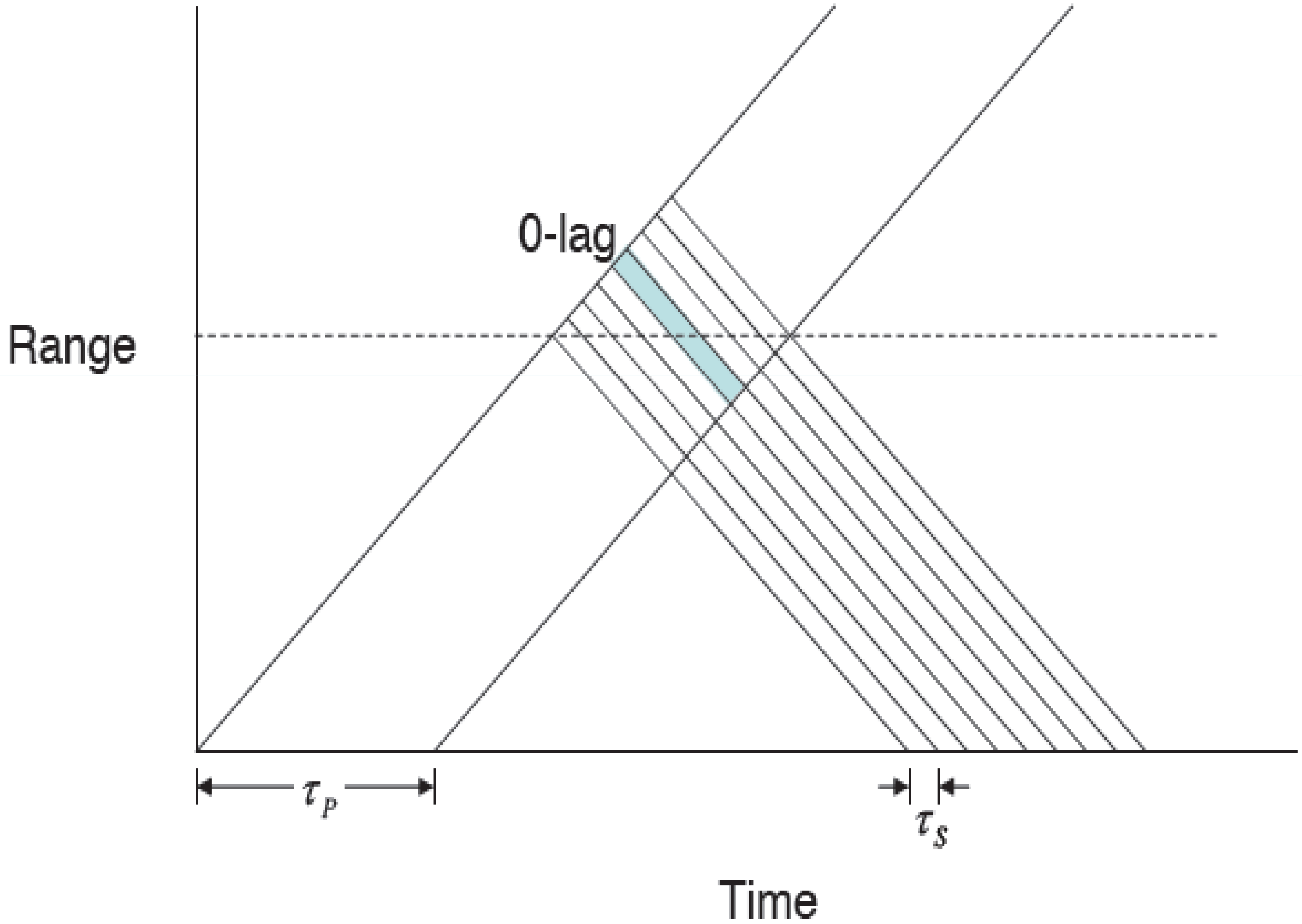
Computing the ACF



