Introduction to Phased arrays

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What is a Phased Array?

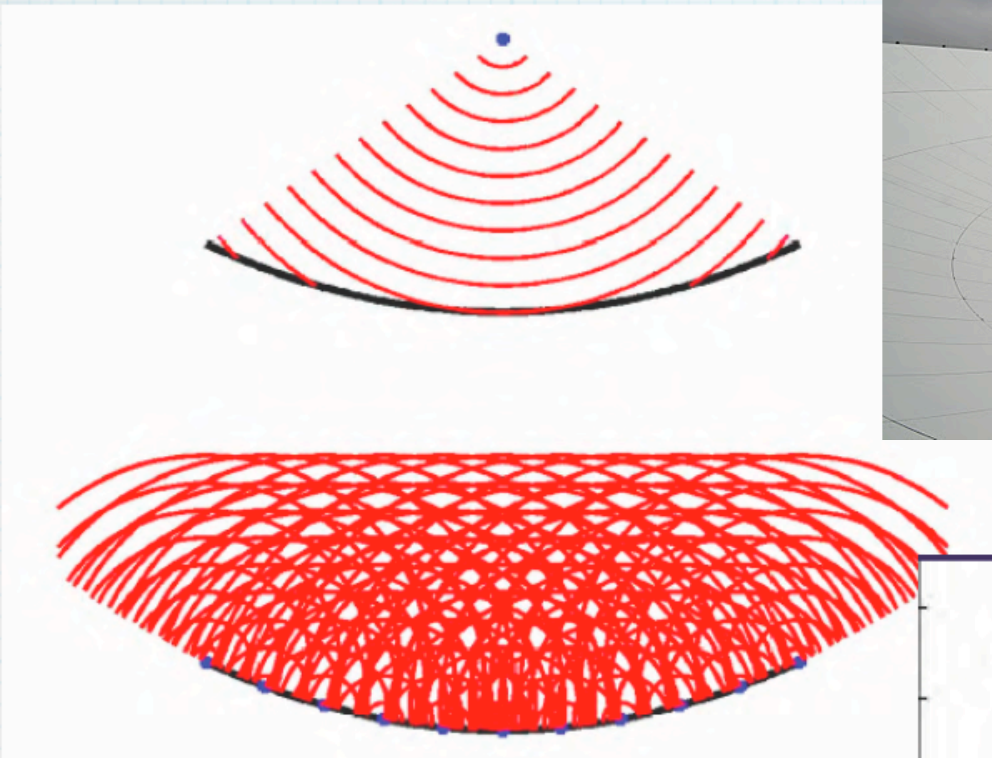
Large, single, steerable dish

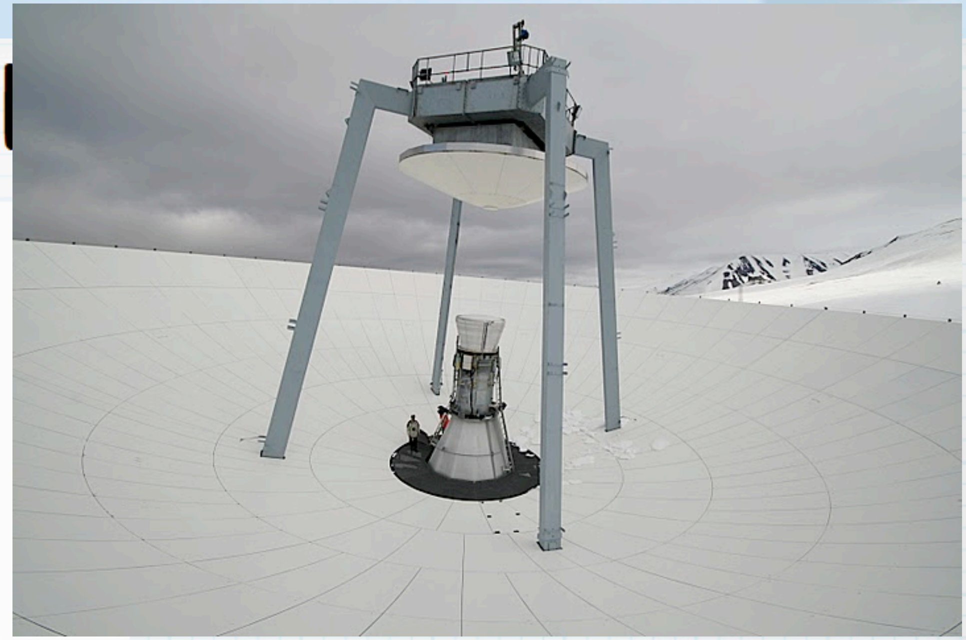
Array of small, discrete elements

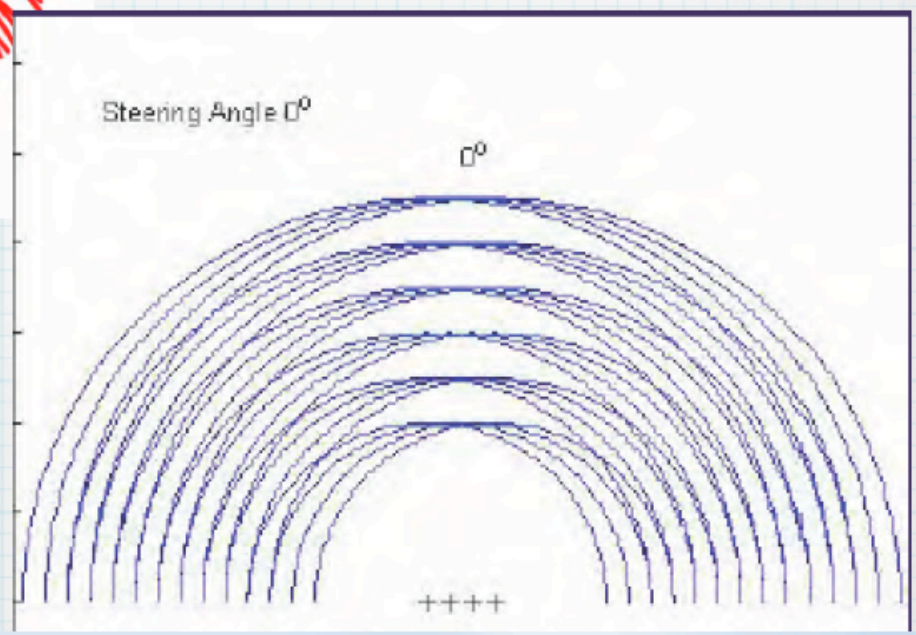
A phased array is a group of antennas whose effective (summed) radiation pattern can be altered by phasing the signals of the individual elements.

By varying the phasing of the different elements, the radiation pattern can be modified to be maximized / suppressed in given directions, within some limits determined by (a) the radiation pattern of the elements, (b) the size of the array, and (c) the configuration of the array.

