

# The short introduction to Incoherent Scatter (IS) Theory

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# Incoherent?...

- **Dictionary:** The property of being coherent
- **Antonym:** Incoherent
- **Incoherent=Random, viz. Incoherent scatter is the process by which radiowaves are randomly scattered by electrons in the ionosphere**
- **Media:** Incoherent=Incomprehensible

# Incoherent?...

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- **Media:** Incoherent=Incomprehensible

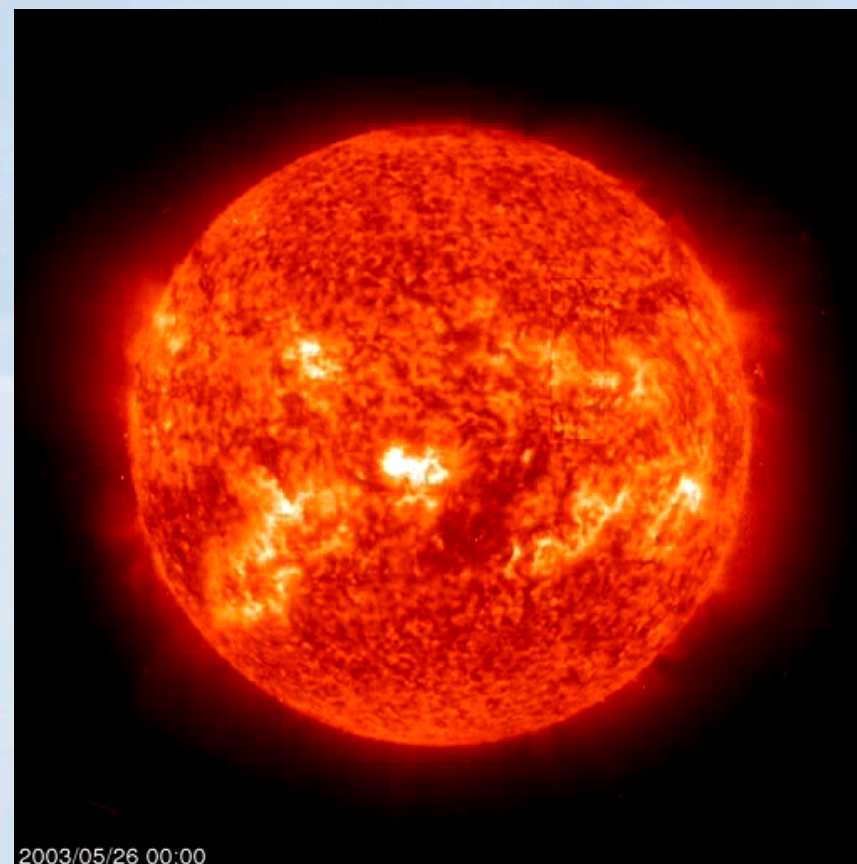
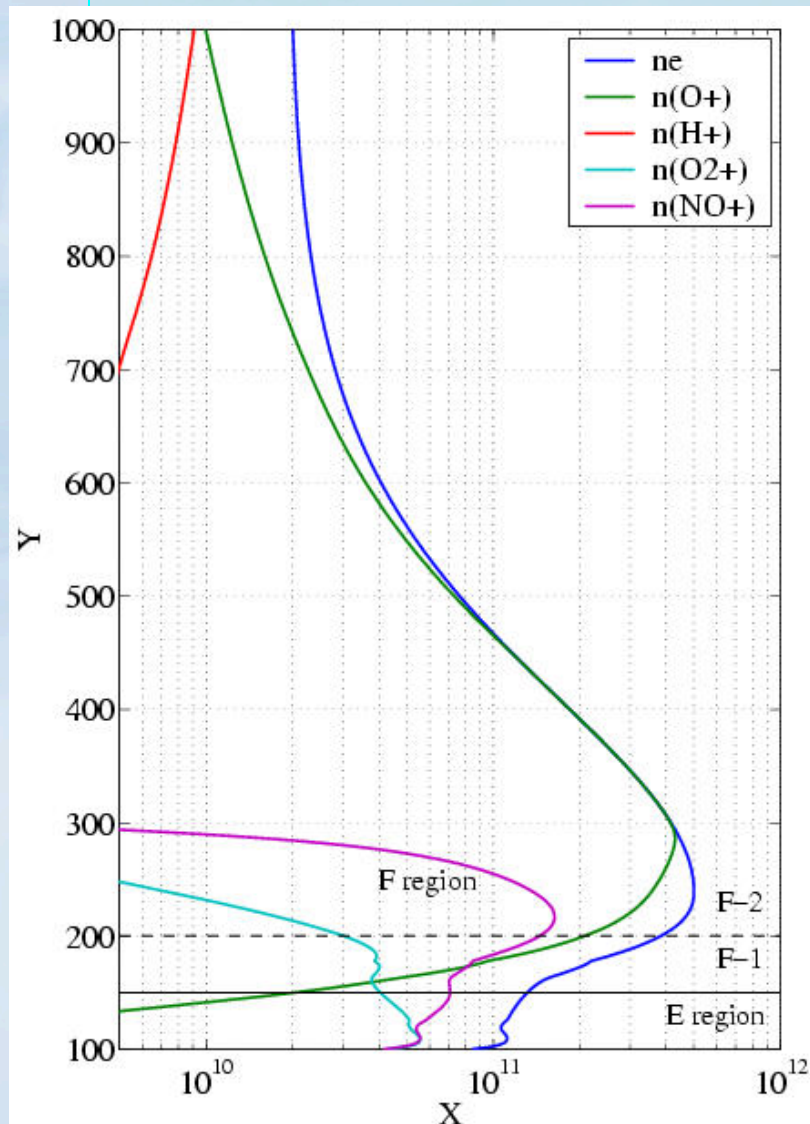
**Incoherent scatter is neither incoherent nor  
incomprehensible**



First: We need an  
Ionosphere...



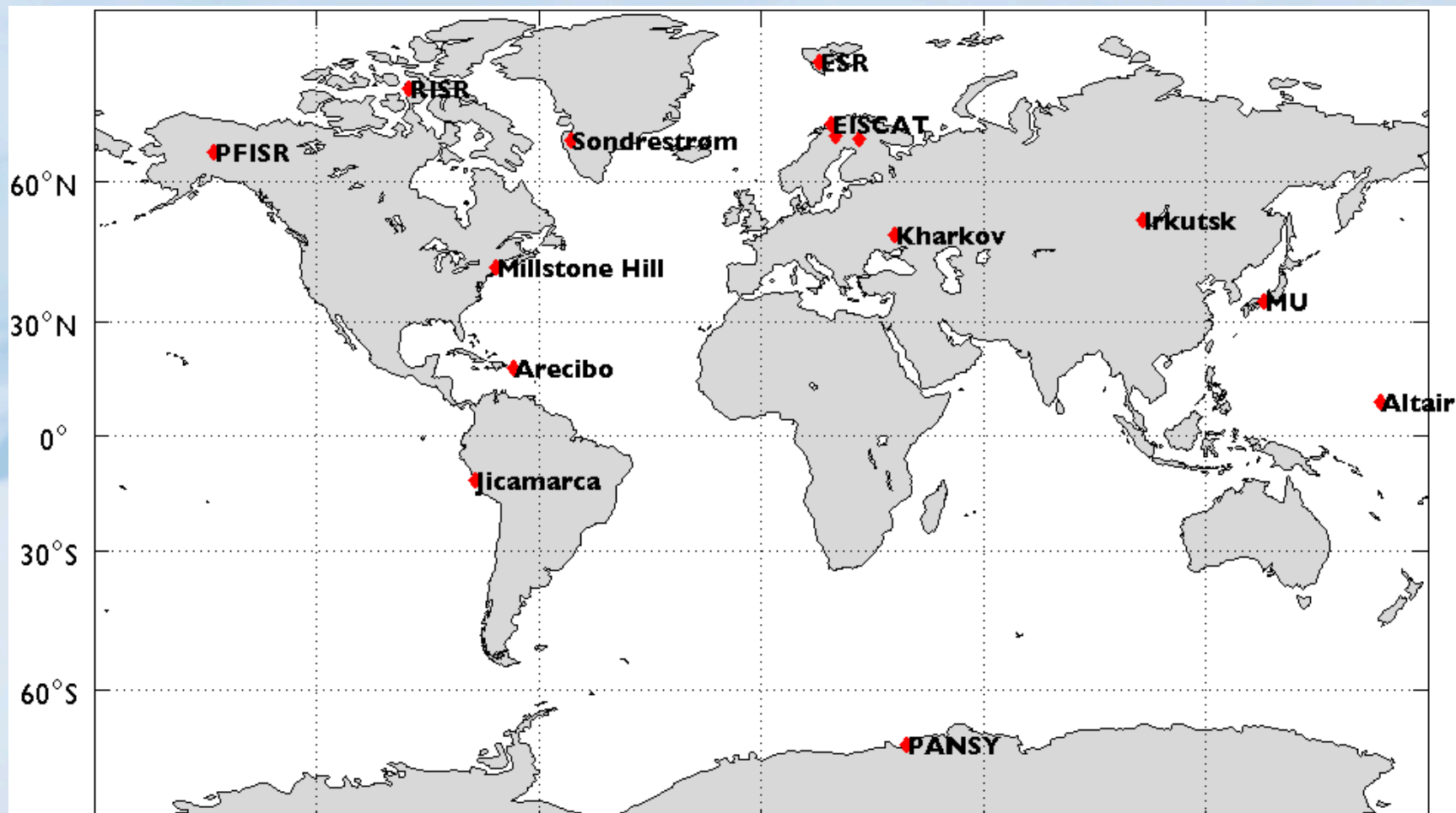
# The Earth's Ionosphere



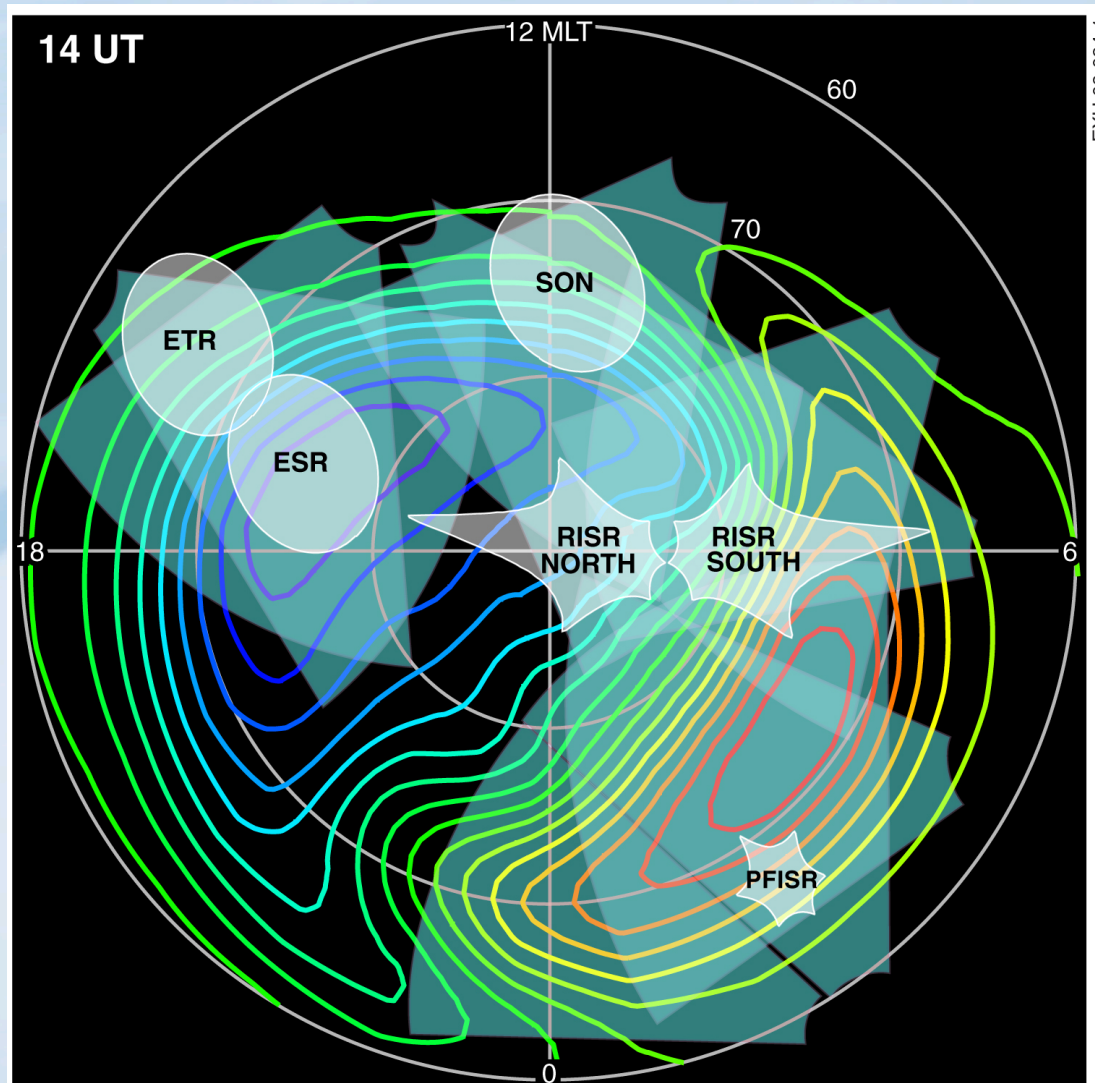


Now we have an ionosphere -  
let's add the Incoherent Scatter  
Radar (ISR) to probe it!