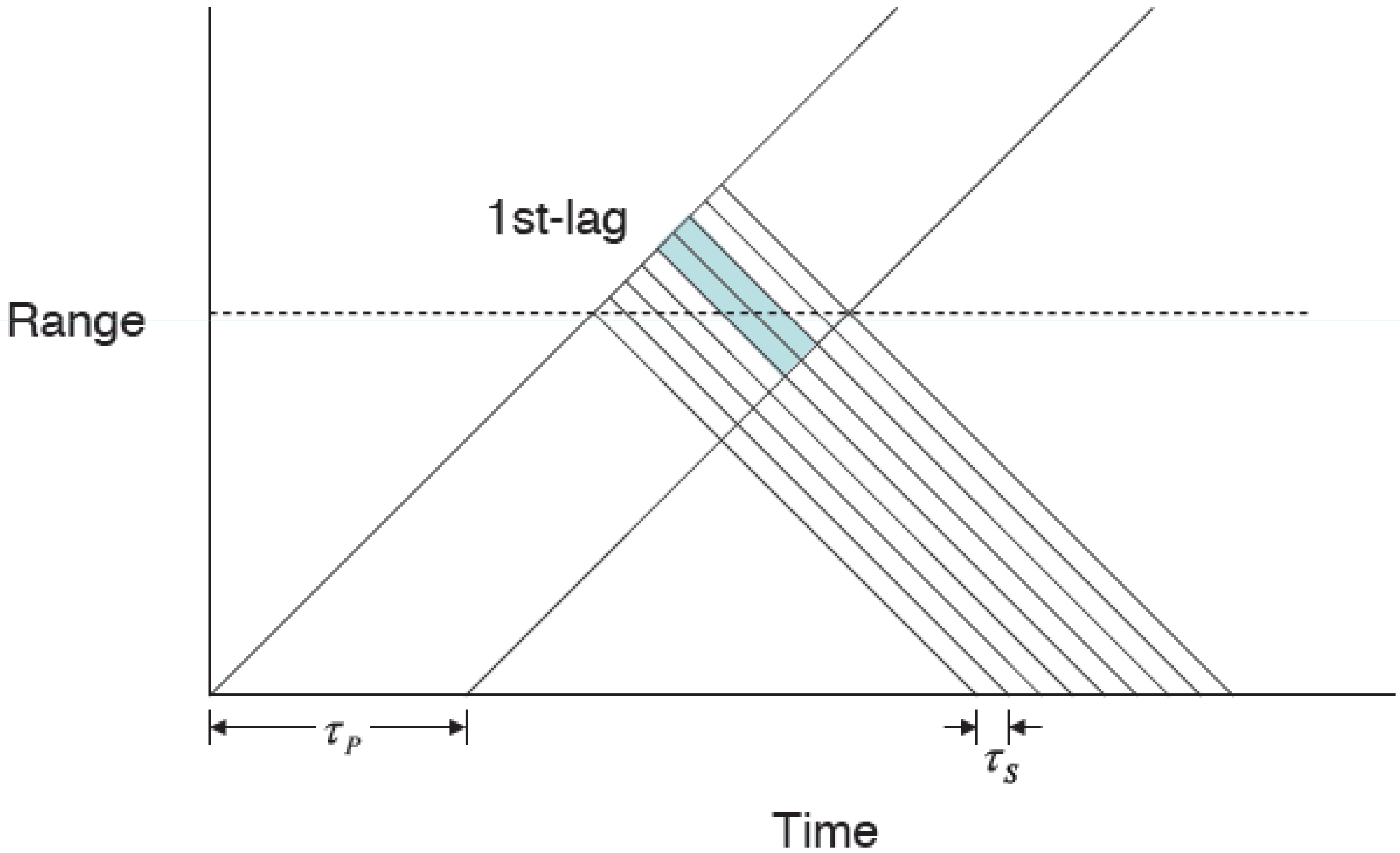
τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