What is a Pl







Array
 - Waves produced by each element interfere constructively in pointing direction

Communication Links

No antenna radiates isotropically

$$P_{inc} = P_t(\mathrm{W})\frac{G_{tx}}{4\pi R^2}$$

Power incident on target (W/m²)

$$P_r = P_{inc} A_{rec}^{eff}$$

Received Power (W)

Effective area (always less than physical area) of receive antenna, related to gain:

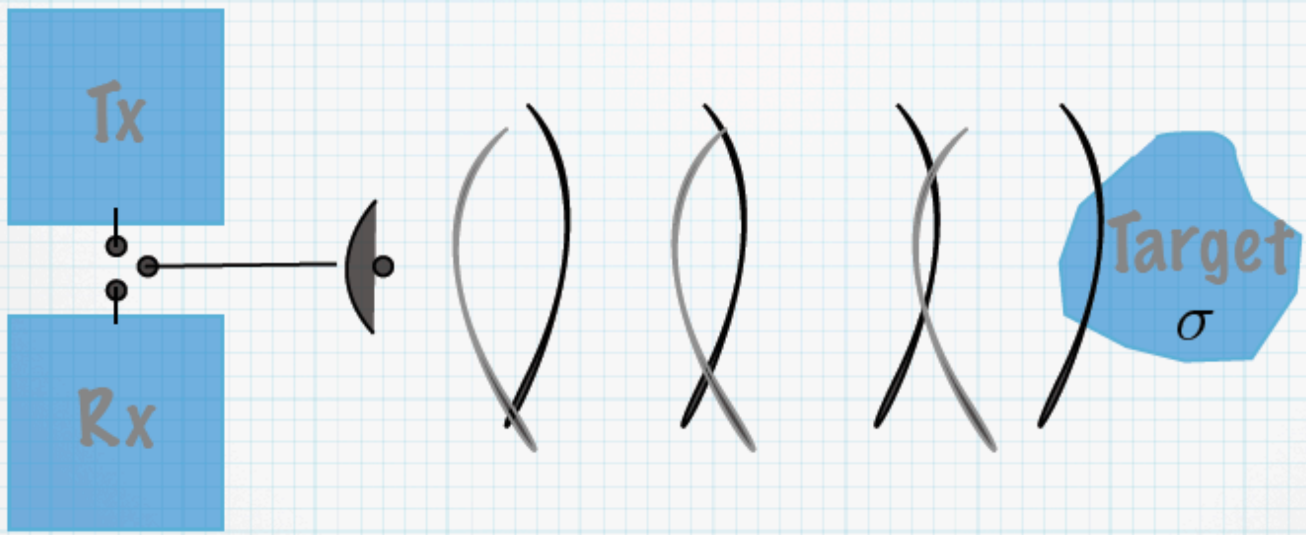
$$G = \frac{4\pi A_{eff}}{\lambda^2}$$

$$G_{tx} = \frac{P_{max}}{P_{ave}} = \frac{P_{max}}{P_{Total}/4\pi R^2}$$

Gain/Directivity (**subtle difference**) is the maximum power density at range R divided by the average power density at range R

“Equations are the devil’s sentences”
- **Stephen Colbert**

Basic Radar



Target has a cross section (isotropic) [m^2]

$$P_{inc} = P_t \frac{G_{tx}}{4\pi R^2} \quad \mathrm{W/m^2}$$ Power incident on target

$$P_{scat} = P_{inc}\sigma_{radar} \quad \mathrm{W}$$ Scattered power

$$P_{rec} = P_{scat}\frac{A_{eff}}{4\pi R^2} \quad \mathrm{W}$$ Received power

$$= P_t \frac{G_{tx} A_{eff} \sigma_{radar}}{16\pi^2 R^4} \quad \mathrm{W}$$ Radar equation

EI

Hertzian Dipole

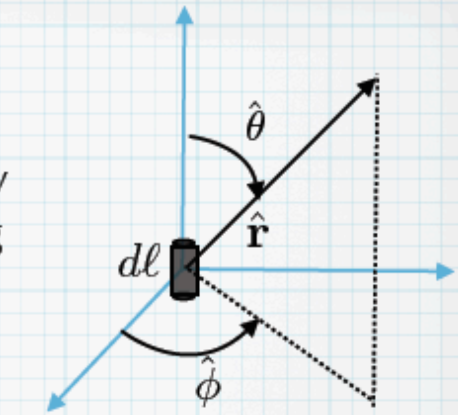
far field — near field

$$H_\phi = I d\ell \sin\theta \frac{1}{4\pi}\left[\frac{jk_0}{r} + \frac{1}{r^2} + 0\right] e^{j(\omega t - k_0 r)}$$

$$E_r = I d\ell \cos\theta \frac{j z_0}{2\pi k_0}\left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

$$E_\theta = I d\ell \sin\theta \frac{j z_0}{4\pi k_0}\left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

Spherically expanding wavefront



$$z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$$

$$k_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \omega\sqrt{\mu_0\epsilon_0}$$

For $r \gg \lambda$, keep terms only linear in r - **far field approximation.**

$$E_\theta \perp H_\phi \perp \hat{\mathbf{r}} \qquad \frac{E_\theta}{H_\phi} = z_0$$

Power flow represented by Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \qquad \langle P_r \rangle = \frac{1}{2}\Re\{P_r\} = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\}\cdot\hat{\mathbf{r}} \quad \mathrm{W/m^2}$$

Hertzian Dipole (2)

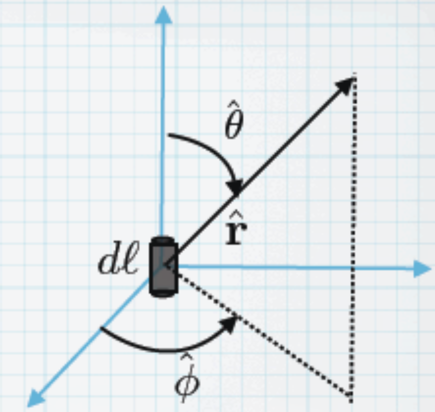
far field near field

Spherically expanding

$$H_\phi = I d\ell \sin\theta \frac{1}{4\pi}\left[\frac{jk_0}{r} + \frac{1}{r^2} + 0\right] e^{j(\omega t - k_0 r)}$$

$$E_r = I d\ell \cos\theta \frac{j z_0}{2\pi k_0}\left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

$$E_\theta = I d\ell \sin\theta \frac{j z_0}{4\pi k_0}\left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$



Directivity pattern:

$$D(\theta,\phi) = \frac{\text{Power Density Radiated In } (\theta,\phi) \text{ Direction}}{\text{Average Power Density}} = 4\pi R^2 \frac{\text{Power Density In } (\theta,\phi)}{\text{Total Power Radiated}}$$

$$\langle P_r \rangle = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\}\cdot\hat{\mathbf{r}} = I^2 z_0 (d\ell)^2 k_0^2 \sin^2\theta \frac{1}{32\pi^2 r^2} \ \mathrm{W/m^2}$$

$$P_{total} = \int_0^{2\pi} d\phi \int_0^{\pi} \langle P_r \rangle r^2 \sin\theta d\theta = z_0 \frac{\pi}{3}\left(\frac{I d\ell}{\lambda}\right)^2 \ \mathrm{W}$$

$$P_{total} = \frac{1}{2} I^2 R_{rad}$$

$$D(\theta,\phi) = \frac{3}{2}\sin^2\theta$$

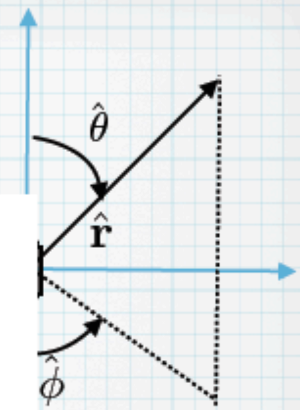
Hertzian Dipole (2)