# Incoherent Scatter Radars of the World



# Map of the north...





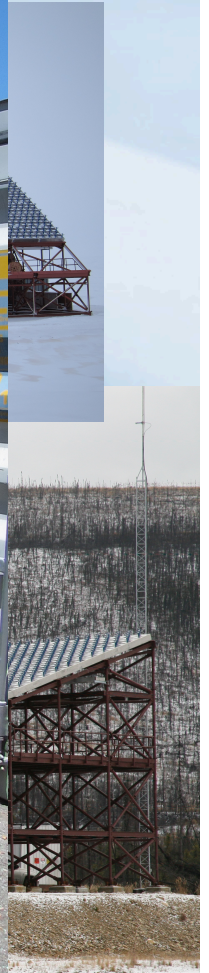
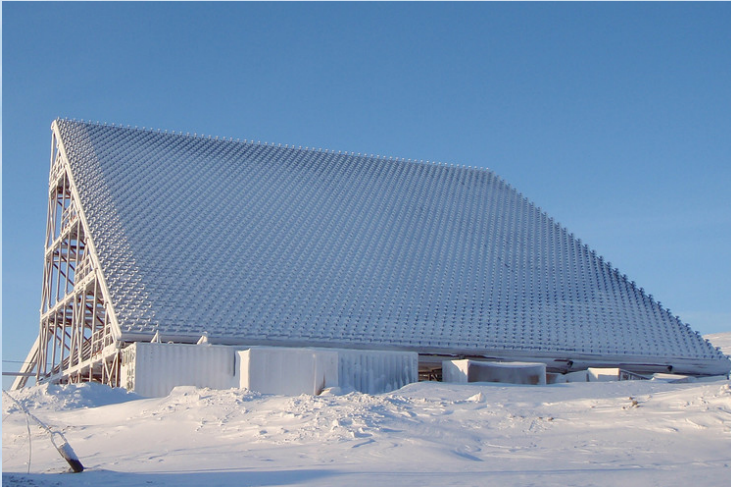


## High latitude Incoherent Scatter Radars....



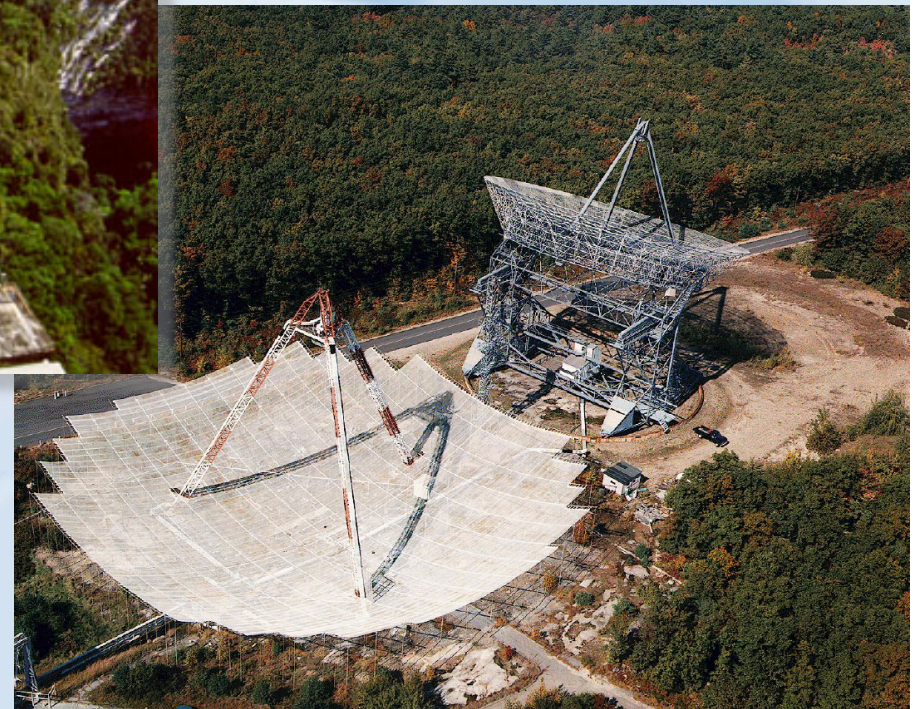


# PFISR (Poker Flat Incoherent Scatter Radar) and RISR-N (Resolute Bay Incoherent Scatter Radar) AMISRs currently





# Mid-Latitude Incoherent Scatter Radars





# Low-Latitude Incoherent Scatter Radars

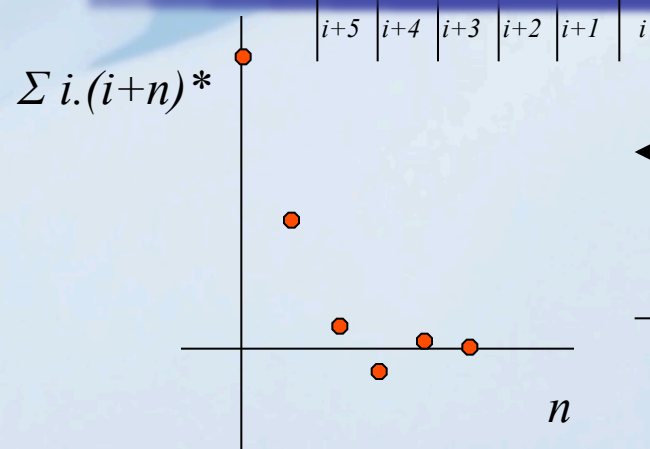
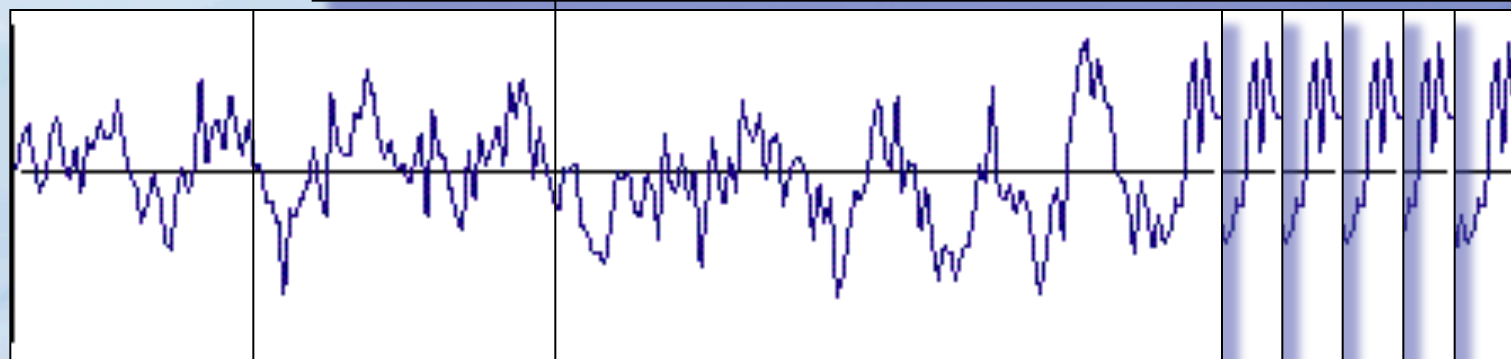
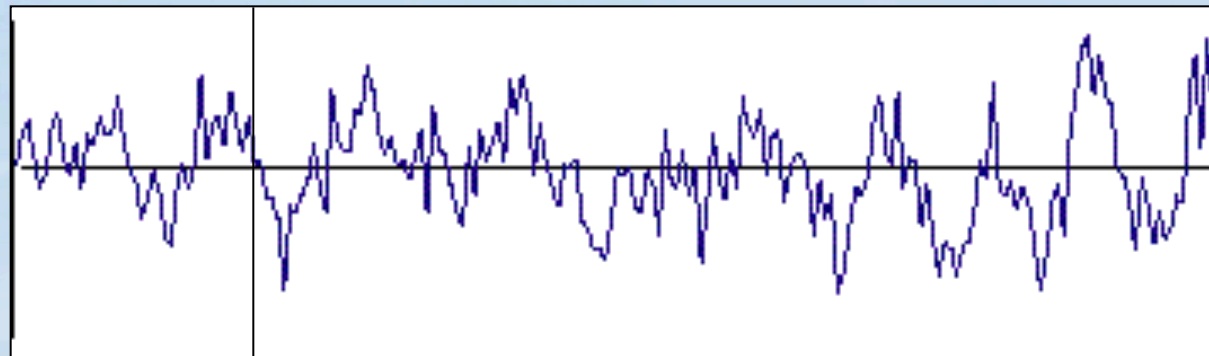


# Questions you might have now:

- Why are incoherent scatter radars (ISRs) so big? Is it a status-thing?
- Why is it called *incoherent* scattering?
- What do the ISR returns look like and why?
- What can ISRs measure?
- Can we get through this before lunch?







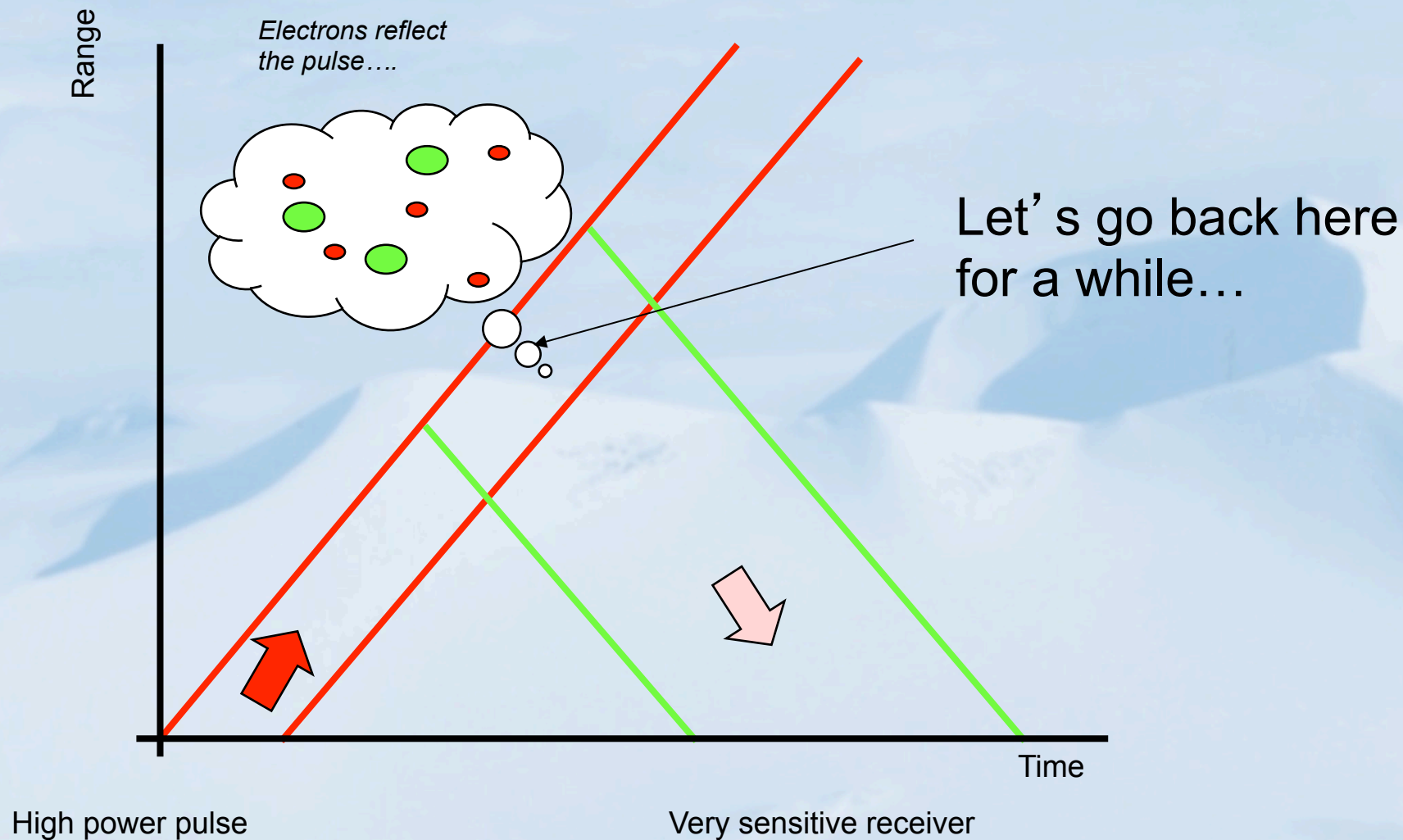
Fourier Transform



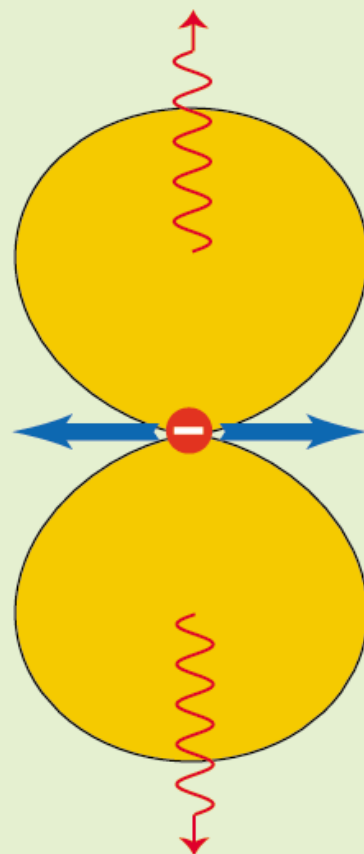




# How ISRs work...



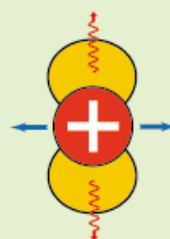
**electron**



$$\mathbf{k} \uparrow$$

$$\mathbf{E} \sin(\omega t - \mathbf{k} \cdot \mathbf{x}) \leftarrow$$

**ion**

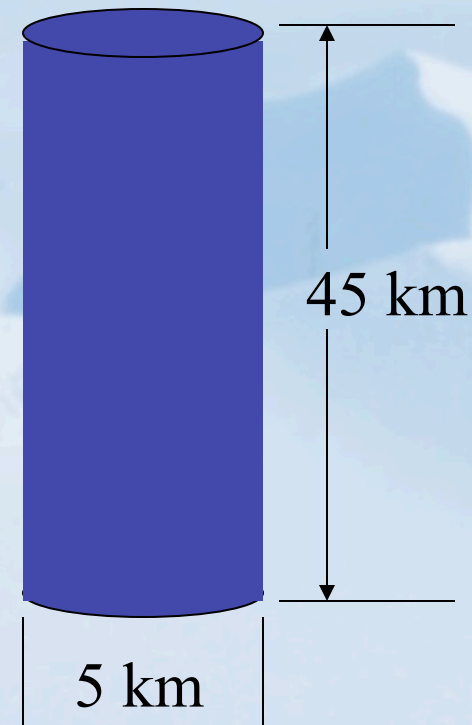


$$\frac{\sigma_{ion}}{\sigma_{ele}} = \left( \frac{m_{ele}}{m_{ion}} \right)^2$$

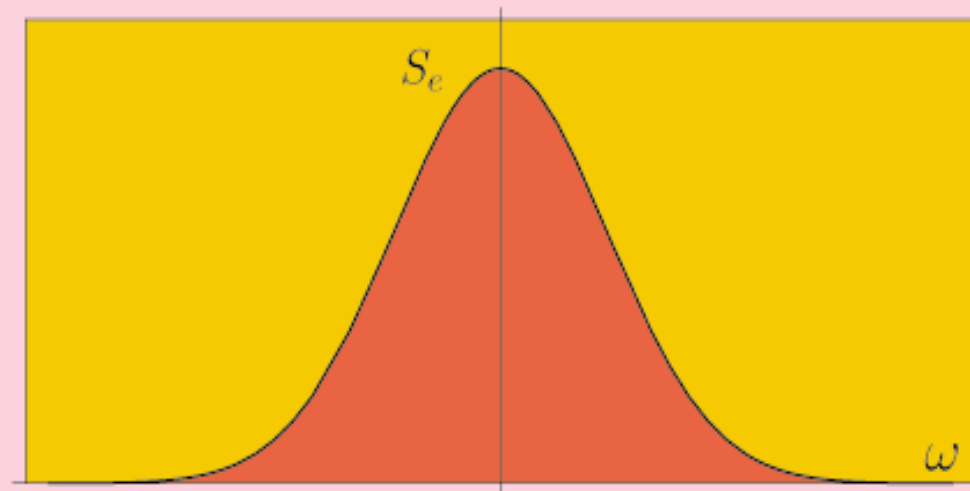
# Total Cross-Section Estimate

Consider an antenna with a 1-degree beam measuring the ionospheric plasma at 300 km range and using a 300 microsecond pulse.