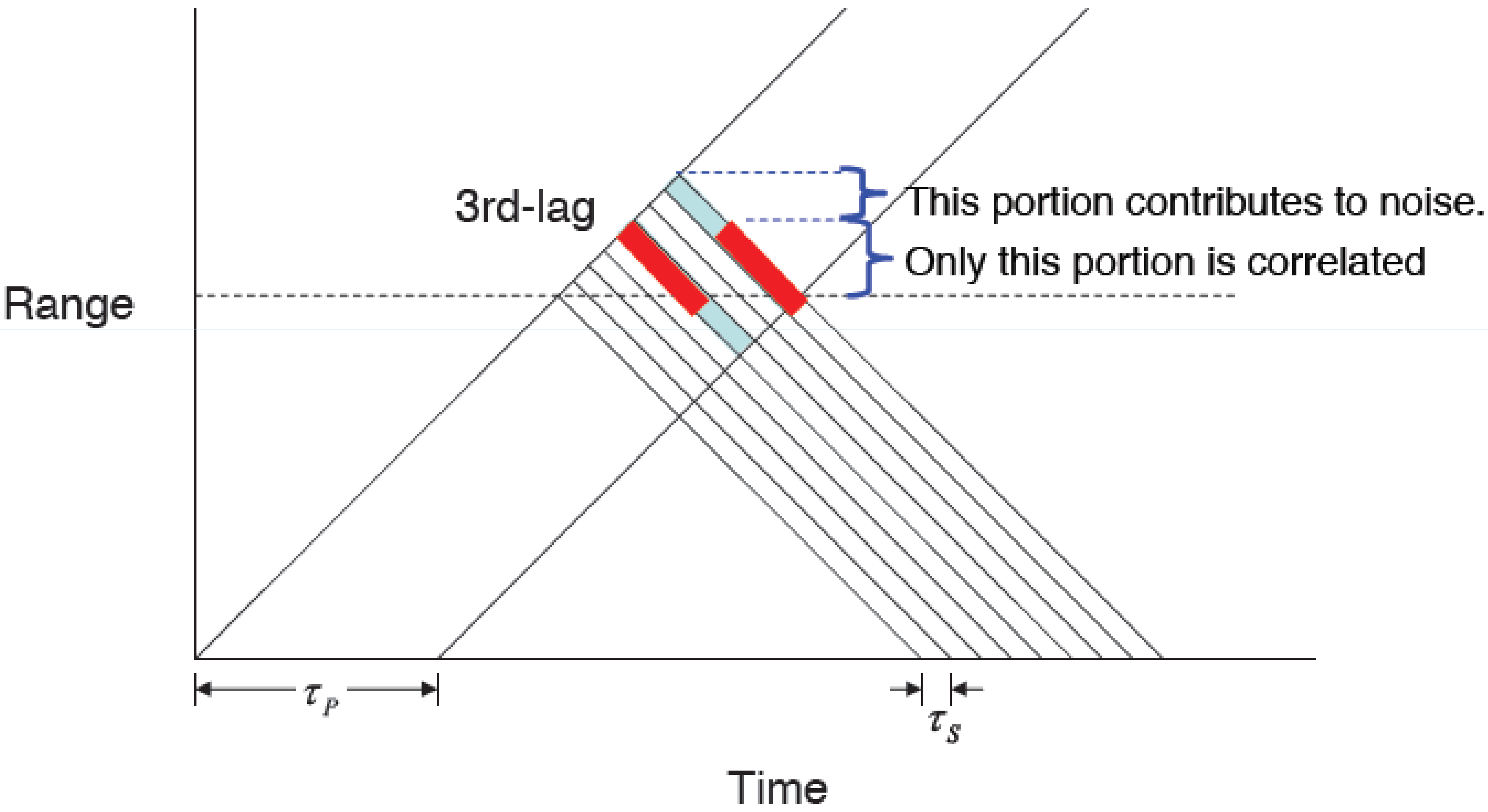
Computing the ACF

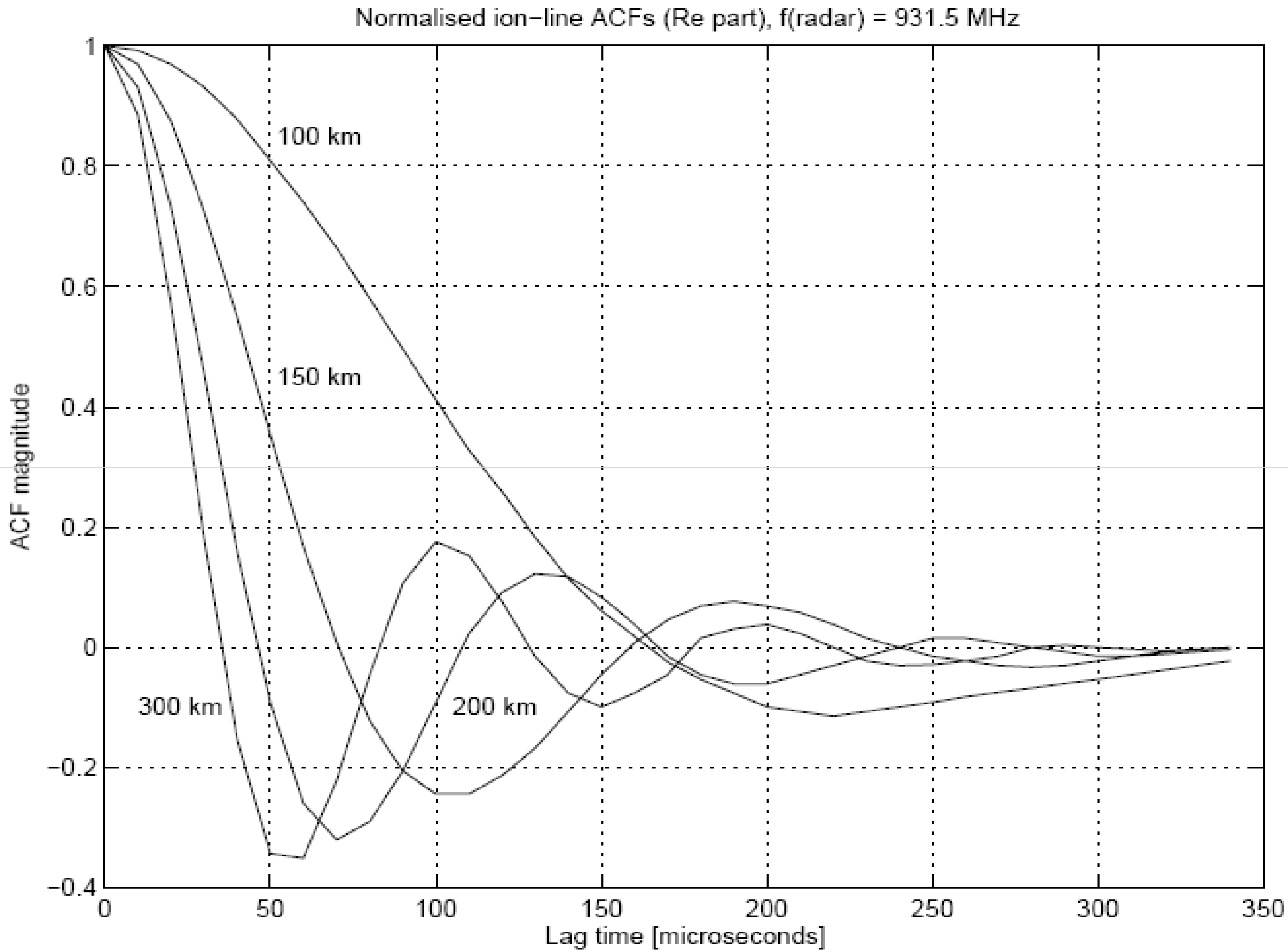


Computing the ACF



Computing the ACF




Normalised ion−line ACFs (Re part), f(radar) = 931.5 MHz
100 km
150 km
200 km
300 km
ACF magnitude
Lag time [microseconds]
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
50
100
150
200
250
300
350