far field near field Spherically

$H_\phi = Id$

$E_r = Id$

$E_\theta = Id$

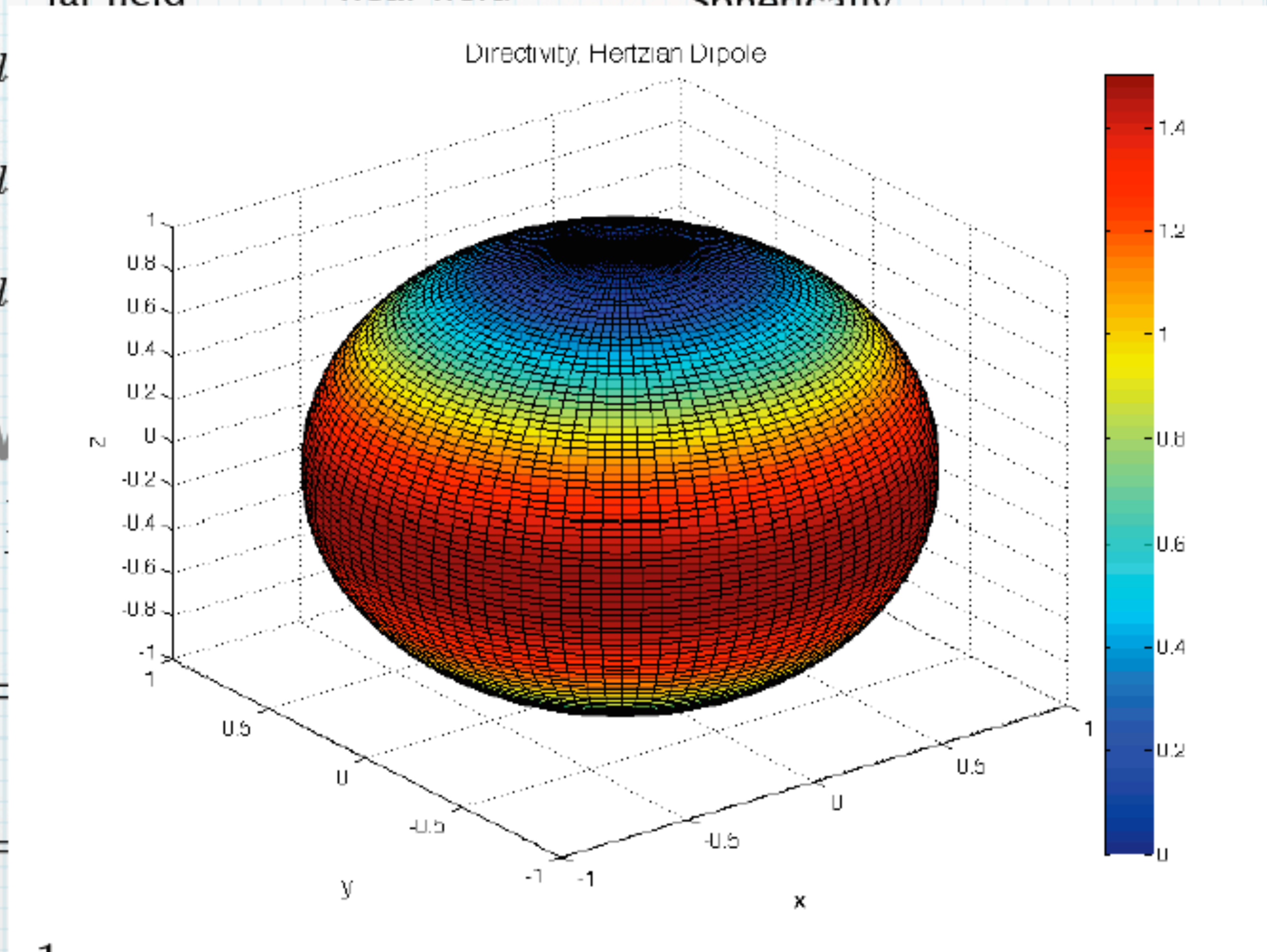


Directiv

$D(\theta,\phi) =$ ...sity In (θ,ϕ) / ...er Radiated

$\langle P_r \rangle =$ ... $\mathrm{V/m^2}$

$P_{total} =$



$$P_{total} = \frac{1}{2} I^2 R_{rad}$$

$$D(\theta,\phi) = \frac{3}{2}\sin^2\theta$$

Directivity Patterns for Dipoles

Hertzian Dipole

$$D(\theta, \phi) = \frac{3}{2} \sin^2 \theta$$

$$\text{HPBW} = 90^\circ$$

Half-Wave Dipole

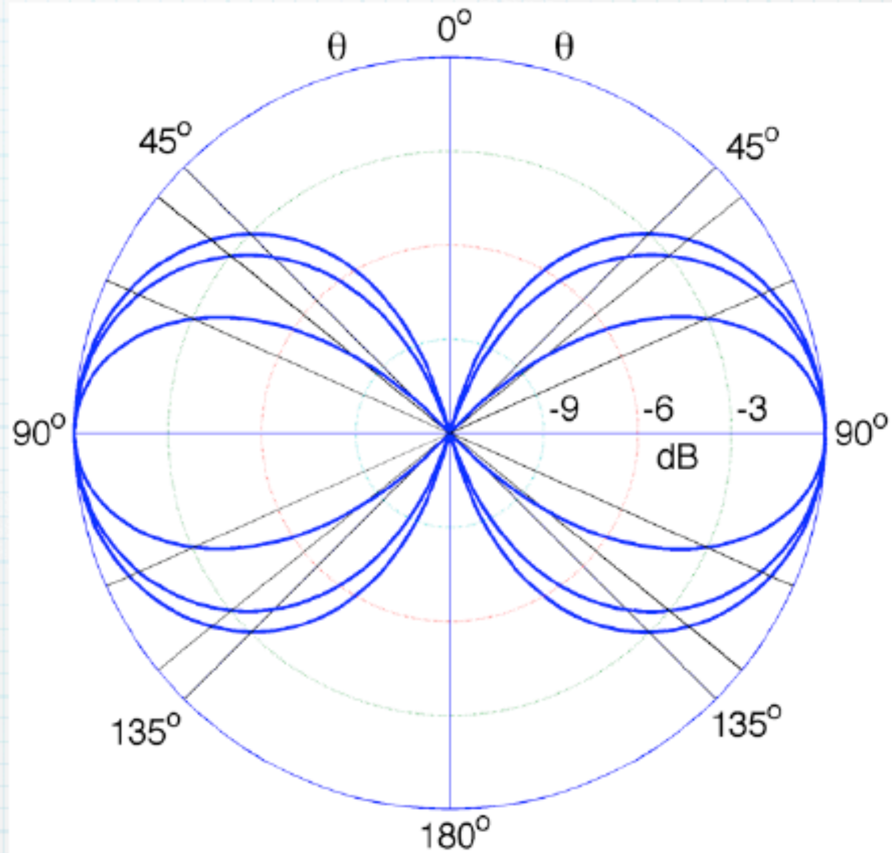
$$D(\theta, \phi) = 1.64 \left[\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right]^2$$