If the electron density is  $10^{12} \text{ m}^{-3}$ , the total number of electrons scattering into a given measurement is  $\sim 8.8 \times 10^{23}$ . This yields a total cross-section of  $88 \text{ mm}^2$  – we need a big radar!



$$S_e(\mathbf{k}, \omega) = N_e \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$



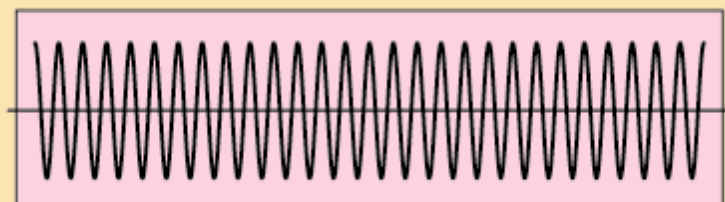
# Incoherent scattering - the short story



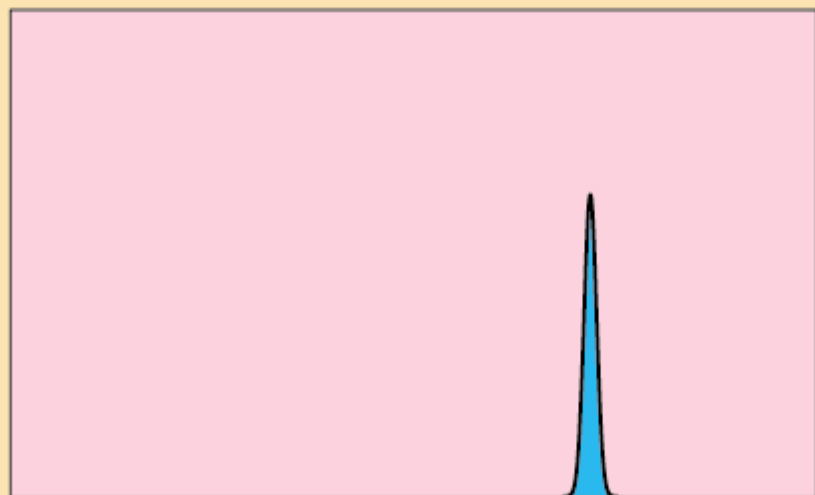
# Incoherent scattering - the short story







time



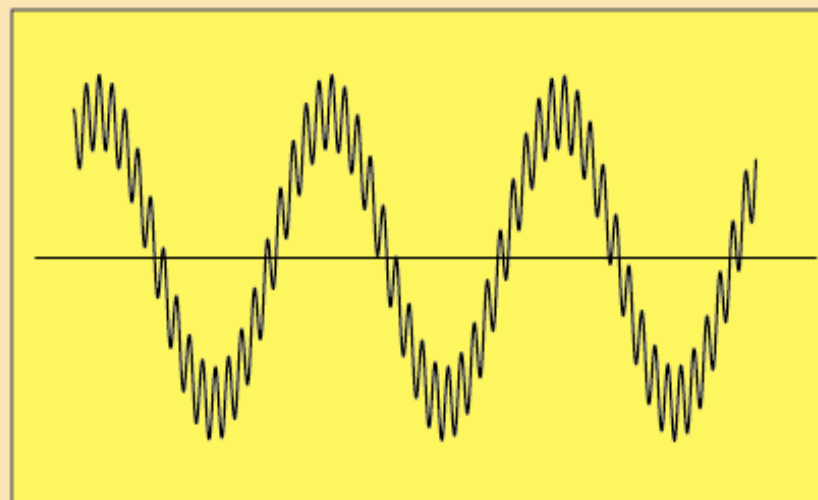
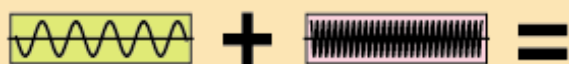
frequency