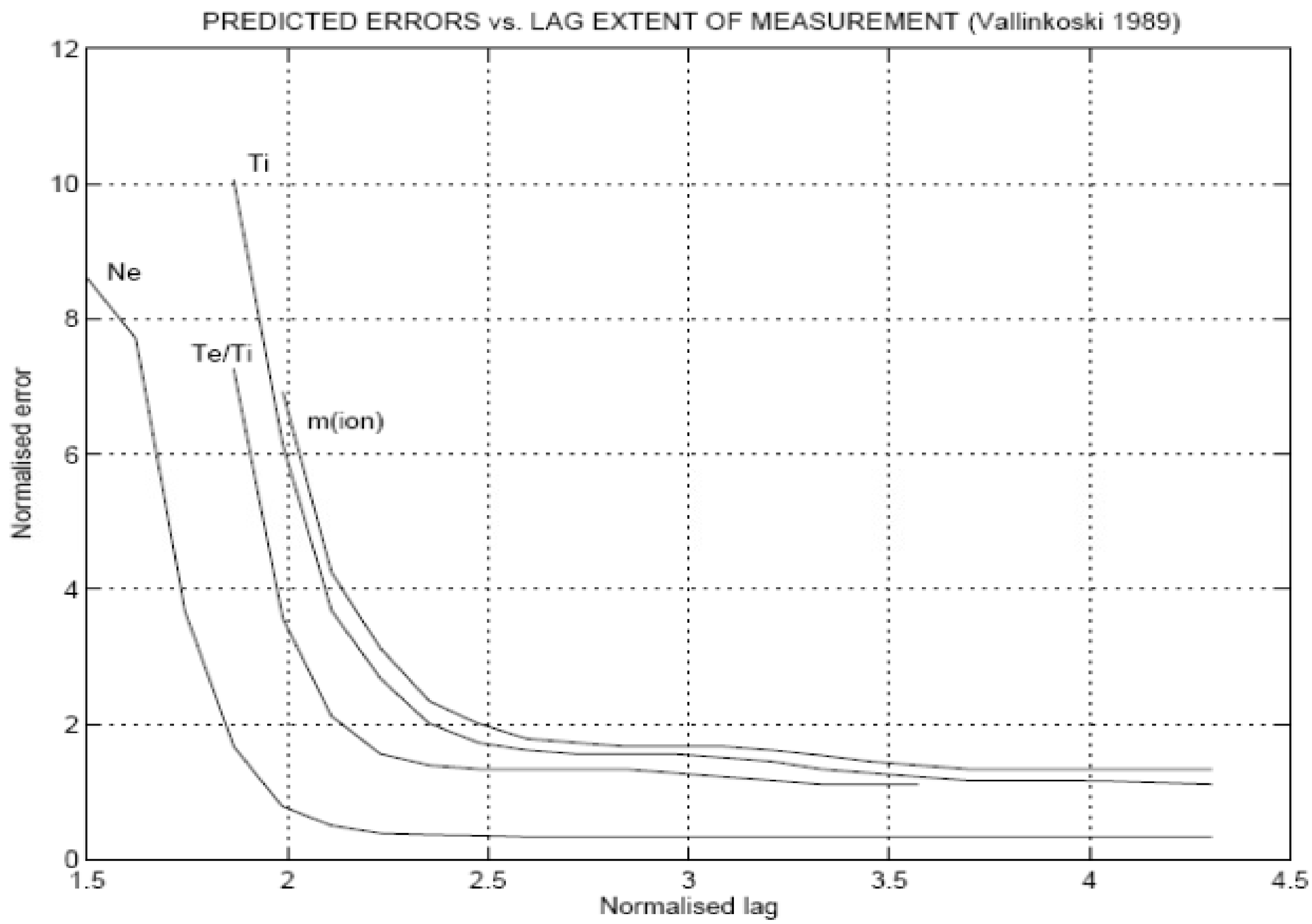
The plasma autocorrelation function, $r_{xx}(\tau)$

is the Fourier transform of the ion line power spectral density. Using the plasma dispersion relation, we can compute model autocorrelation functions for different combinations of N_e, T_e and T_i

An estimate of the target r_{xx} at *lag time* $n\tau_0$ can be computed from the time series of complex amplitude samples, $s(t)$, output from the receiver:

$$r_{xx}(n\tau_0) = s\,(t)\,s^*(t + n\tau_0)$$

Intuitively, it may appear natural to continue sampling at a given range for so long that the ACF has decayed almost to zero. To see if that helps at all, let us first look at how the different plasma parameters influence the ACF at different lag times :


PREDICTED ERRORS vs. LAG EXTENT OF MEASUREMENT (Vallinkoski 1989)
Normalised error
Normalised lag
Ti
Ne
Te/Ti
m(ion)
0
2
4
6
8
10
12
1.5
2
2.5
3
3.5
4
4.5

Partial derivatives of the plasma dispersion function:

$\partial r_{xx}(\tau) / \partial Ne$

$\partial r_{xx}(\tau) / \partial T_i$

$\partial r_{xx}(\tau) / \partial (T_e/T_i)$

$\partial r_{xx}(\tau) / \partial m_i$

$\partial r_{xx}(\tau) / \partial \nu_{in}$

are shown in terms of τ/τ_0 , where τ_0 , the *plasma correlation time*, is the time to the first zero crossing of the ACF of a undamped ion-acoustic wave with wavelength = Λ = ½ λ_{radar}