$$\text{HPBW} \approx 78^\circ$$

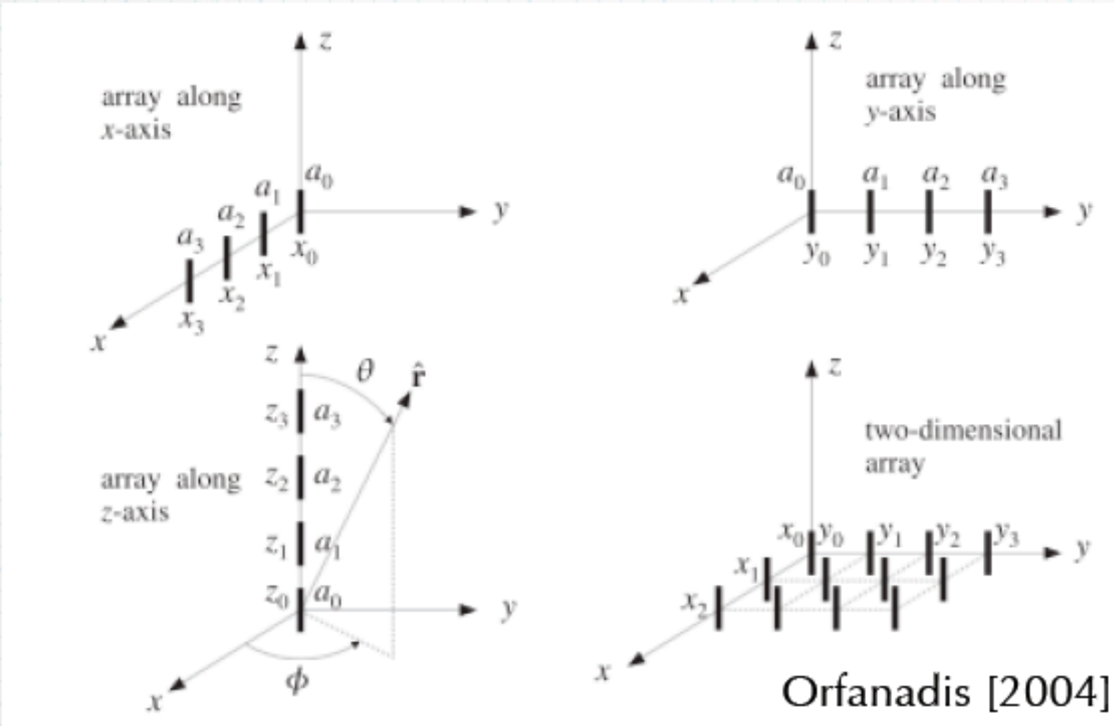
Full-Wave Dipole

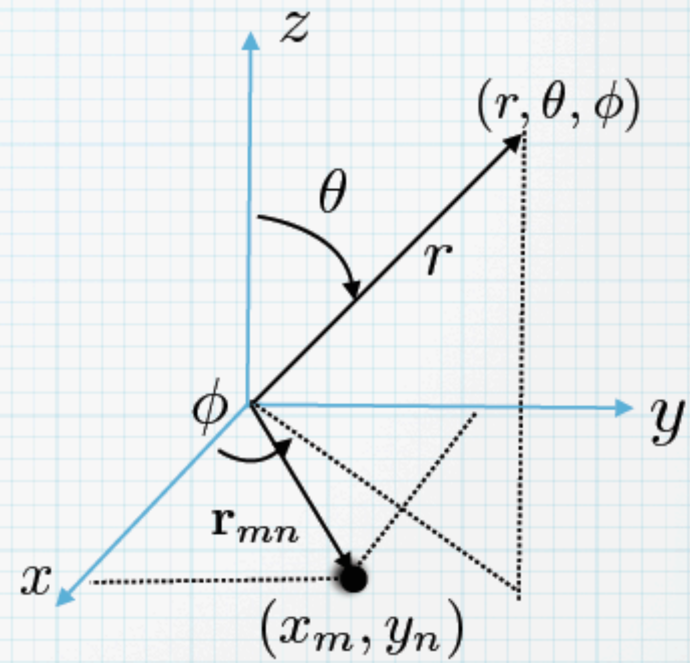
$$D(\theta, \phi) = 2.41 \left| \frac{\cos(\pi \cos \theta) - 1}{\sin \theta} \right|^2$$

$$\text{HPBW} \approx 48^\circ$$



Antenna Arrays





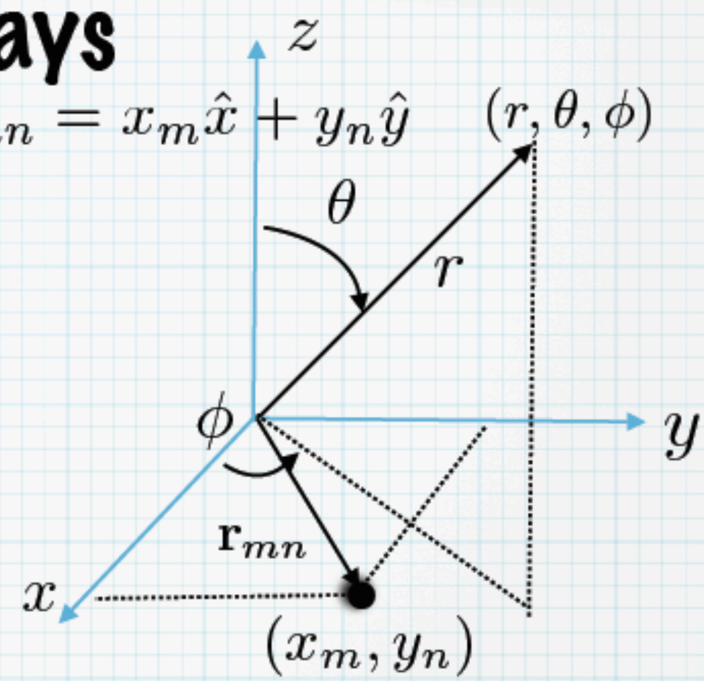
$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$

Assumptions:

1. Far field
   - parallel rays, 1/r amplitude dependence
2. No mutual coupling between elements (will discuss later)
3. A "reference" element radiates from the origin
4. All elements/radiators are identical, max radiation in z direction (broadside)

Antenna Arrays

$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$



Reference element at origin will produce a vector electric field at point (r, θ, ϕ)

$$\mathbf{E}_{00} = I_{00}(E_\theta \hat{\theta} + E_\phi \hat{\phi})$$

Constant

Fields due to *mn*th element is:

$$\begin{aligned} \mathbf{E}_{mn} &= I_{mn}(E_\theta \hat{\theta} + E_\phi \hat{\phi}) e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}} \\ &= I_{mn}(E_\theta \hat{\theta} + E_\phi \hat{\phi}) e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)} \end{aligned}$$

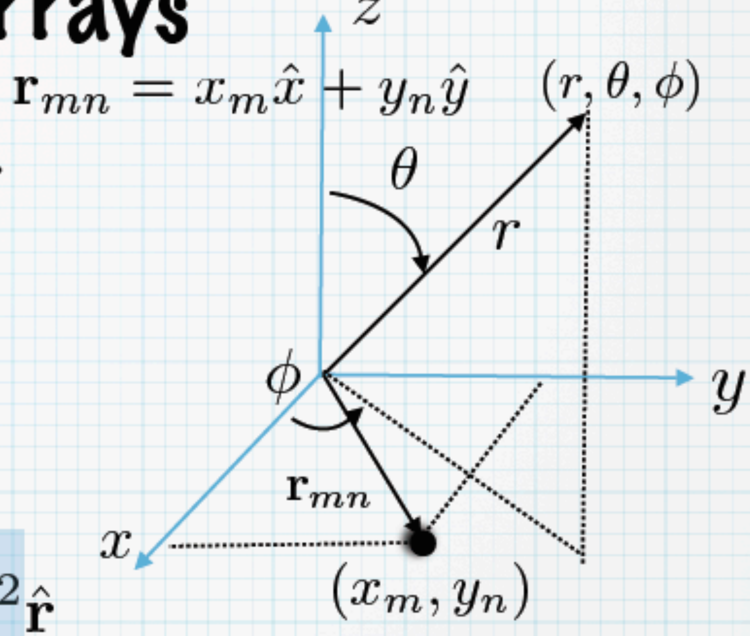
Total vector field at (r, θ, ϕ)

$$\mathbf{E} = (E_\theta \hat{\theta} + E_\phi \hat{\phi}) \sum_m \sum_n I_{mn} e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$