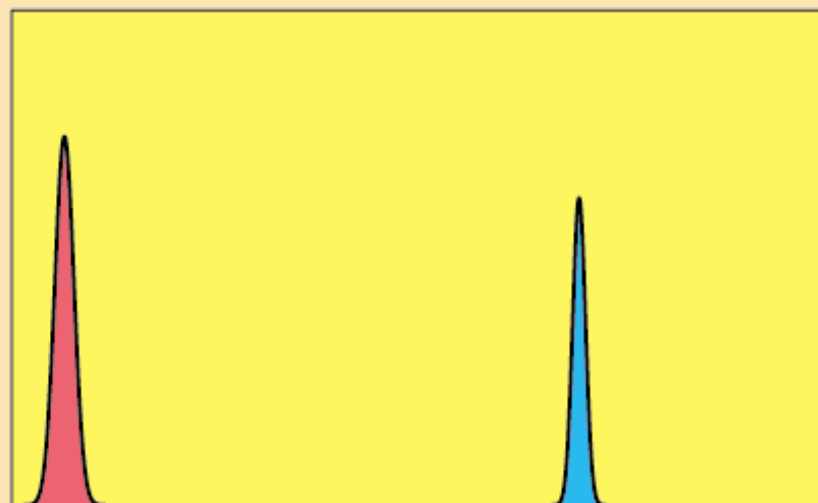
time



frequency

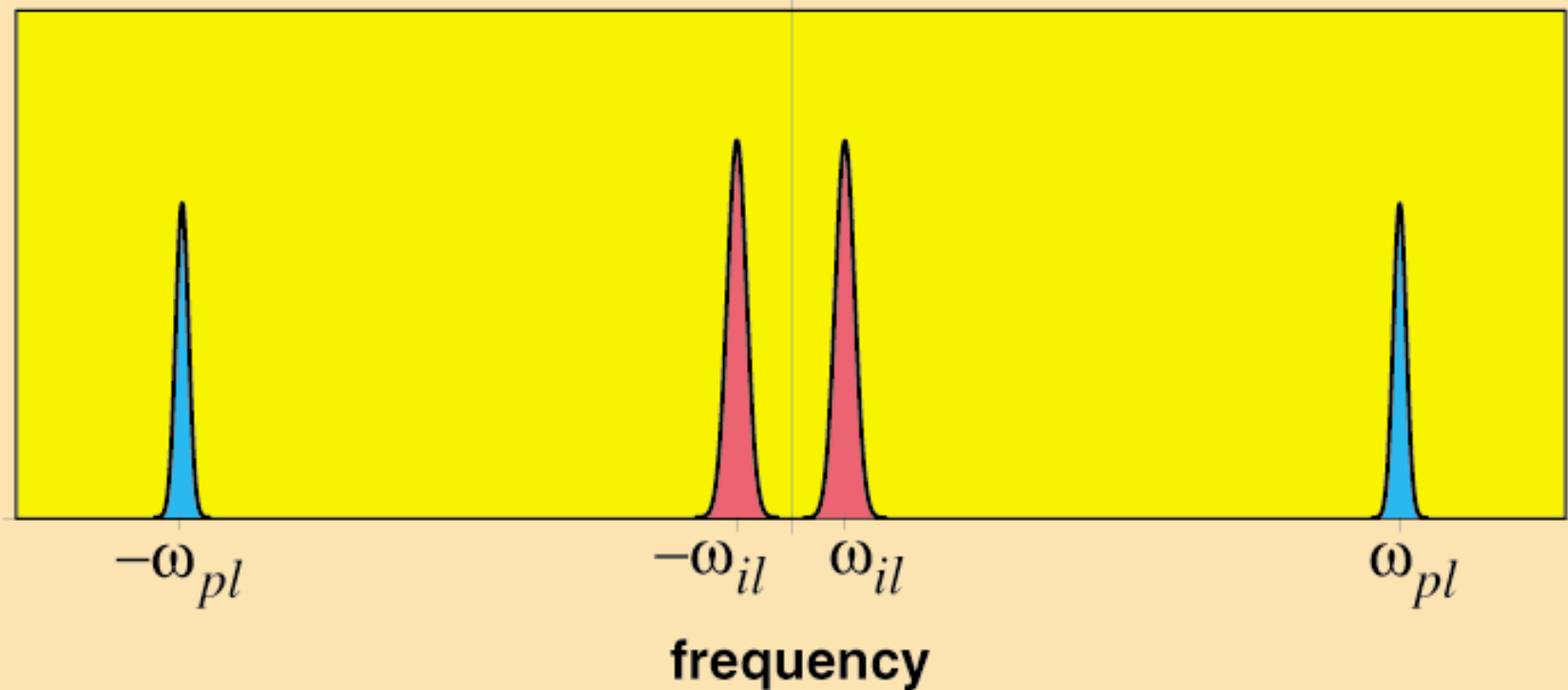


time

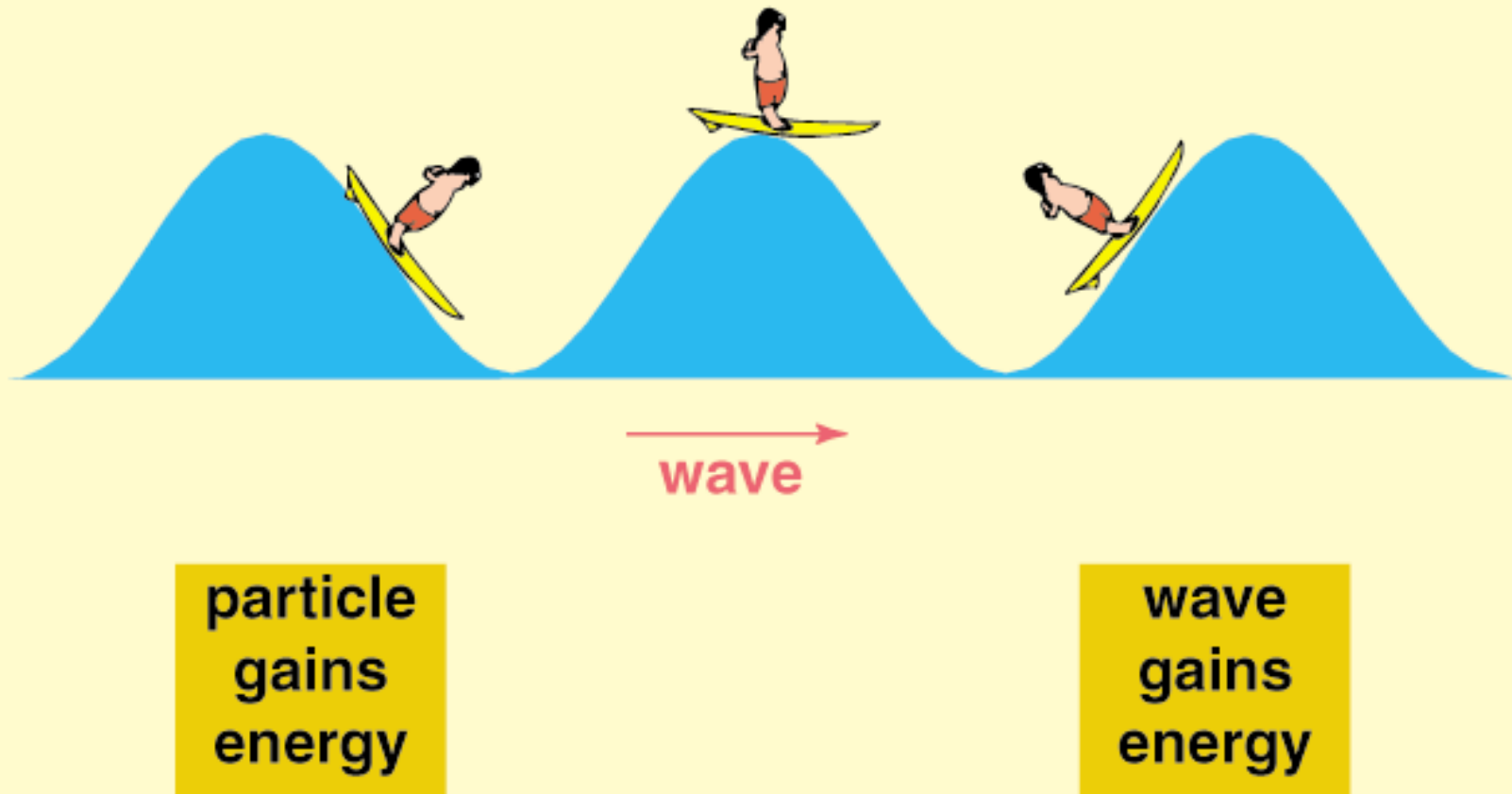


frequency

# Plasma Wave Approach (cont' d)



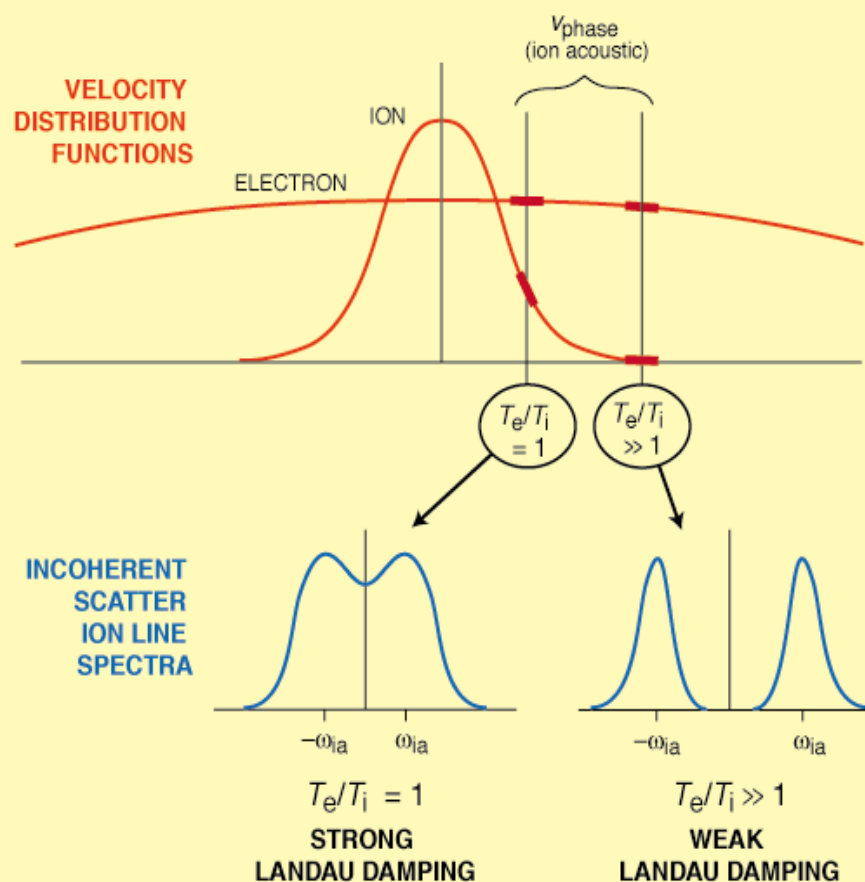
## Landau wave-particle interactions

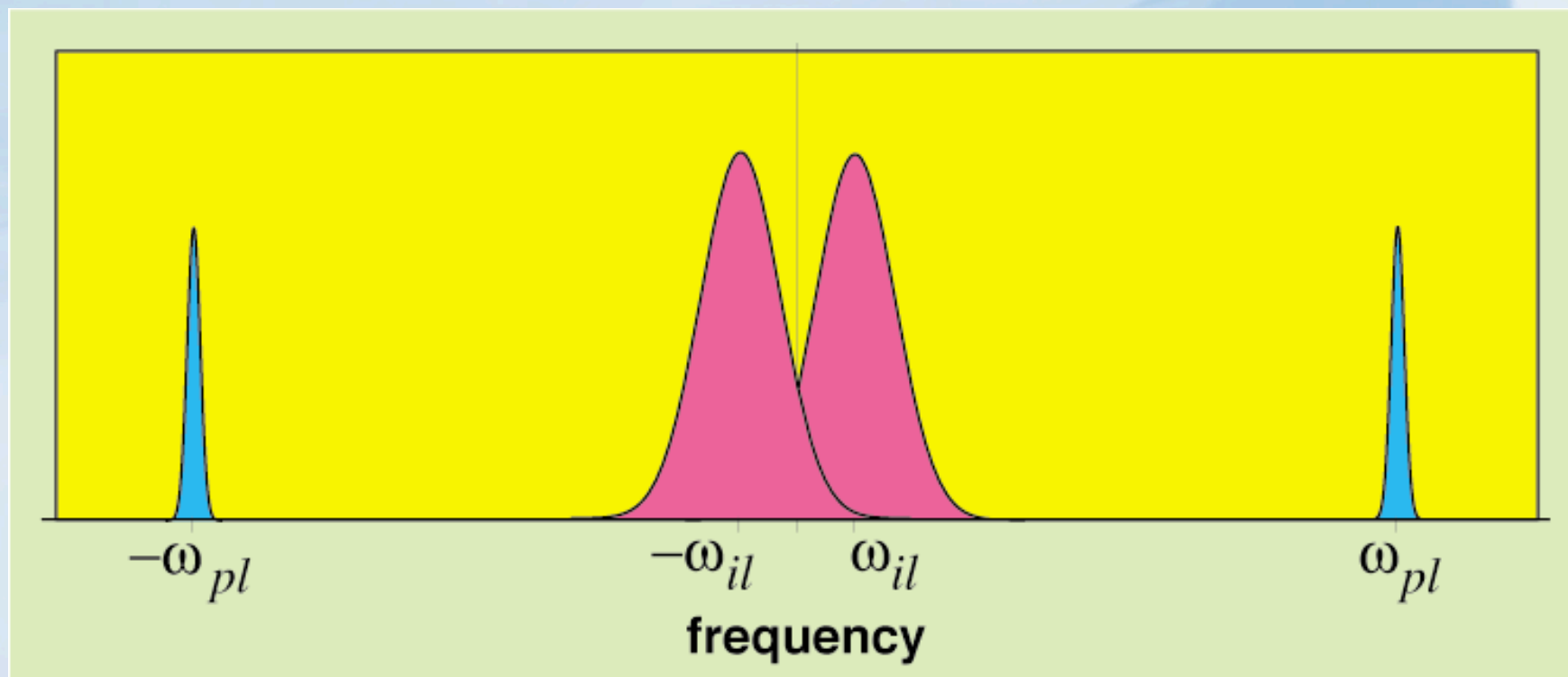


## THE EFFECT OF LANDAU DAMPING ON THE INCOHERENT SCATTER ION LINE SPECTRUM

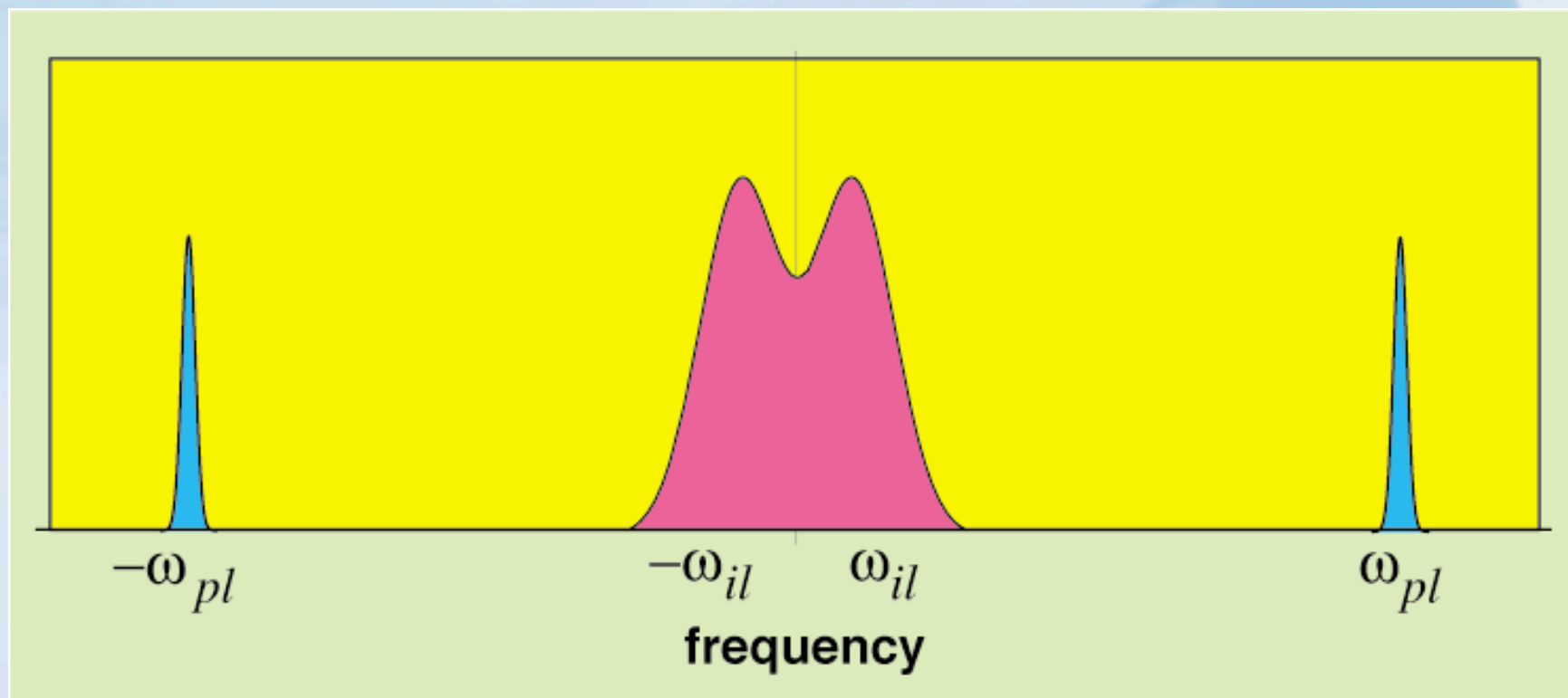
ION-ACOUSTIC  
DISPERSION  
EQUATION

$$\omega_{ia} = k v_{\text{phase}} = k \left( \frac{T_e + 3T_i}{m_i} \right)^{1/2}$$









## Incoherent Scattering Spectrum

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

Plasma line

Ion line

electric susceptibility  $\chi_{e,i}(\mathbf{k}, \omega)$

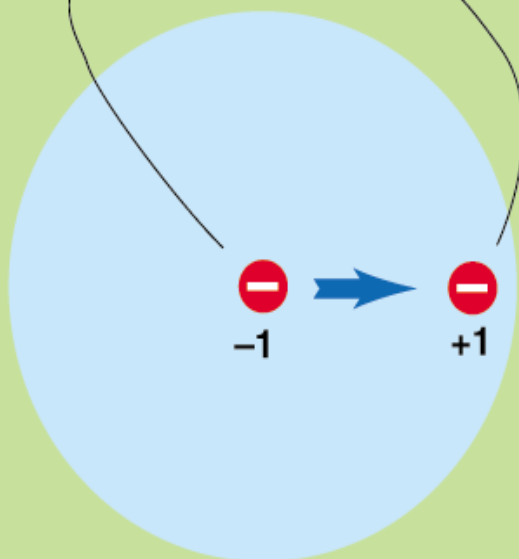
dielectric constant function  $\epsilon(\mathbf{k}, \omega)$

velocity distribution function  $f_{e,i}(\mathbf{v})$

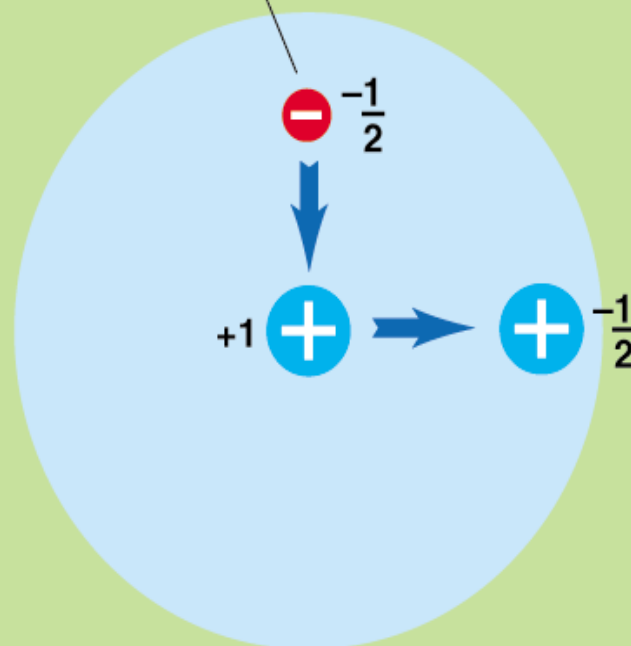
### Plasma Line $S_{PL}(\mathbf{k}, \omega)$

### Ion Line $S_{IL}(\mathbf{k}, \omega)$

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

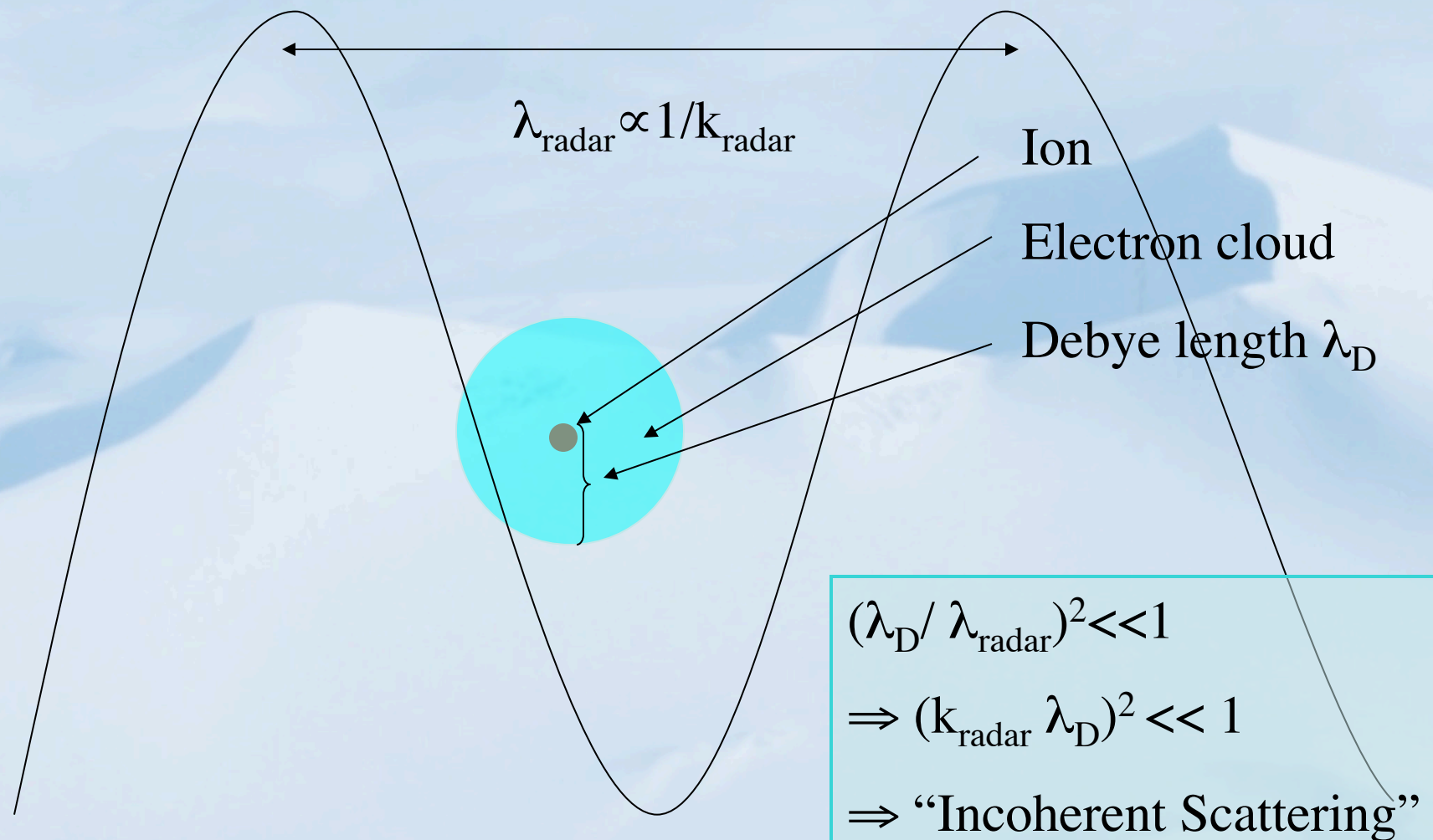


electron with cloud



ion with cloud

# Debye length dependence



**Plasma Line**  $S_{PL}(\mathbf{k}, \omega)$

**Ion Line**  $S_{IL}(\mathbf{k}, \omega)$

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

**Plasma Line**  $S_{PL}(\mathbf{k}, \omega)$

**Ion Line**  $S_{IL}(\mathbf{k}, \omega)$

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

$$\epsilon(\mathbf{k}, \omega) = 0$$

$$\omega_{pl}(k) \approx \omega_{pe}(1 + 3\lambda_D^2 k^2)$$

$$\omega_{ia}(k) \approx k \sqrt{\frac{T_e + 3T_i}{m_i}}$$

**Plasma Line**  $S_{PL}(\mathbf{k}, \omega)$

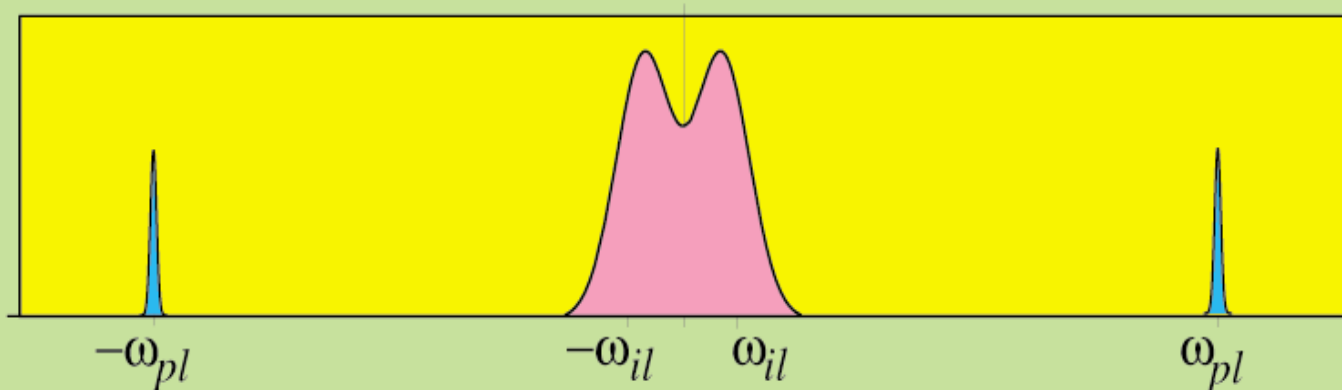
**Ion Line**  $S_{IL}(\mathbf{k}, \omega)$