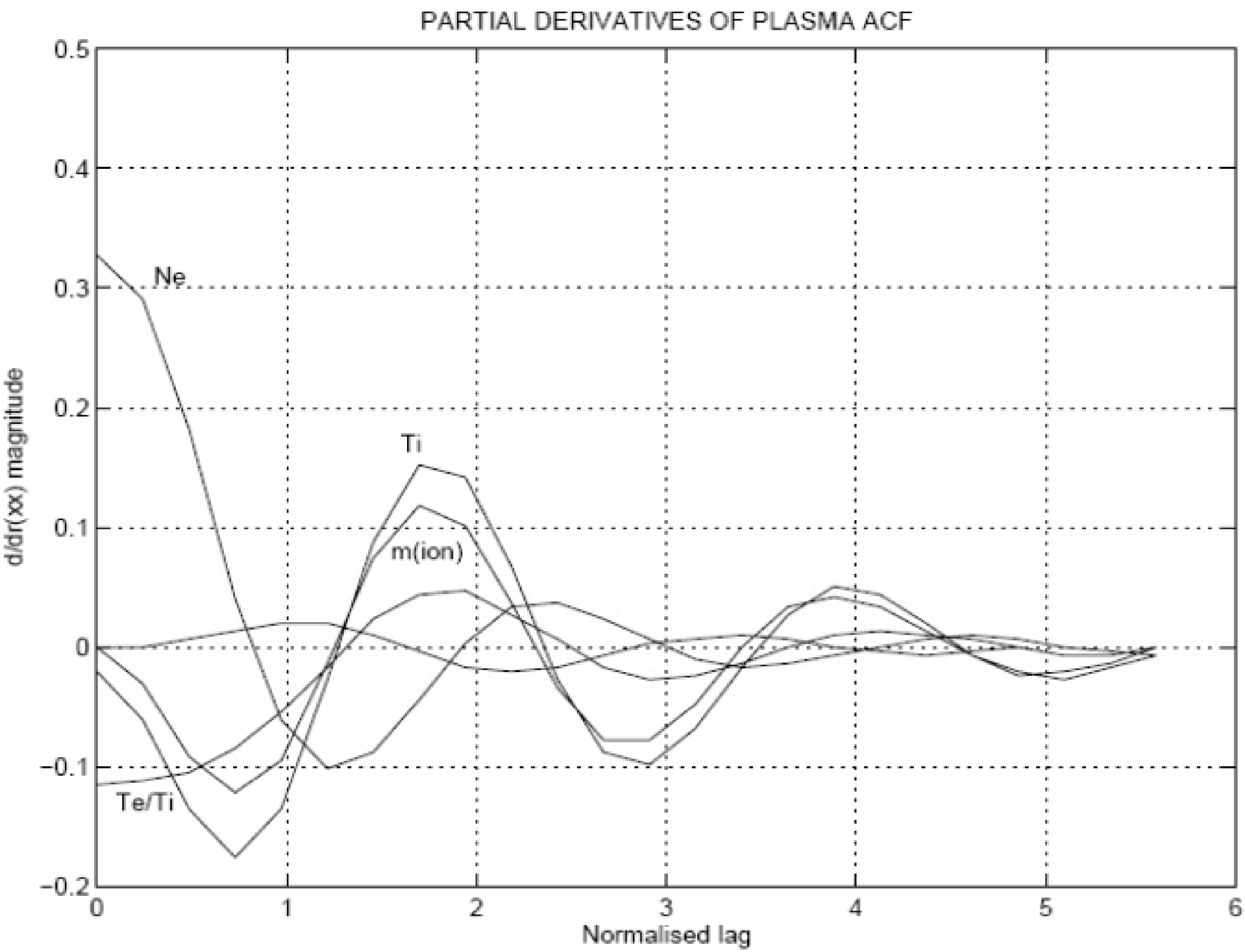
NOTE: $\partial r_{xx}(\tau) / \partial T_i$ and $\partial r_{xx}(\tau) / \partial m_i$ are almost linearly dependent...