Element Factor

Array Factor

Antenna Arrays



$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$

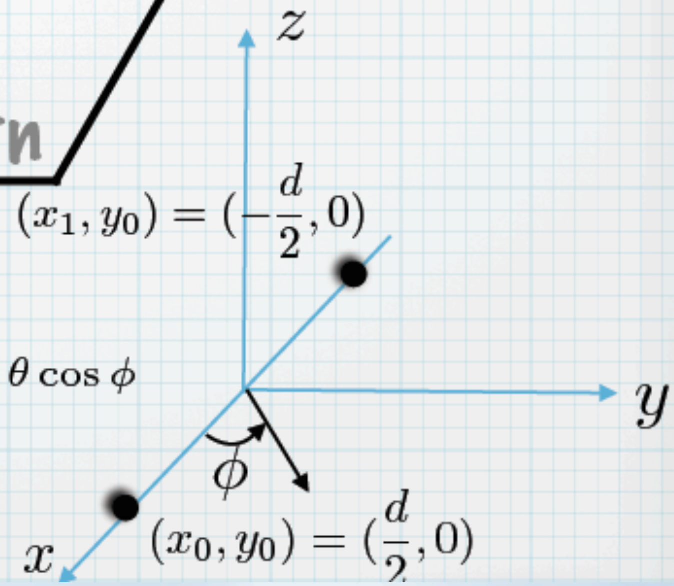
Poynting vector

$$\mathbf{P} = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\} = \frac{1}{2z_0}|\mathbf{E}|^2\hat{\mathbf{r}}$$

$$= \underbrace{\frac{1}{2z_0}(|E_\theta|^2 + |E_\phi|^2)}_{\text{Element Pattern}}\underbrace{|F_{array}|^2}_{\text{Array Pattern}}\hat{\mathbf{r}}$$