$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

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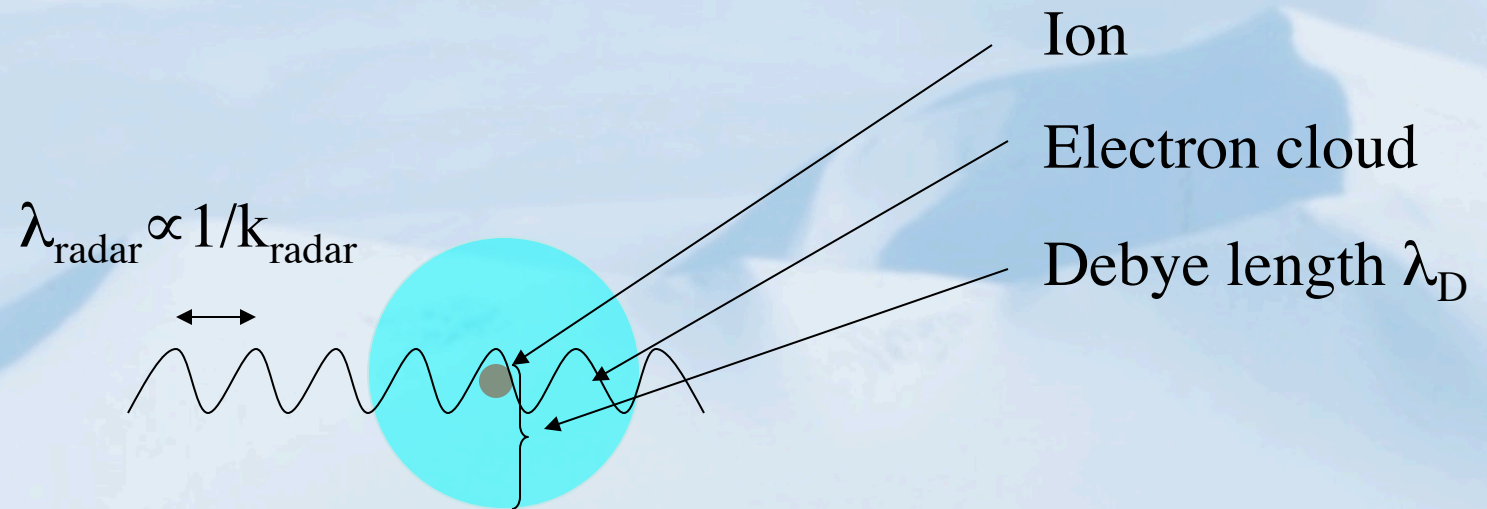
$$\omega_{pl}(k) \approx \omega_{pe}(1 + 3\lambda_D^2 k^2)$$

$$\omega_{ia}(k) \approx k \sqrt{\frac{T_e + 3T_i}{m_i}}$$





# Debye length dependence



$$(\lambda_D / \lambda_{\text{radar}})^2 > 1$$

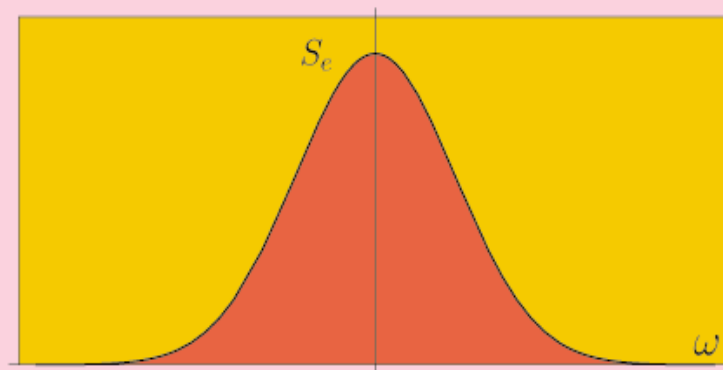
$$\Rightarrow (k_{\text{radar}} \lambda_D)^2 > 1$$

$\Rightarrow$  No collective interactions

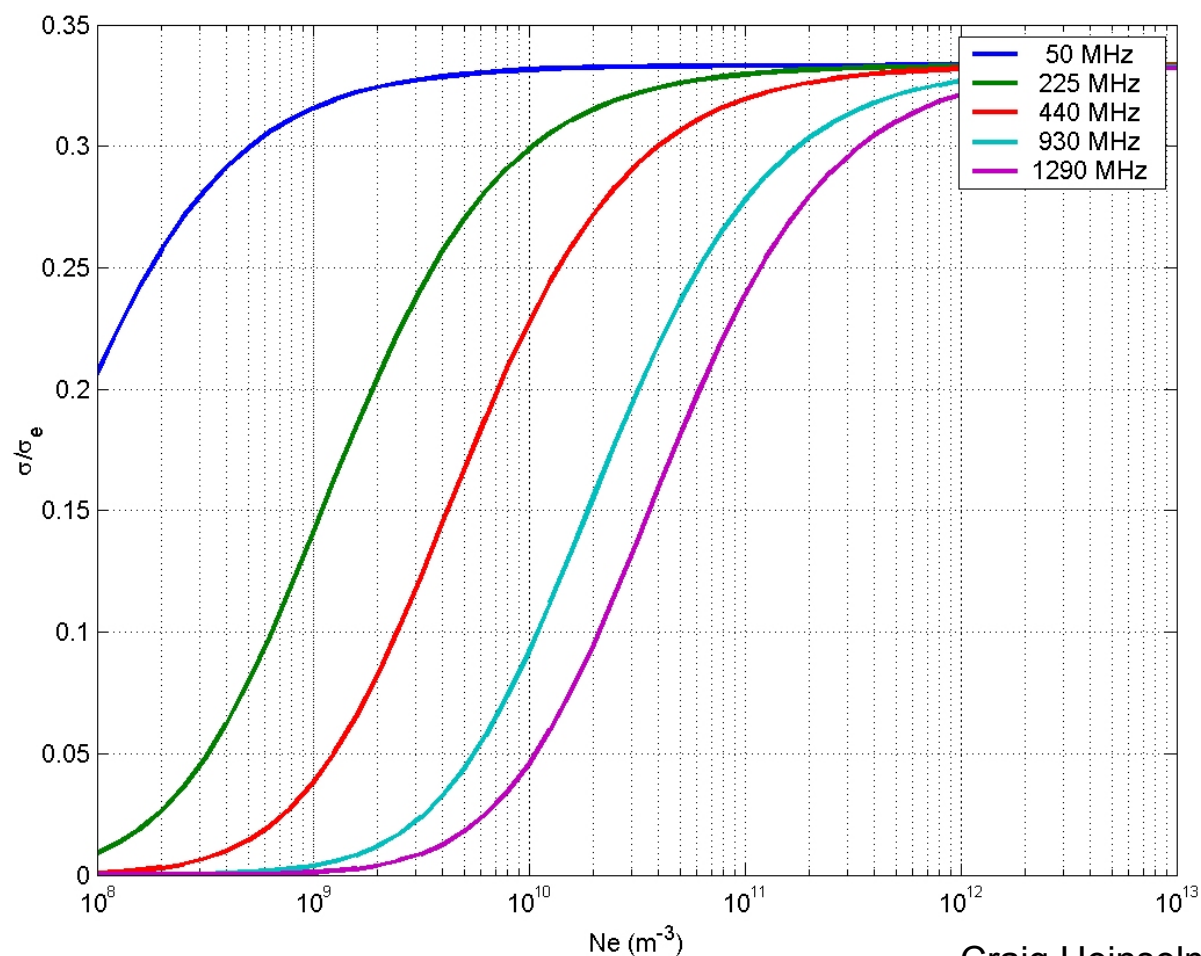
**no collective interactions**

~~$$S_e(\mathbf{k}, \omega) = N_e \left| 1 - \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) + N_i \left| \frac{\chi_e(\mathbf{k}, \omega)}{\epsilon(\mathbf{k}, \omega)} \right|^2 \int d\mathbf{v} f_i(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$~~

$$S_e(\mathbf{k}, \omega) = N_e \int d\mathbf{v} f_e(\mathbf{v}) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$



# Debye Length Dependencies

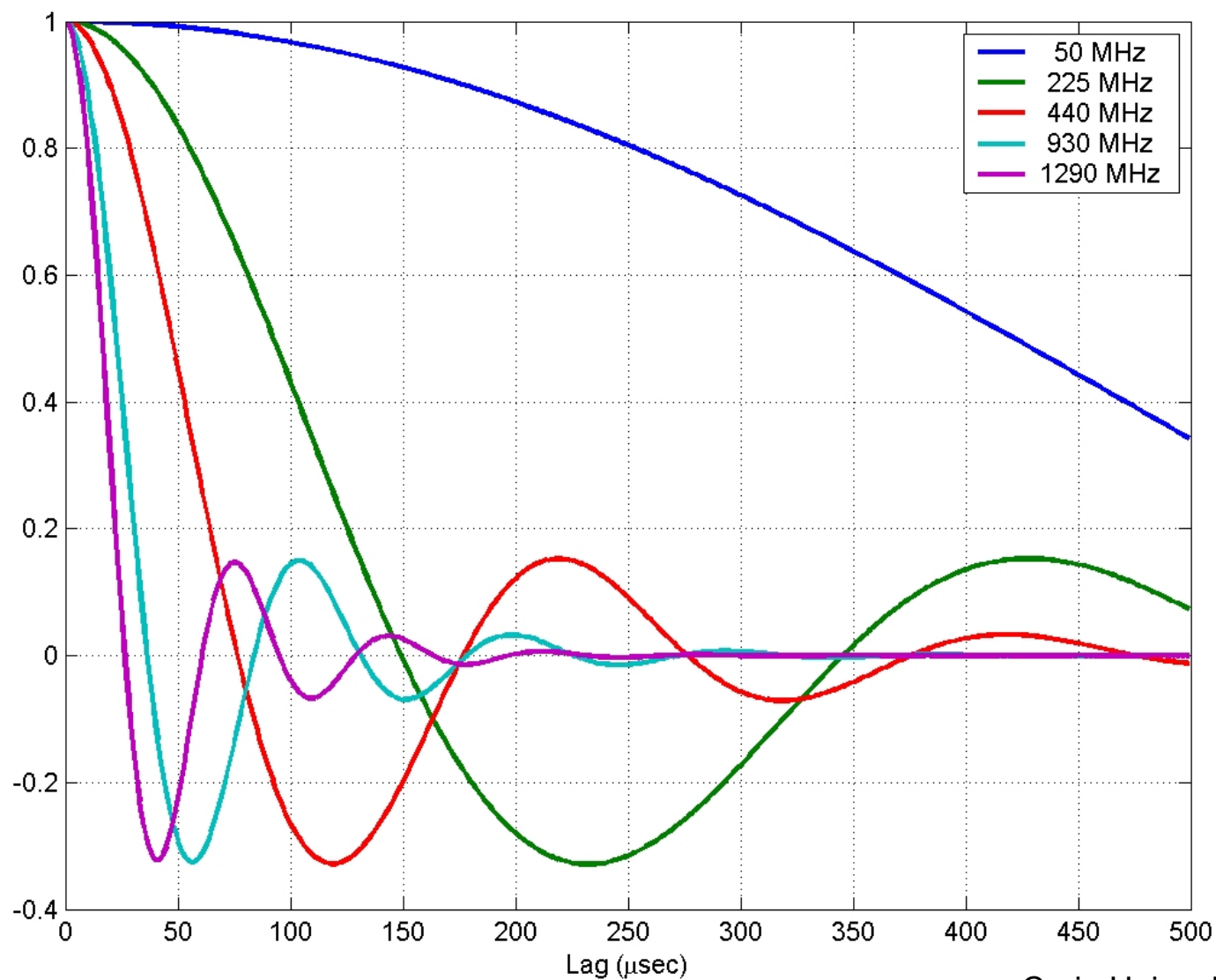


## Parameters

Ti: 1000 K

Te: 2000 K

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### Parameters

Ne:  $10^{12} \text{ m}^{-3}$

Ti: 1000 K

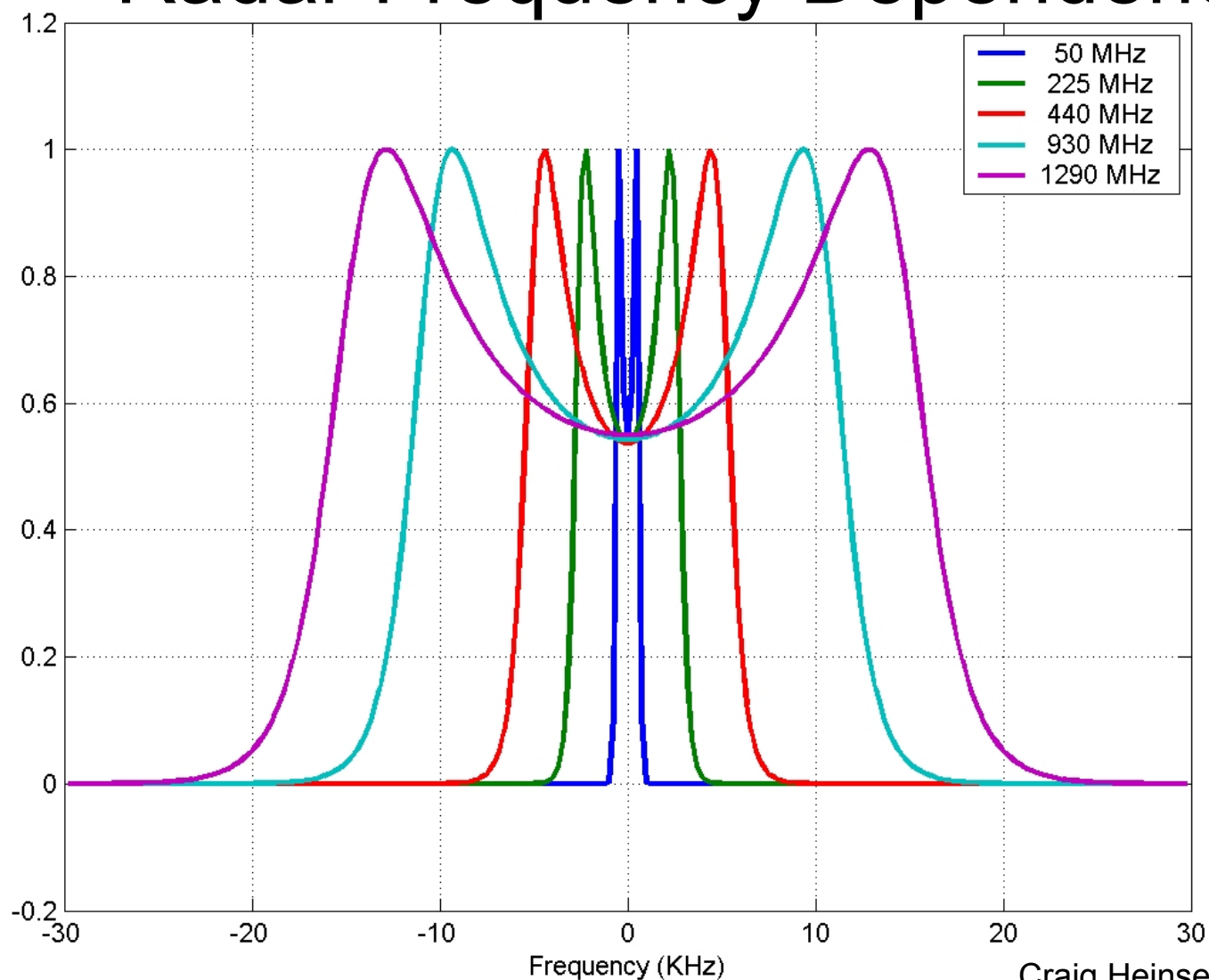
Te: 2000 K

Comp: 100% O<sup>+</sup>

$\nu_{in}$ :  $10^{-6} \text{ KHz}$

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# Radar Frequency Dependencies



## Parameters

Ne:  $10^{12} \text{ m}^{-3}$

Ti: 1000 K

Te: 2000 K

Comp: 100% O<sup>+</sup>

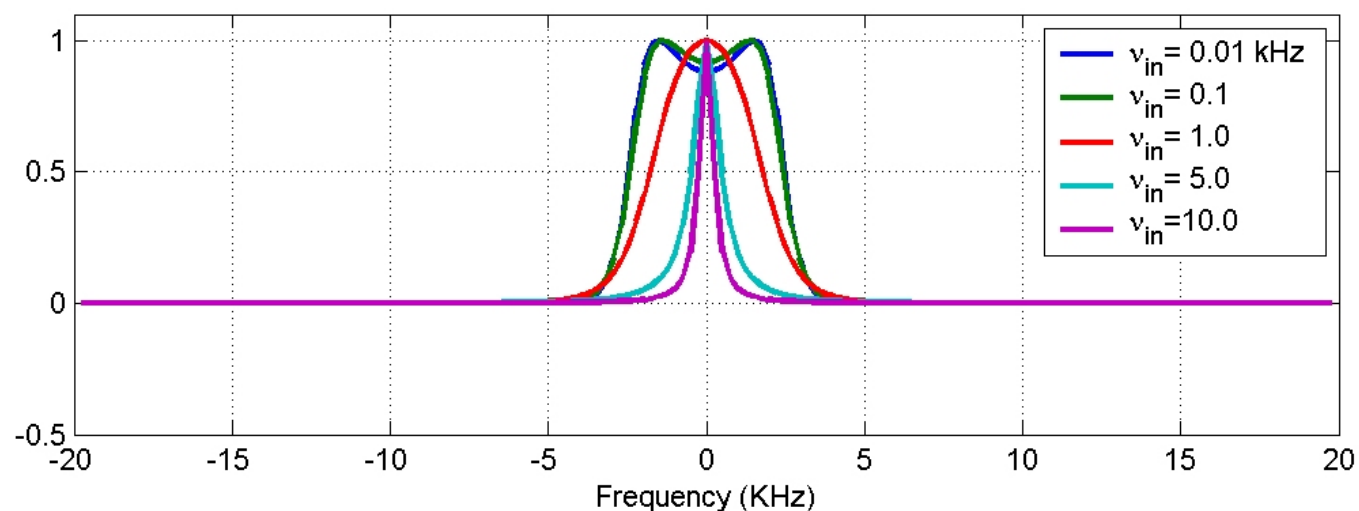
$\nu_{in}$ :  $10^{-6} \text{ KHz}$

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With the frequency of the radar chosen (which is a one time thing!), how does the spectra depend on geophysical parameters?