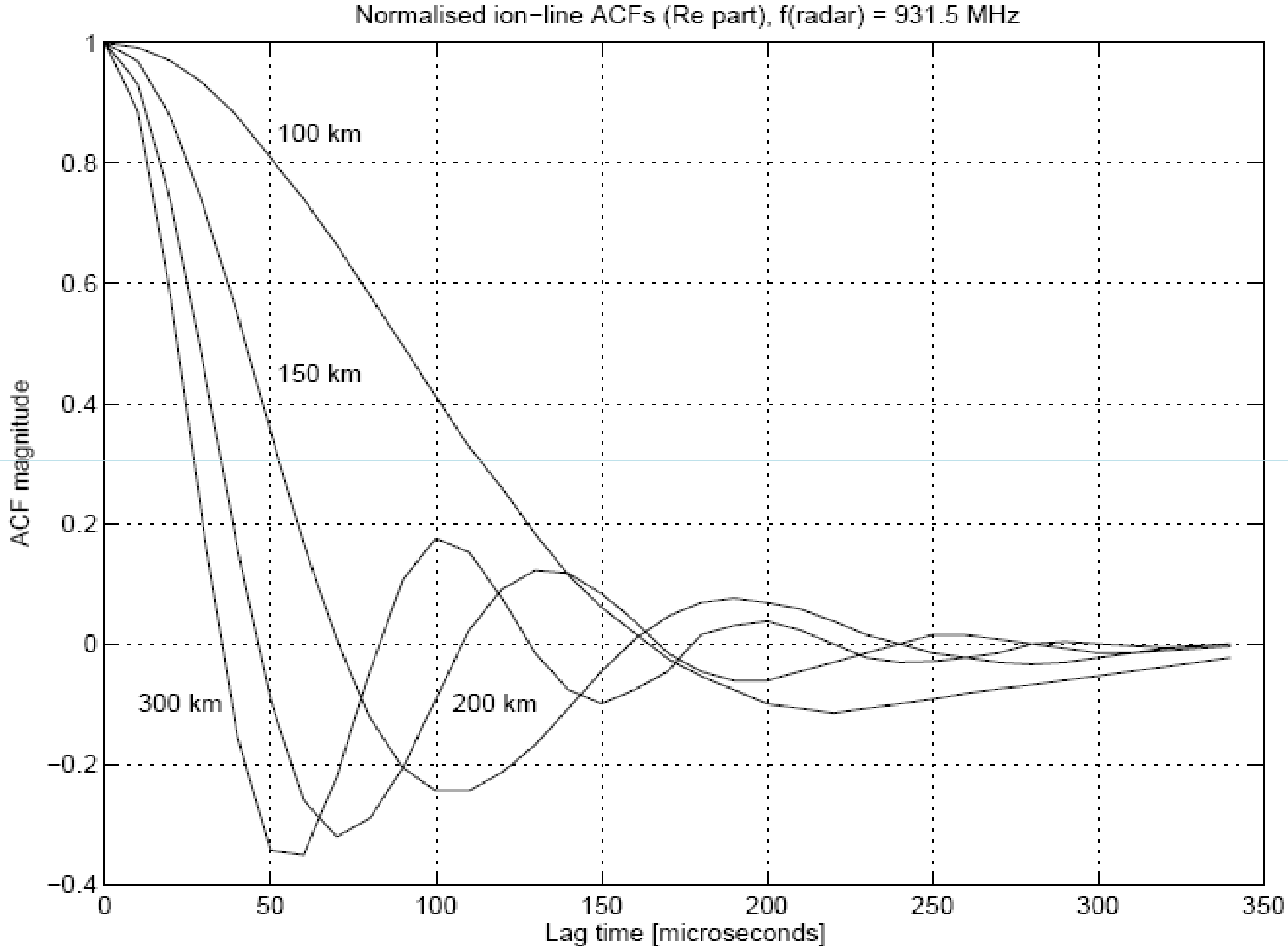
ACF estimate extent and errors

The next figure (from *Vallinkoski 1989*) shows how the errors of the different plasma parameters behave as functions of lag extent when measurement data are fitted to a five-parameter plasma model.

Comparing this to the previous figure , we see that *as the lag extent is increased to the point where the partial derivative of a given parameter goes through a complete cycle*, the error in that parameter suddenly drops dramatically.

If one is satisfied with slightly less than ultimate accuracy, extending the measurement to τ/τ_0 = 2.5 should be sufficient. By about τ/τ_0 = 3.5, all errors have settled down to their asymptotic value.


PARTIAL DERIVATIVES OF PLASMA ACF
d/dr(xx) magnitude
Normalised lag
Ne
Ti
m(ion)
Te/Ti
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
0
1
2
3
4
5
6