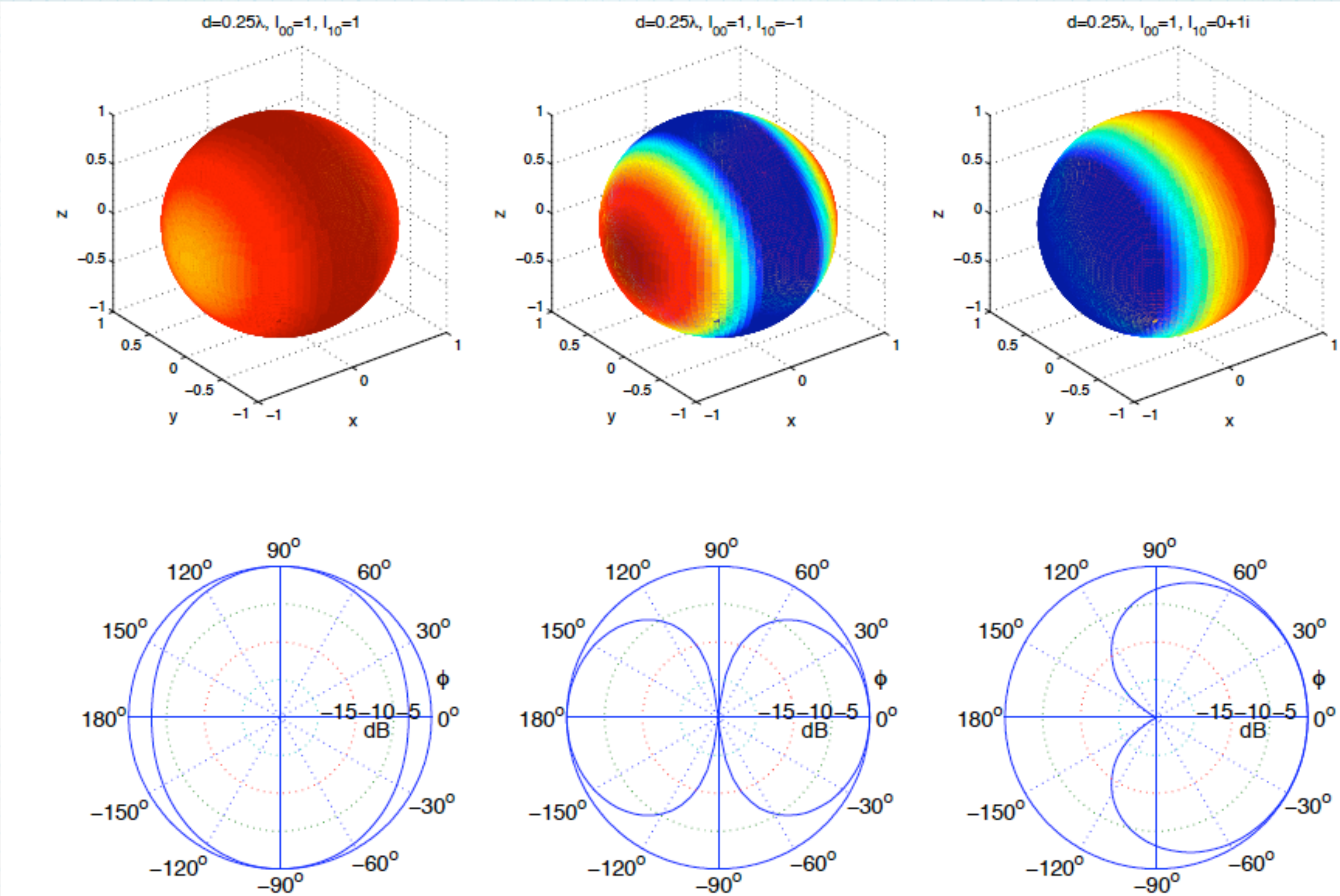
Element Pattern Array Pattern

Simple Two Element Array

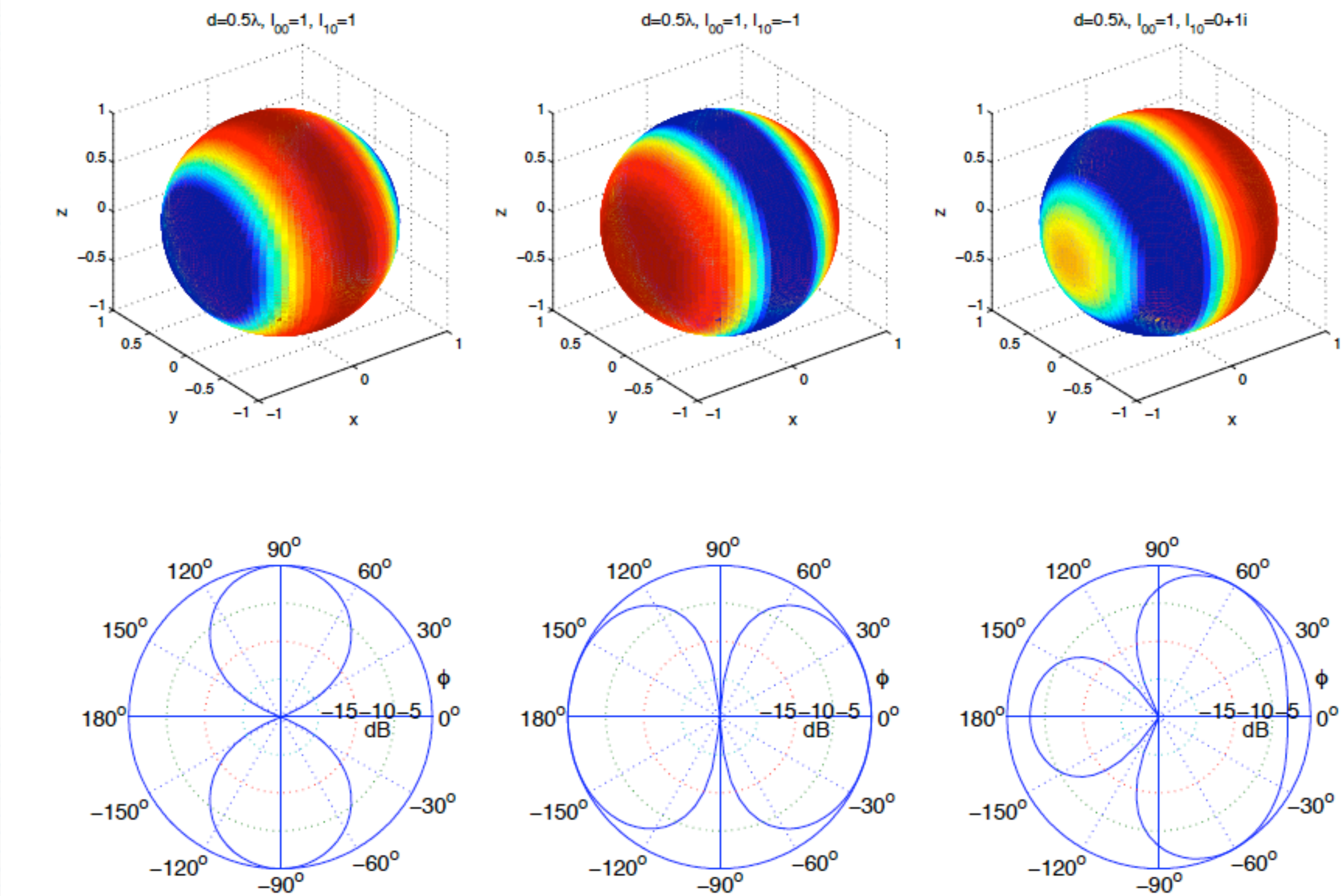


$$F_{array} = I_{00}e^{jk(d/2)\sin\theta\cos\phi} + I_{10}e^{-jk(d/2)\sin\theta\cos\phi}$$