# Ion-Neutral Collision Frequency



## Parameters

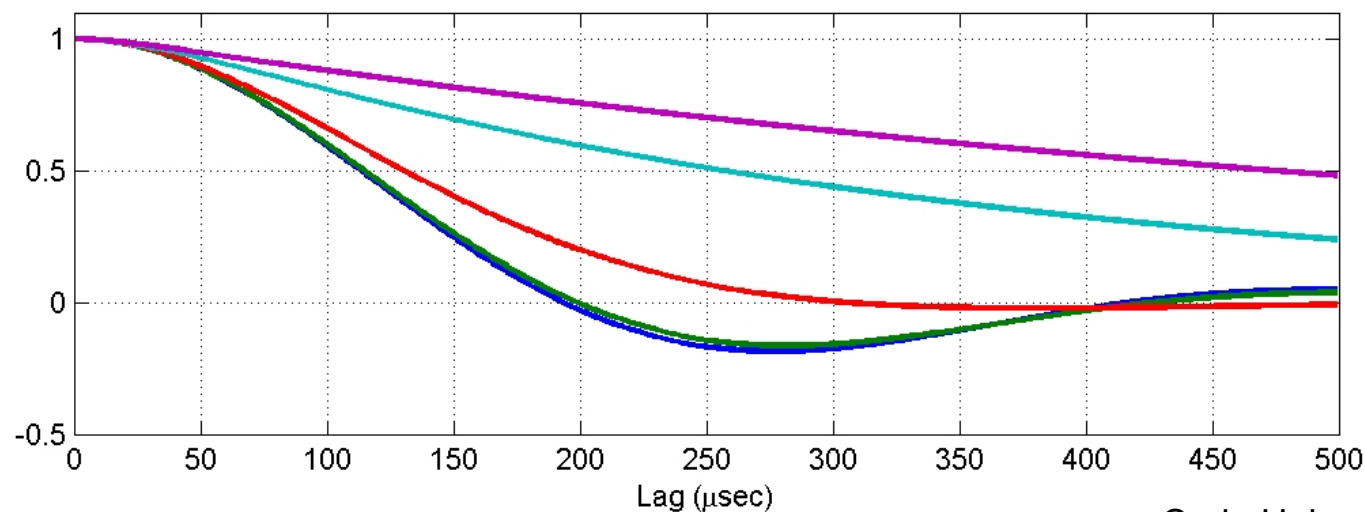
Freq: 449 MHz

Ne:  $10^{12} \text{ m}^{-3}$

Ti: 500 K

Te: 500 K

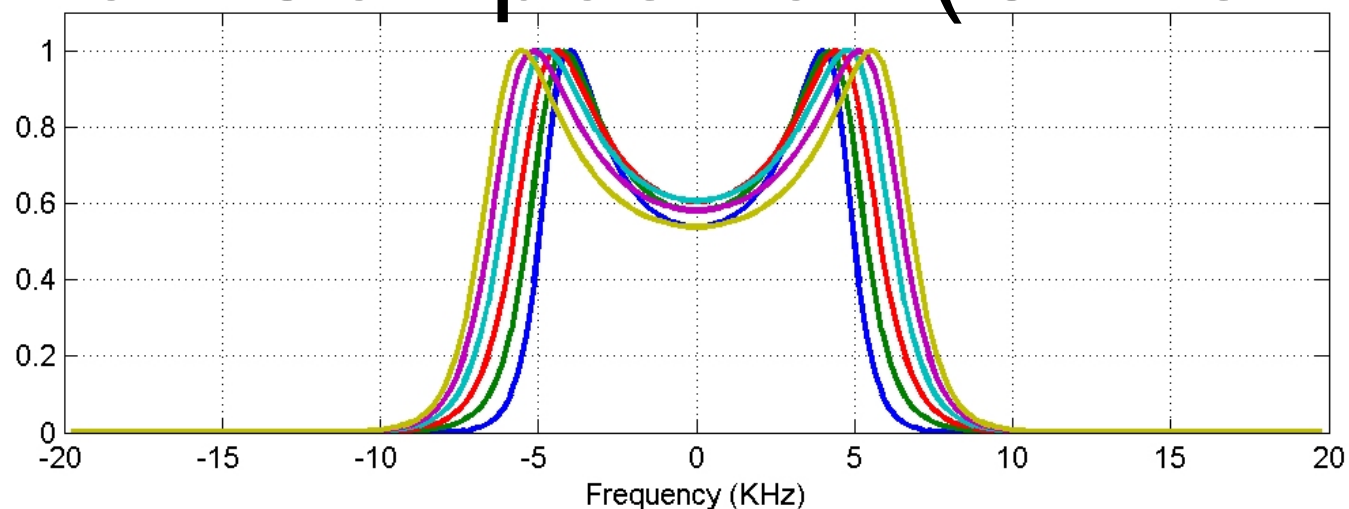
Comp: 100%  $\text{NO}^+$



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# Ion Composition ( $O^+$ vs. $NO^+$ )



## Parameters

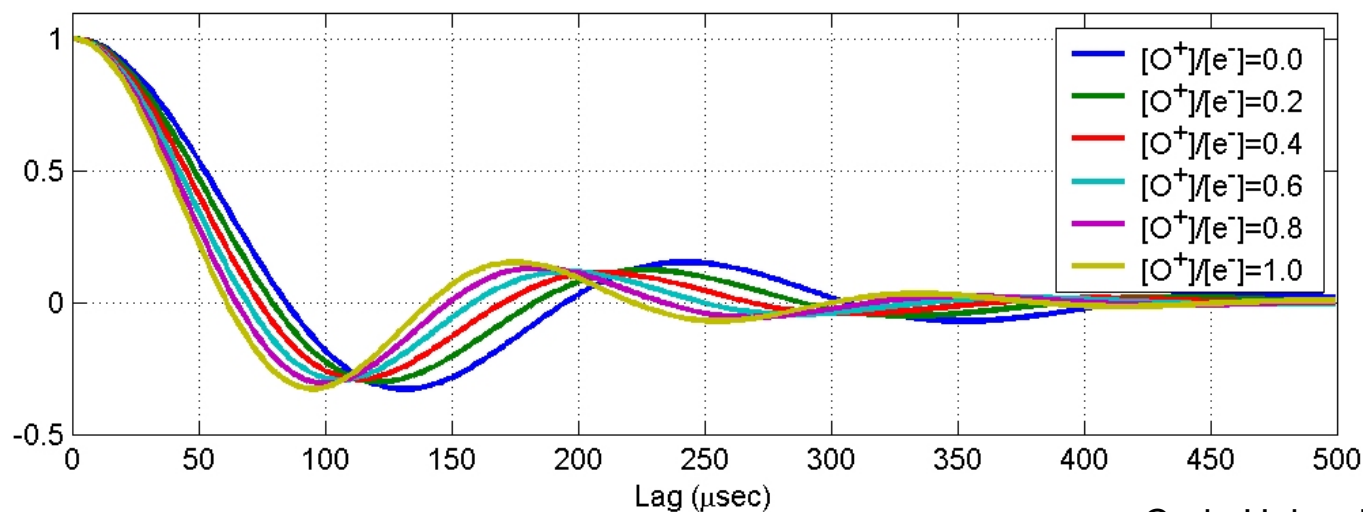
Freq: 449 MHz

Ne:  $10^{12} \text{ m}^{-3}$

Ti: 1500 K

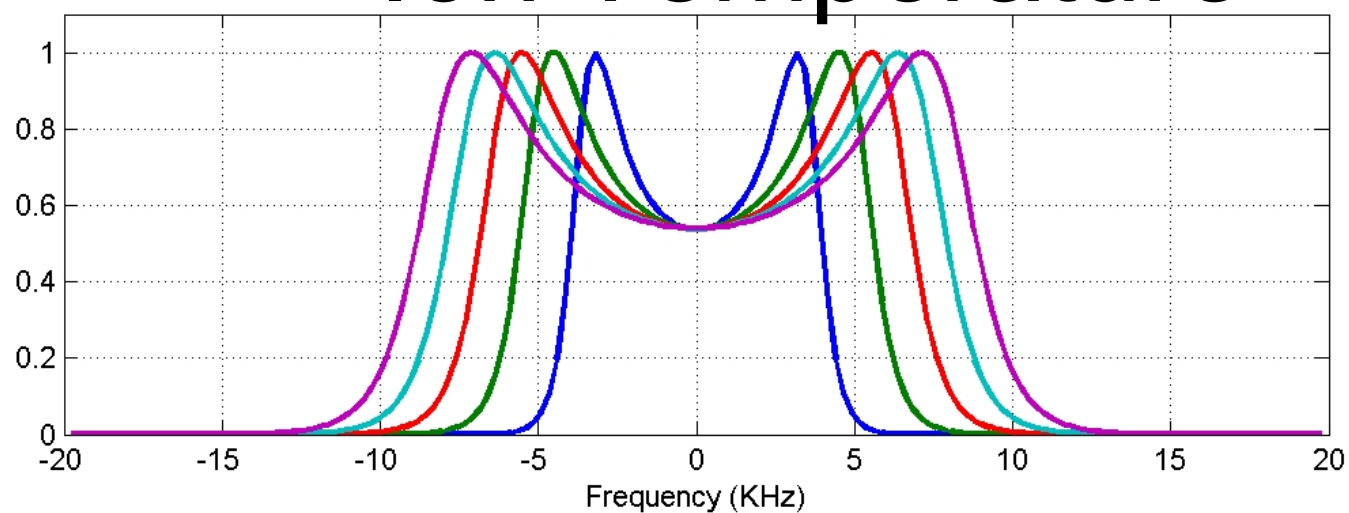
Te: 3000 K

$\nu_{in}$ :  $10^{-6} \text{ KHz}$



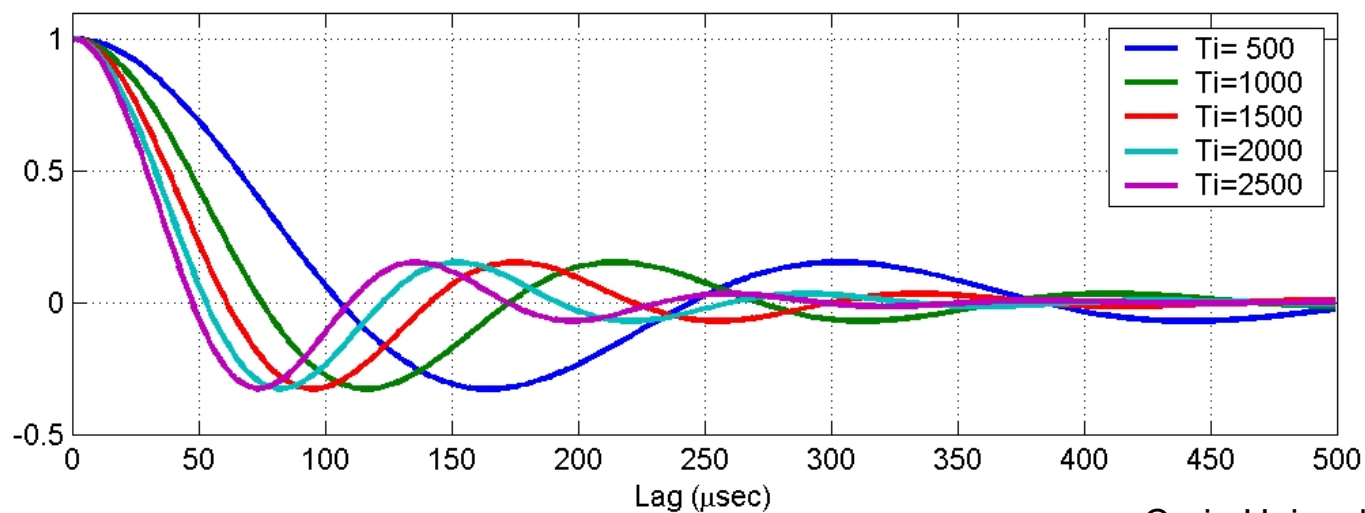
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# Ion Temperature



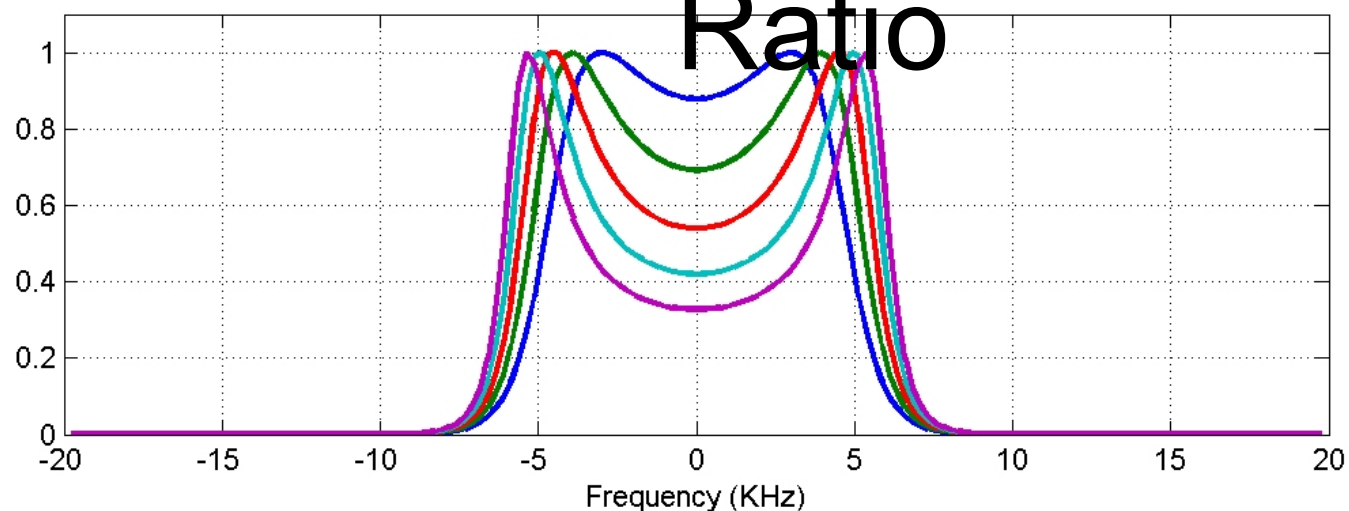
## Parameters

Freq: 449 MHz  
 Ne:  $10^{12} \text{ m}^{-3}$   
 Te:  $2 \cdot T_i$   
 Comp: 100% O<sup>+</sup>  
 $\nu_{in}$ :  $10^{-6} \text{ KHz}$