Normalised ion−line ACFs (Re part), f(radar) = 931.5 MHz
ACF magnitude
Lag time [microseconds]
100 km
150 km
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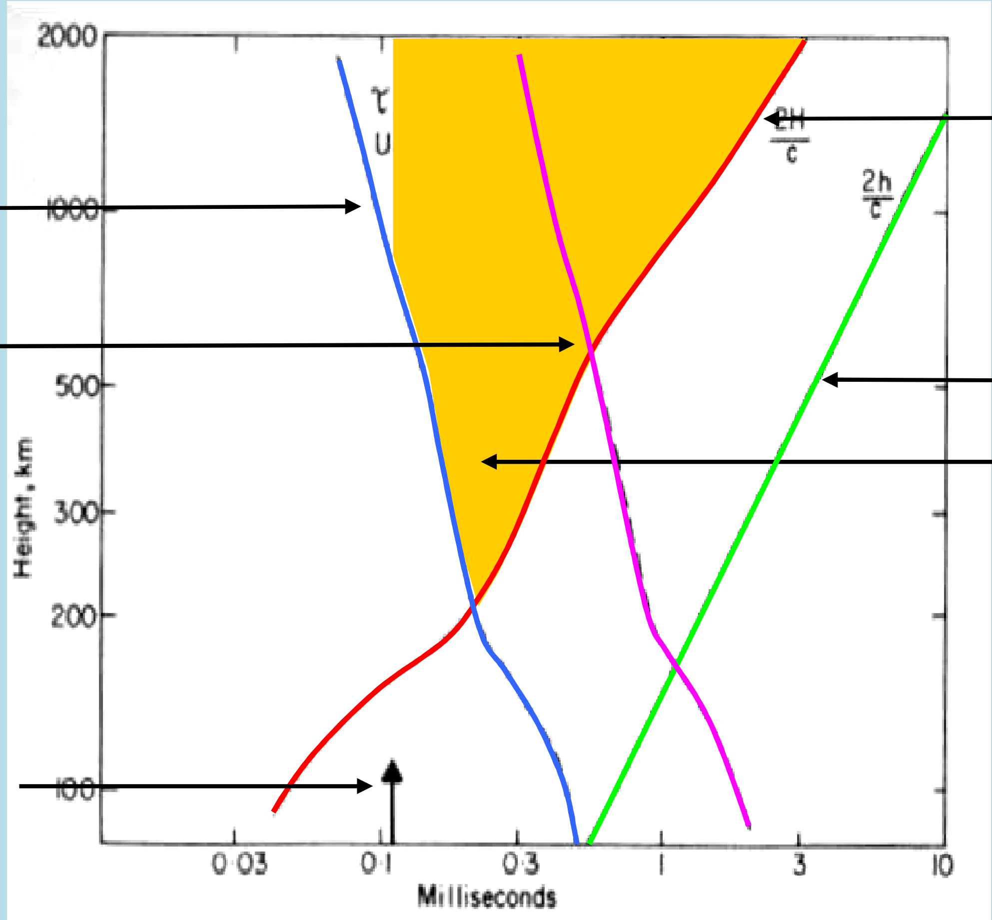
300 km
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
50
100
150
200
250
300
350

Constraining factors for an incoherent scatter radar experiment

ionospheric correlation time, τ_1 for UHF

ionospheric correlation time, τ_1 for VHF

Minimum pulse length obtainable from transmitter



Pulse length giving resolution equal to the scale-height.

Time of flight for radar pulse

Possible values for UHF experiment

Some constraining factors for incoherent scatter experiments, shown as functions of height for typical ionospheric conditions.

Clutter Removal

Not all radar signals have the same correlation time. This can be an advantage in separating signals you want from signals that you don't want.

In particular, sometimes ground scatter from features such as mountains ends up at the same range delays as signals of interest – e.g. the E region. This radar clutter obscures the desired ionospheric signal and can be many orders of magnitude larger.

However, the clutter can have a much longer correlation time (many pulses) compared with the < 1 pulse typical of incoherent scatter. This can be exploited to subtract the clutter at the voltage level.

Bibliography

ISR tutorial material:

- *http://www.eiscat.se/groups/Documentation/CourseMaterials/*

Radar signal processing

- Mahafza, *Radar Systems Analysis and Design Using MATLAB*
- Skolnik, *Introduction to Radar Systems*
- Peebles, *Radar Principles*
- Levanon, *Radar Principles*
- Blahut, *Theory of Remote Image Formation*
- Curlander, *Synthetic Aperture Radar: Systems and Signal Analysis*

Background (Electromagnetics, Signal Processing:

- Ulaby, *Fundamentals of Engineering Electromagnetics*
- Cheng, *Field and Wave Electromagnetics*
- Oppenheim, Willsky, and Nawab, *Signals and Systems*
- Mitra, *Digital Signal Processing: A Computer-based Approach*

For fun:

- http://mathforum.org/johnandbetty/