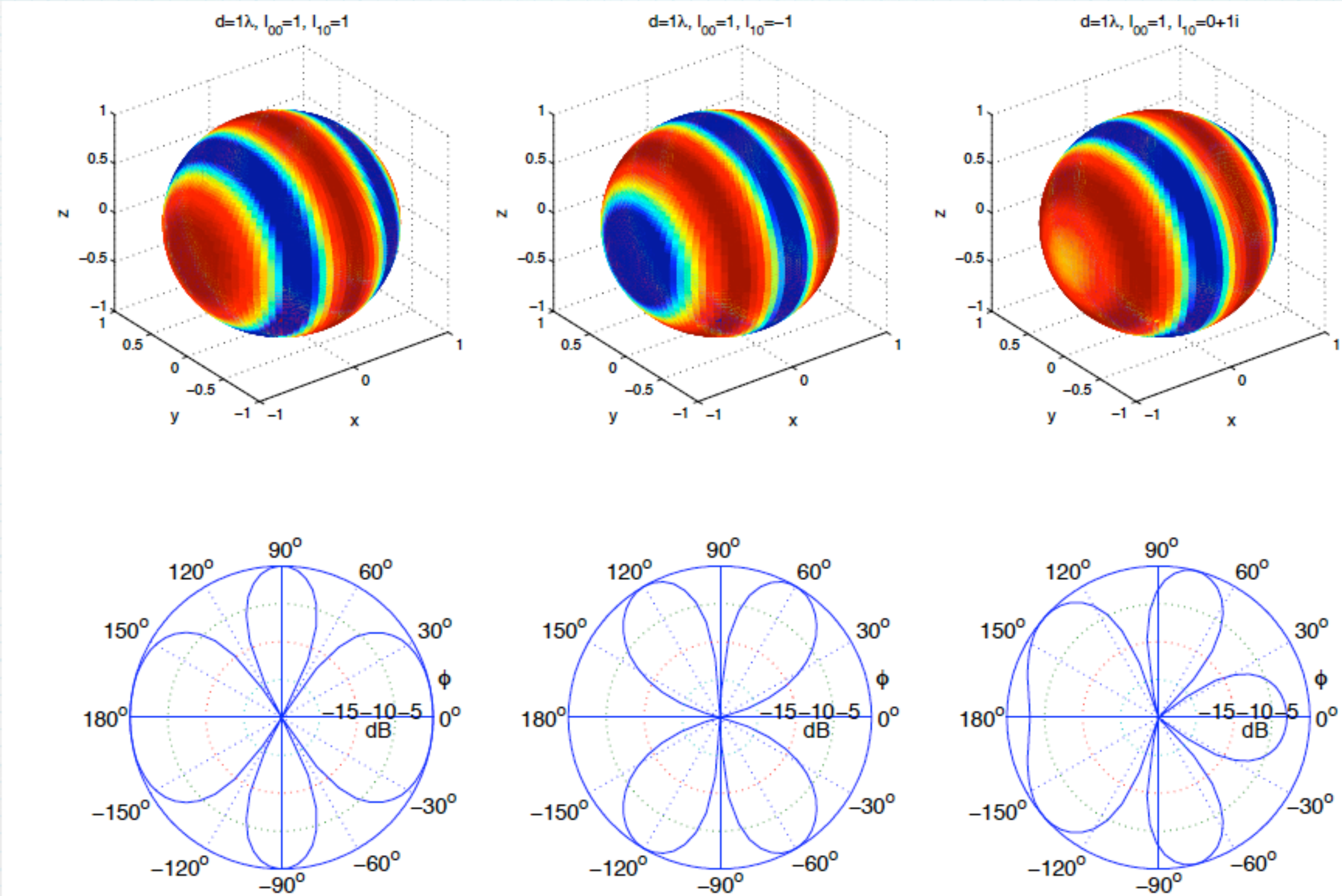
Two-element Array



Two-element Array

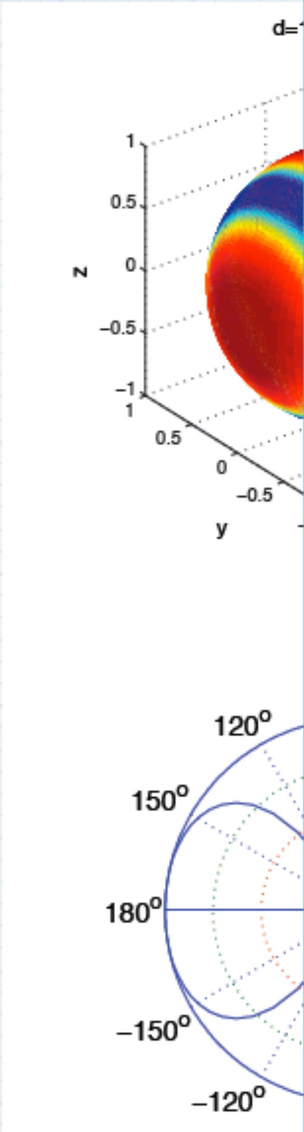


Two-element Array

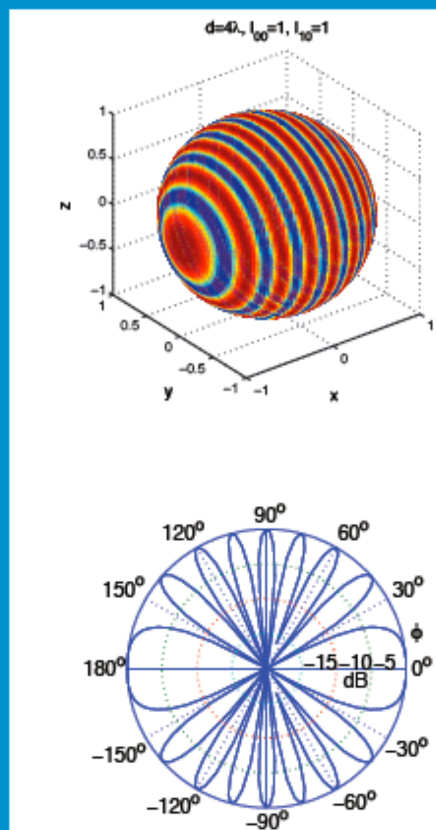


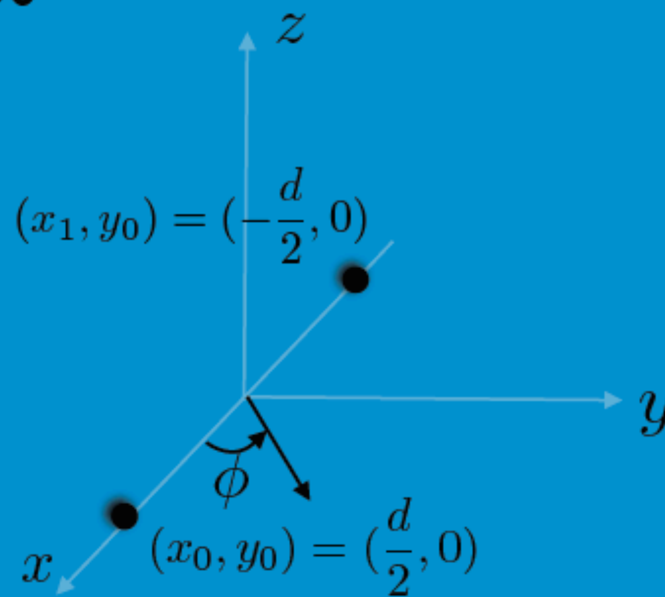
E

Two-element Array



- In phase - "broadside array"
- Out of phase - "endfire array"
- Steered from broadside - larger main lobe
- d>=1/2 - grating lobes





Array Steering

$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)}$$

If the element constants have no phase angles, beam maximum will be in direction:

$$x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi = 0 \longrightarrow \theta = 0$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin\theta_0\cos\phi_0 + y_n \sin\theta_0\sin\phi_0 + (\psi_m + \psi_n) = 0$$

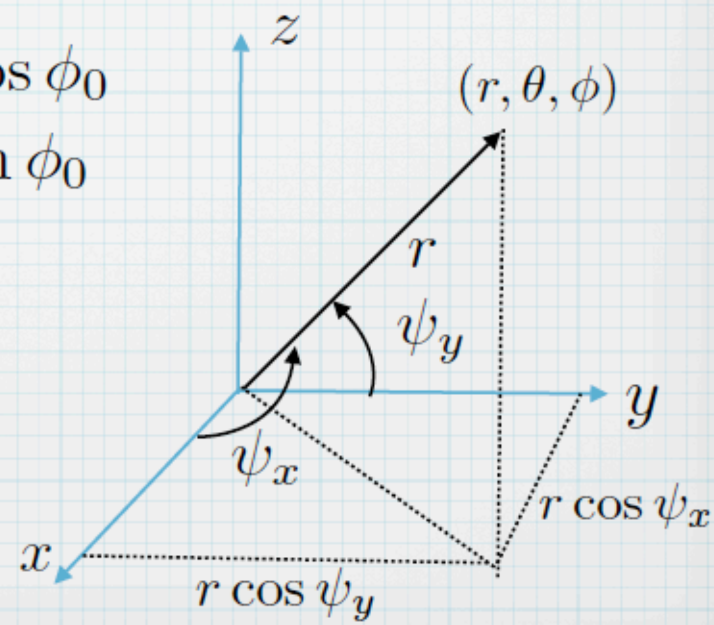
$$\psi_m = -kx_m \sin\theta_0\cos\phi_0$$

$$\psi_n = -ky_n \sin\theta_0\sin\phi_0$$

Direction cosines:

$$\cos\psi_{x0} = \sin\theta_0\cos\phi_0$$

$$\cos\psi_{y0} = \sin\theta_0\sin\phi_0$$



Array Steering

$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta \sin\phi)}$$

$$\boxed{F_{array} \quad = \quad \sum_{m,n} I_{mn} e^{jkx_m(\cos\psi_x - \cos\psi_{x0})} e^{jky_n(\cos\psi_y - \cos\psi_{y0})}}$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin\theta_0 \cos\phi_0 + y_n \sin\theta_0 \sin\phi_0 + (\psi_m + \psi_n) = 0$$

$$\psi_m = -kx_m \sin\theta_0 \cos\phi_0$$

$$\psi_n = -ky_n \sin\theta_0 \sin\phi_0$$

Direction cosines:

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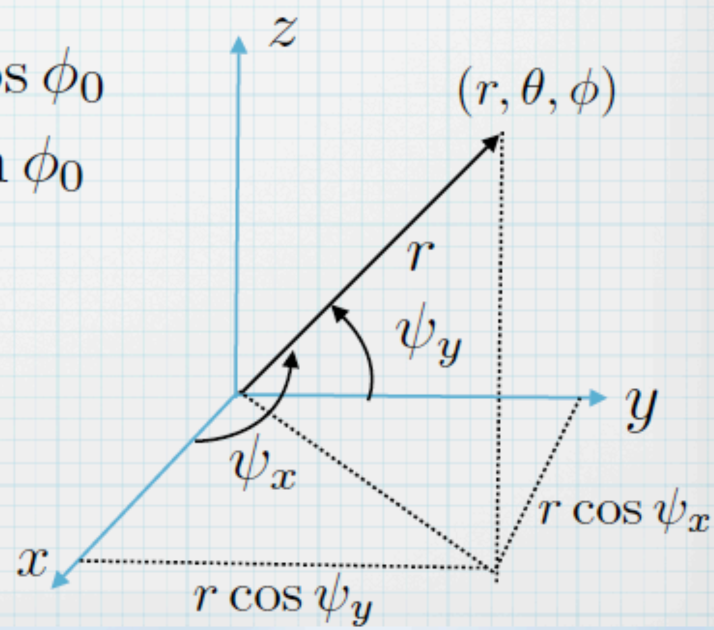


Directive Gain of Antenna Array

Recall:

$$D(\theta,\phi)=\frac{\text{Power Density Radiated In }(\theta,\phi)\text{ Direction}}{\text{Average Power Density}}=4\pi R^2\frac{\text{Power Density In }(\theta,\phi)}{\text{Total Power Radiated}}$$

$$\langle P_r\rangle=\frac{1}{2}\Re\left\{\mathbf{E}\times\mathbf{H}\right\}\cdot\hat{\mathbf{r}}=\frac{1}{2z_0}|\mathbf{E}|^2|F_{array}|^2=P_{el}|F_{array}|^2$$

$$P_{total}=\int_0^{2\pi}d\phi\int_0^{\pi}P_{el}|F_{array}|^2r^2\sin\theta d\theta$$