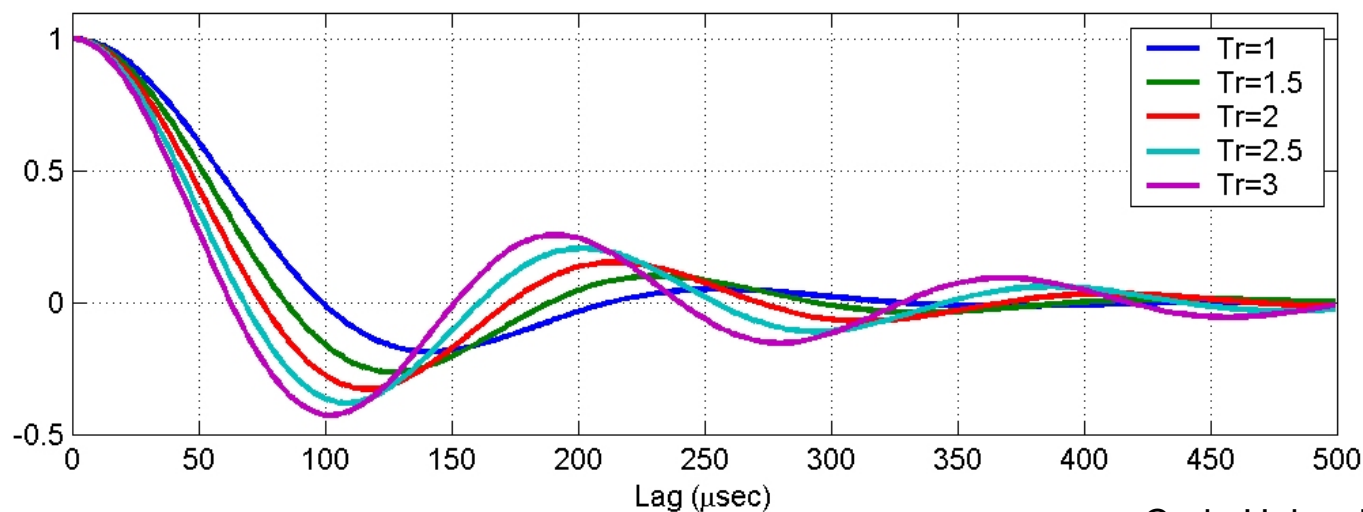
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# Electron/Ion Temperature Ratio



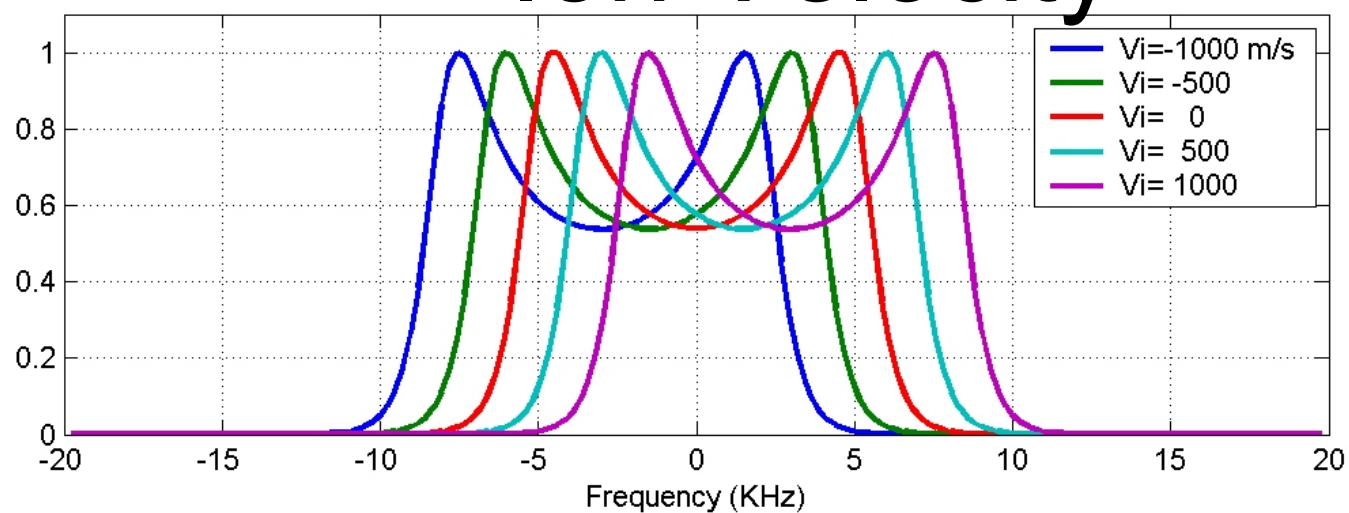
## Parameters

Freq: 449 MHz  
 Ne:  $10^{12} \text{ m}^{-3}$   
 Ti: 1000 K  
 Comp: 100% O<sup>+</sup>  
 $\nu_{in}$ :  $10^{-6} \text{ KHz}$



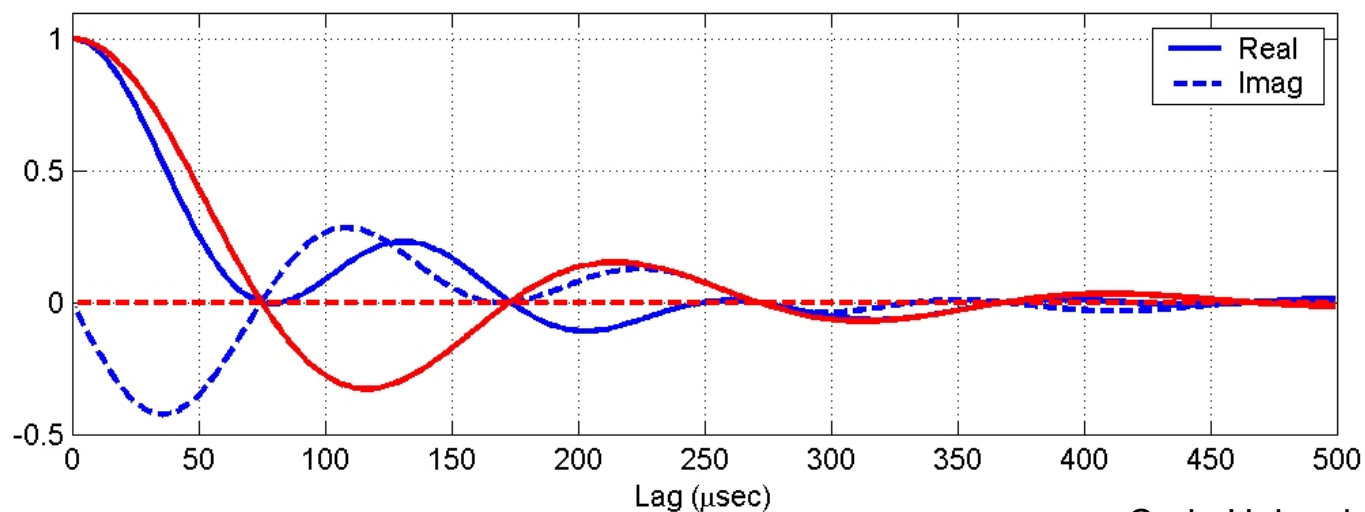
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# Ion Velocity



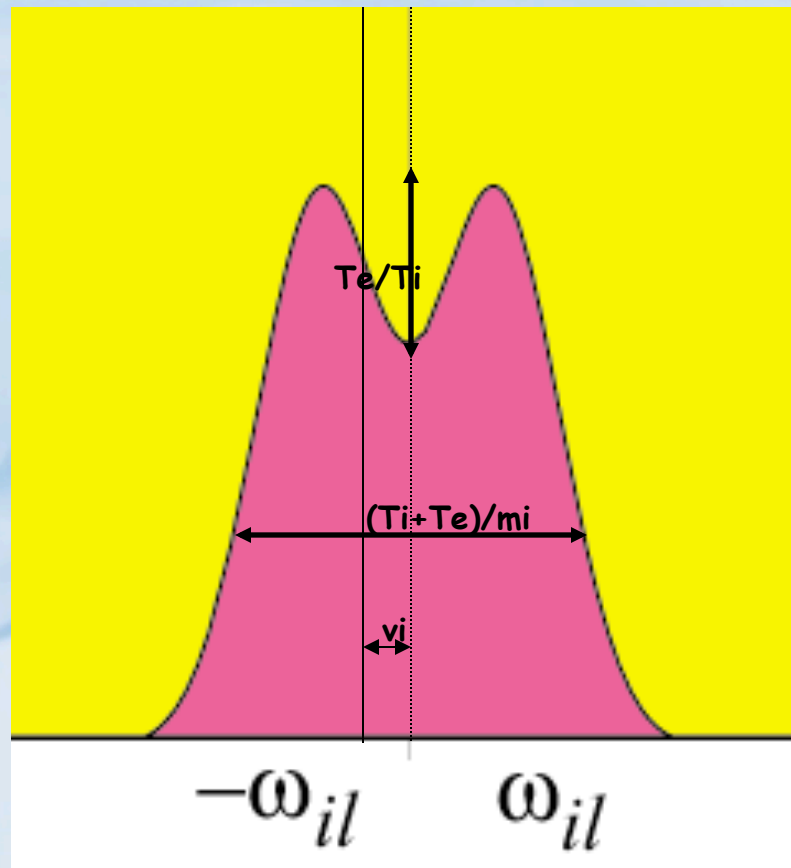
## Parameters

Freq: 449 MHz  
 Ne:  $10^{12} \text{ m}^{-3}$   
 Ti: 1000 K  
 Te: 2000 K  
 Comp: 100% O<sup>+</sup>  
 $v_{in}$ :  $10^{-6}$  KHz



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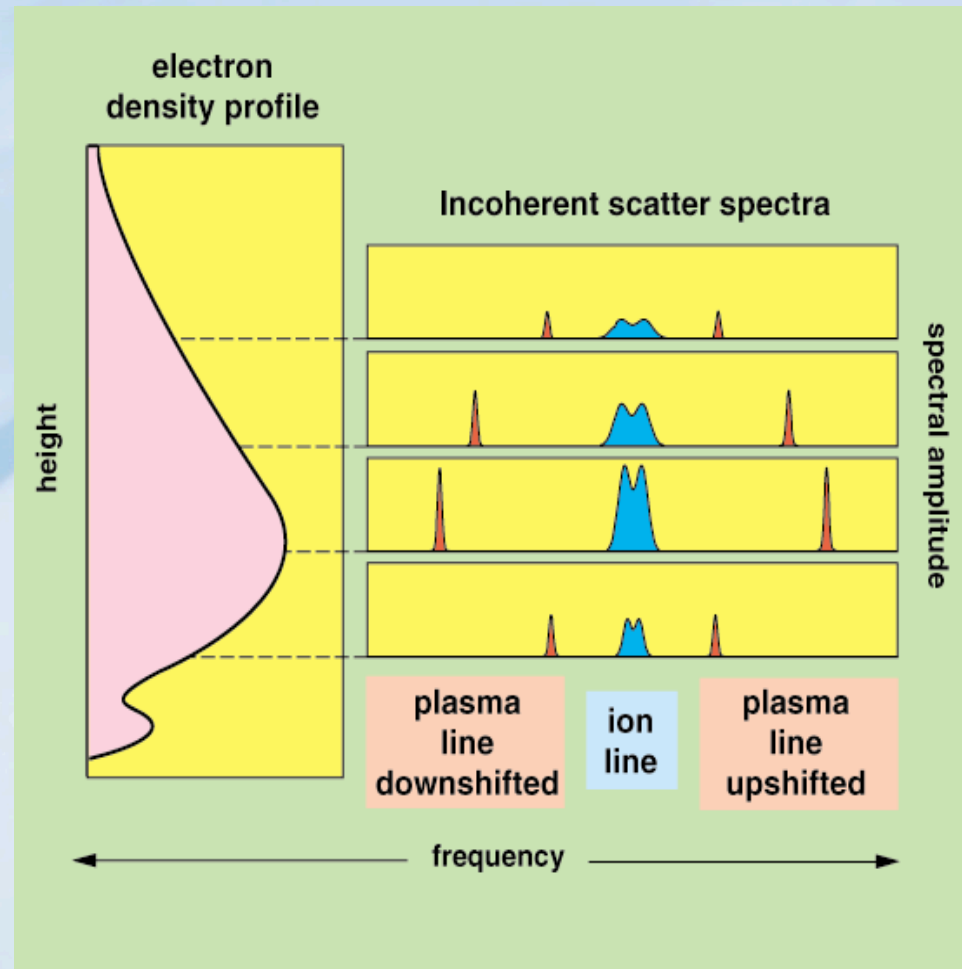
...or to sum up...



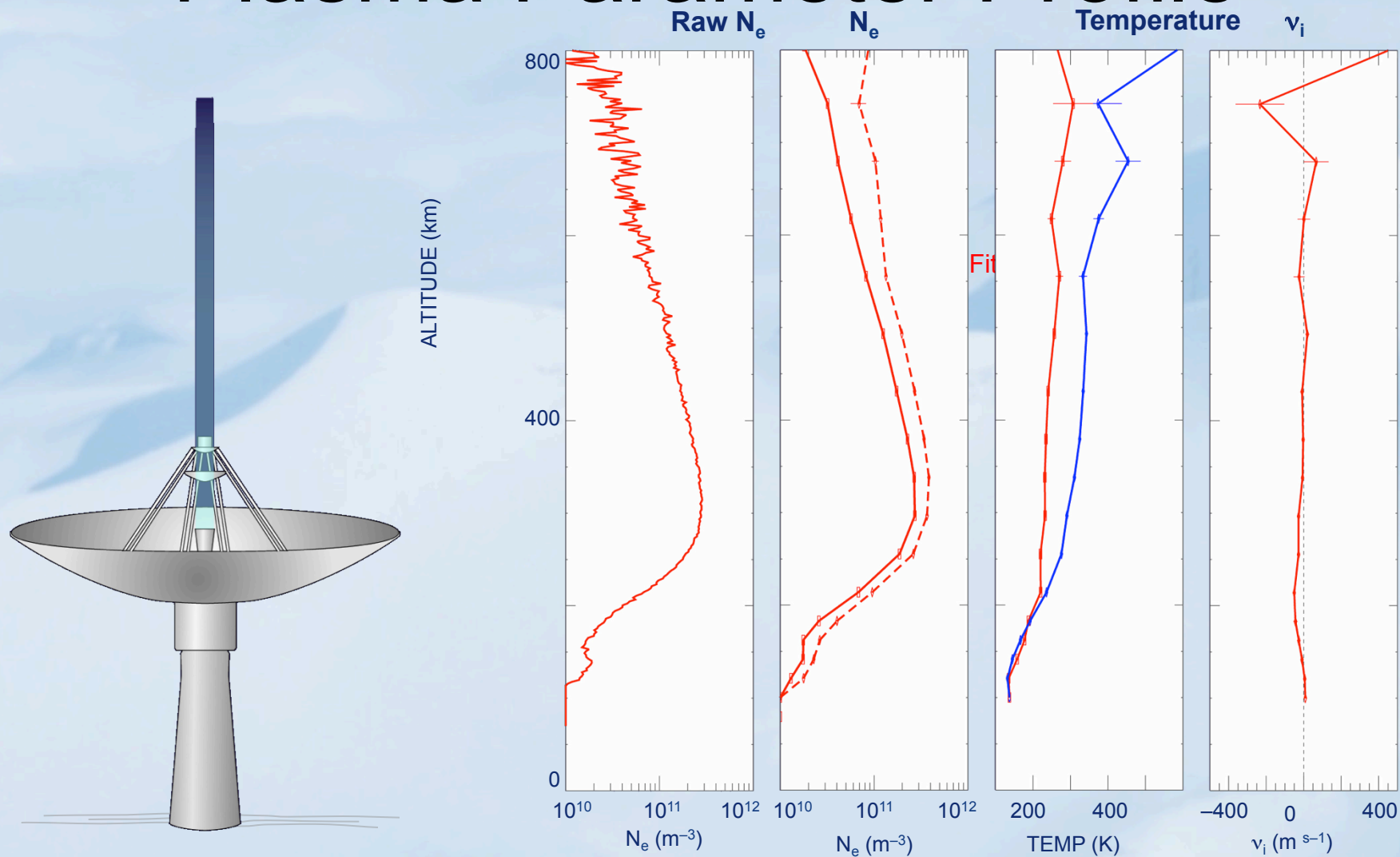
- Ion (and electron) temperature ( $T_i$  and  $T_e$ ) to ion mass ( $m_i$ ) ratio from the width of the spectra
- Electron to ion temperature ratio ( $T_e/T_i$ ) from “peak\_to\_valley” ratio
- Electron (= ion) density from total area (corrected for temperatures)
- Ion velocity ( $v_i$ ) from the Doppler shift



# Spectral space as a function of altitude



# Plasma Parameter Profile





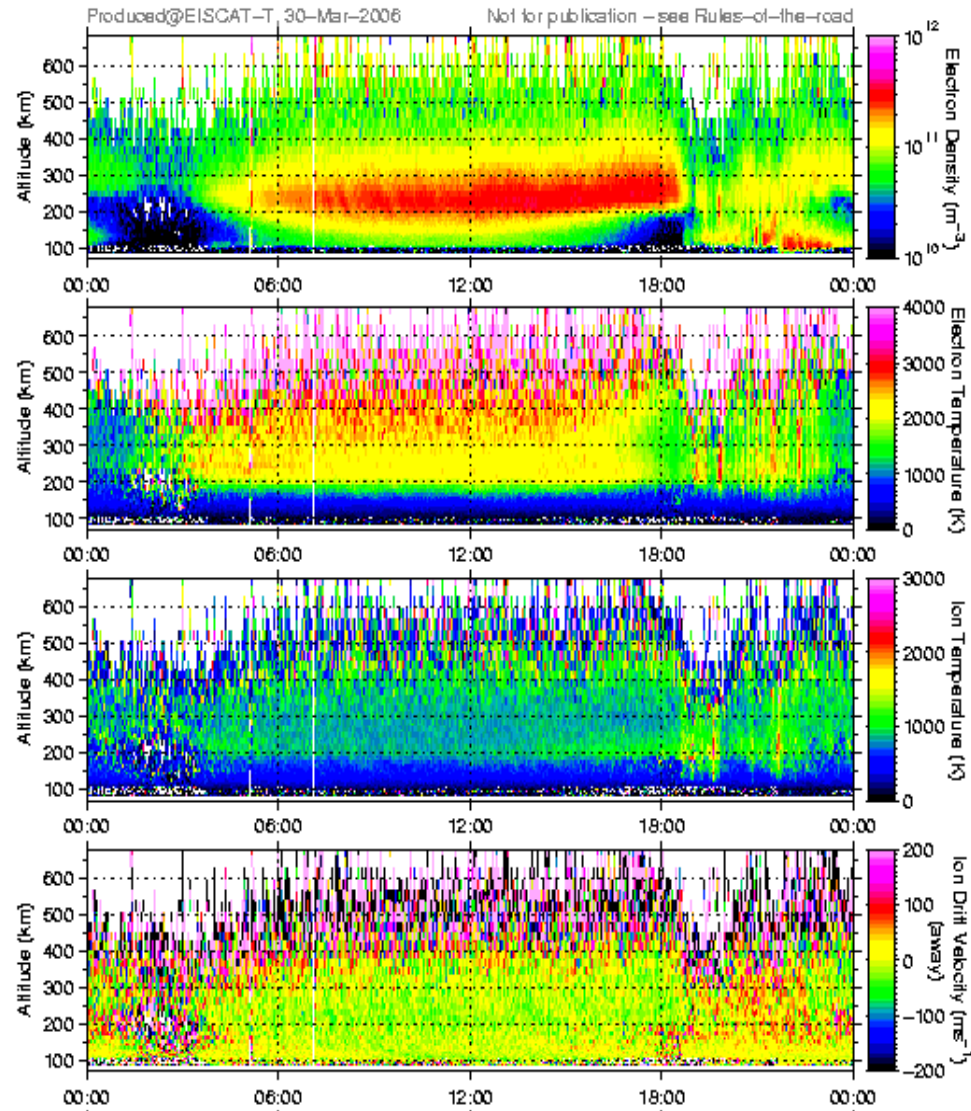
# EISCAT Scientific Association

## EISCAT UHF RADAR

CP, uhf, tau2pl, 29 March 2006

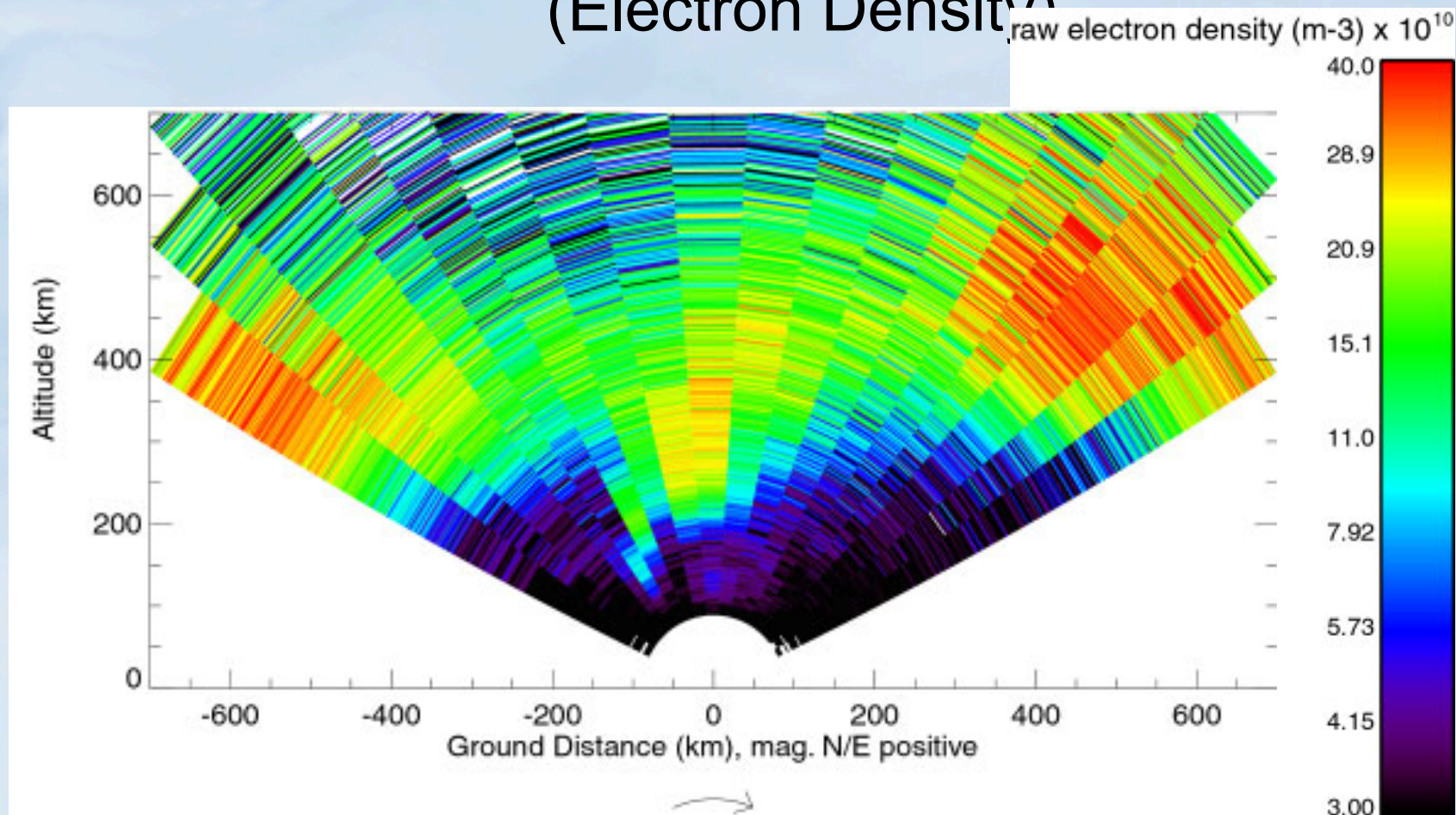
Produced@EISCAT-T, 30-Mar-2006

Not for publication - see Rules-of-the-road



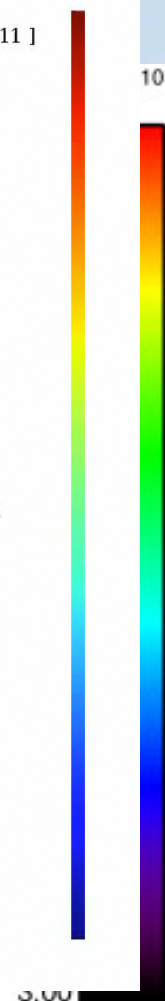
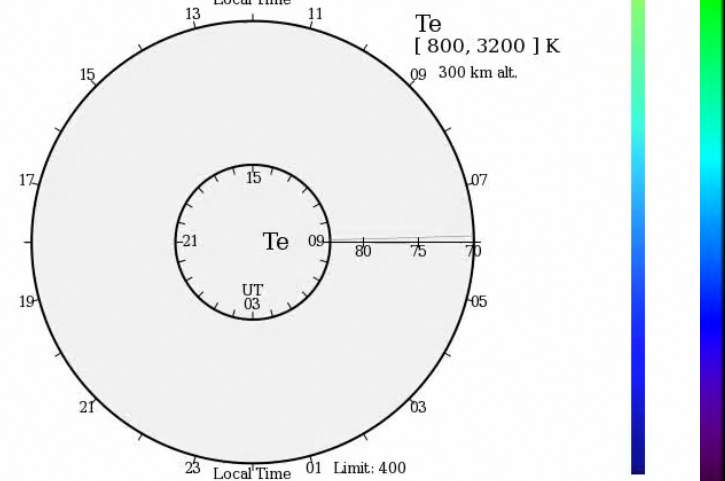
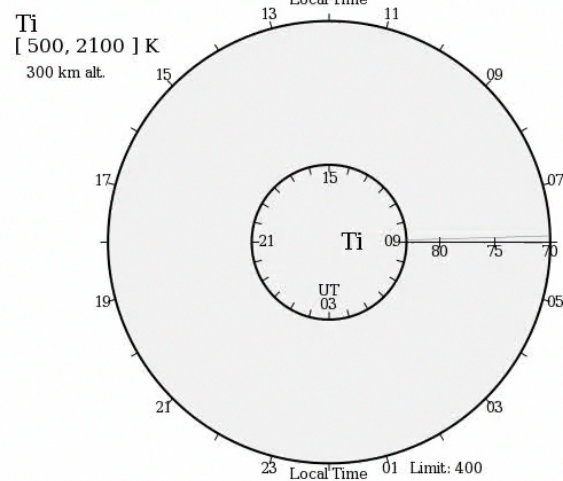
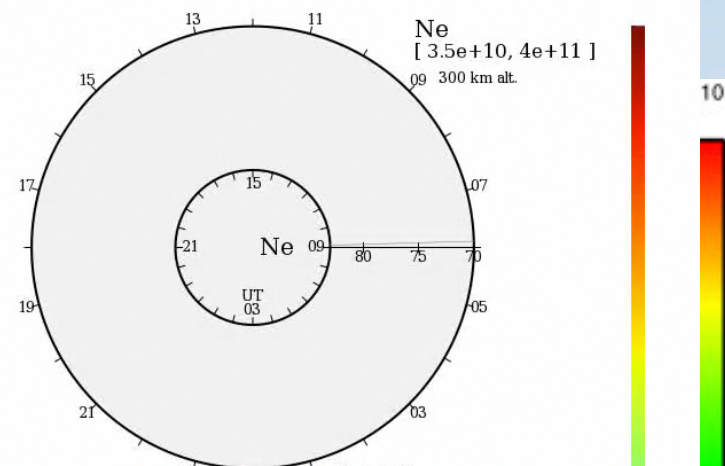
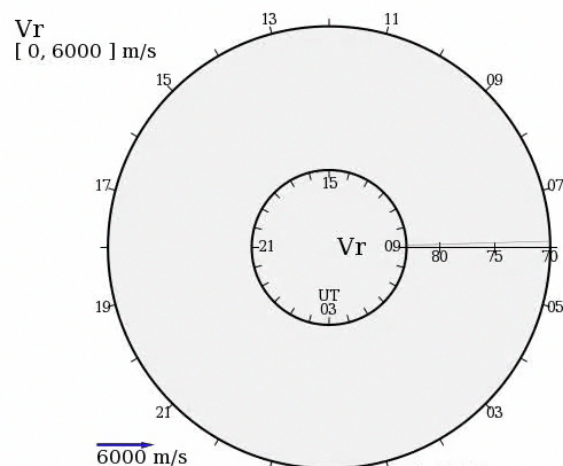
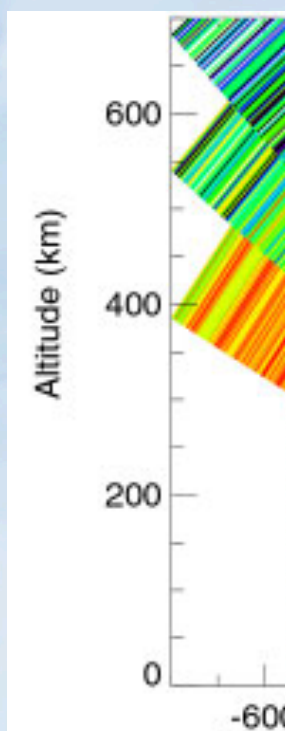
# Sondrestrom Radar View

(Electron Density)





# Sondrestrom Radar View



And this is the level data we  
will work on in the MADRIGAL  
session...