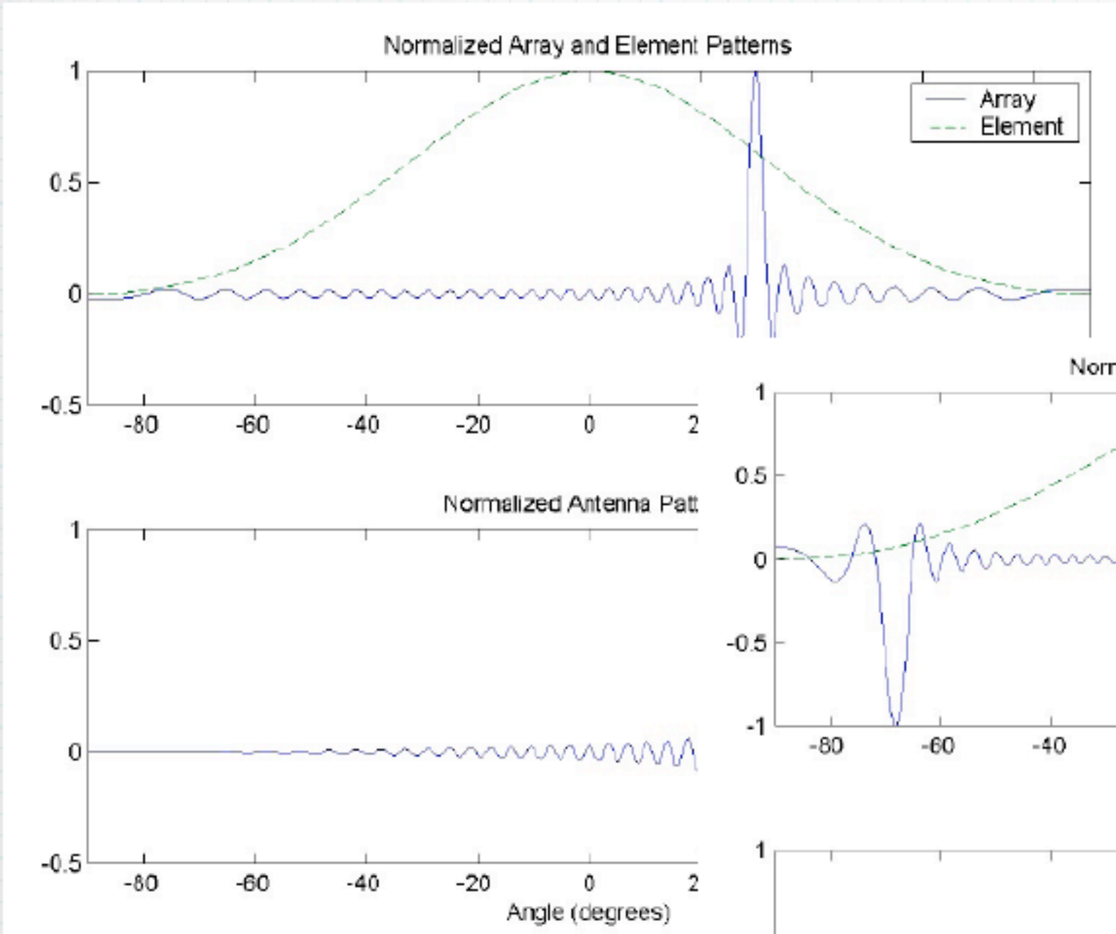
$$D(\theta,\phi)=4\pi r^2\frac{P_{el}|F_{array}|^2}{\int_0^{2\pi}d\phi\int_0^{\pi}P_{el}|F_{array}|^2r^2\sin\theta d\theta}$$

If element pattern is much broader than array pattern,

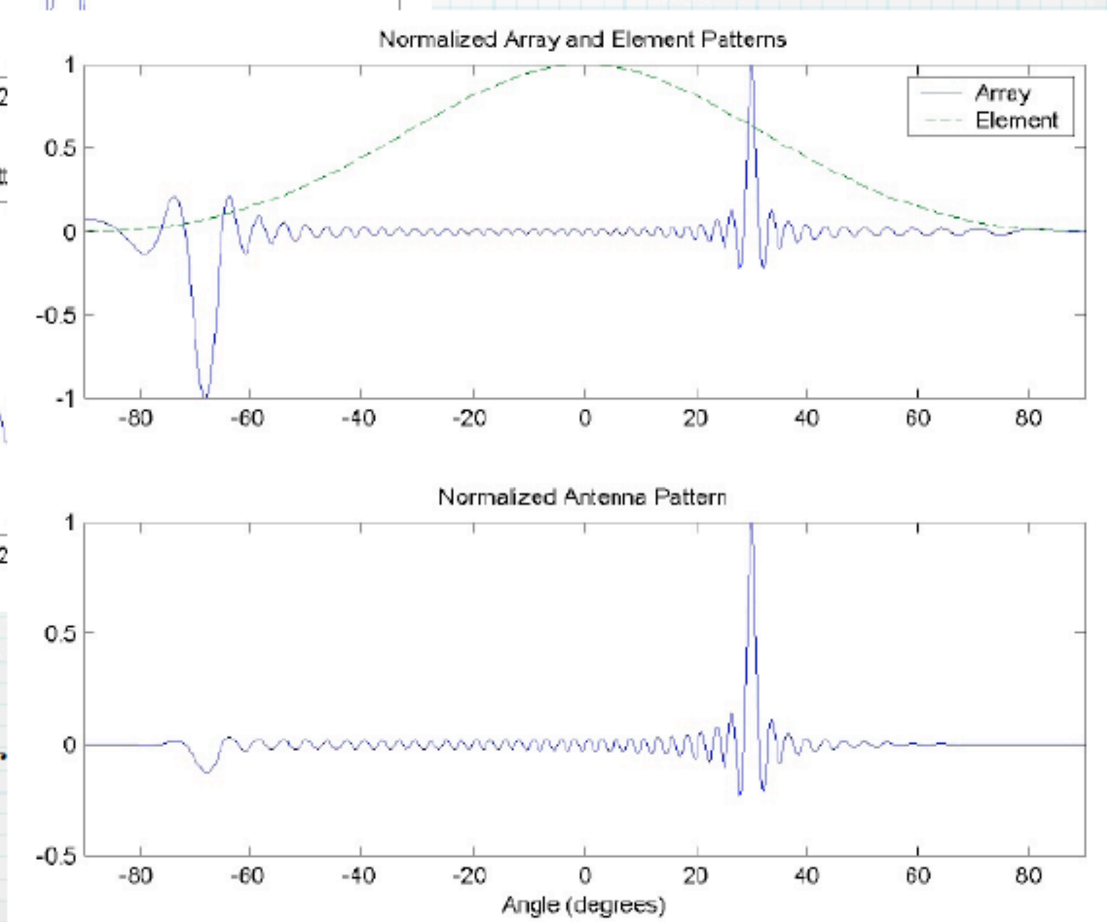
Element pattern doesn't matter.

$$D(\theta,\phi)=4\pi r^2\frac{|F_{array}|^2}{\int_0^{2\pi}d\phi\int_0^{\pi}|F_{array}|^2r^2\sin\theta d\theta}$$

Directive Gain of Antenna Array



n $= 4\pi R^2 \frac{\text{Power Density In } (\theta,\phi)}{\text{Total Power Radiated}}$



$$D(\theta,\phi) = 4\pi r$$

Visible Region and Grating Lobes

Recall (1d array pointed broadside):

$$F_{array} = \sum_m I_m e^{jkdm \cos \psi_x} = \sum_m I_m e^{jm\gamma}$$

Can see that: $\boxed{-kd \leq \gamma \leq kd}$

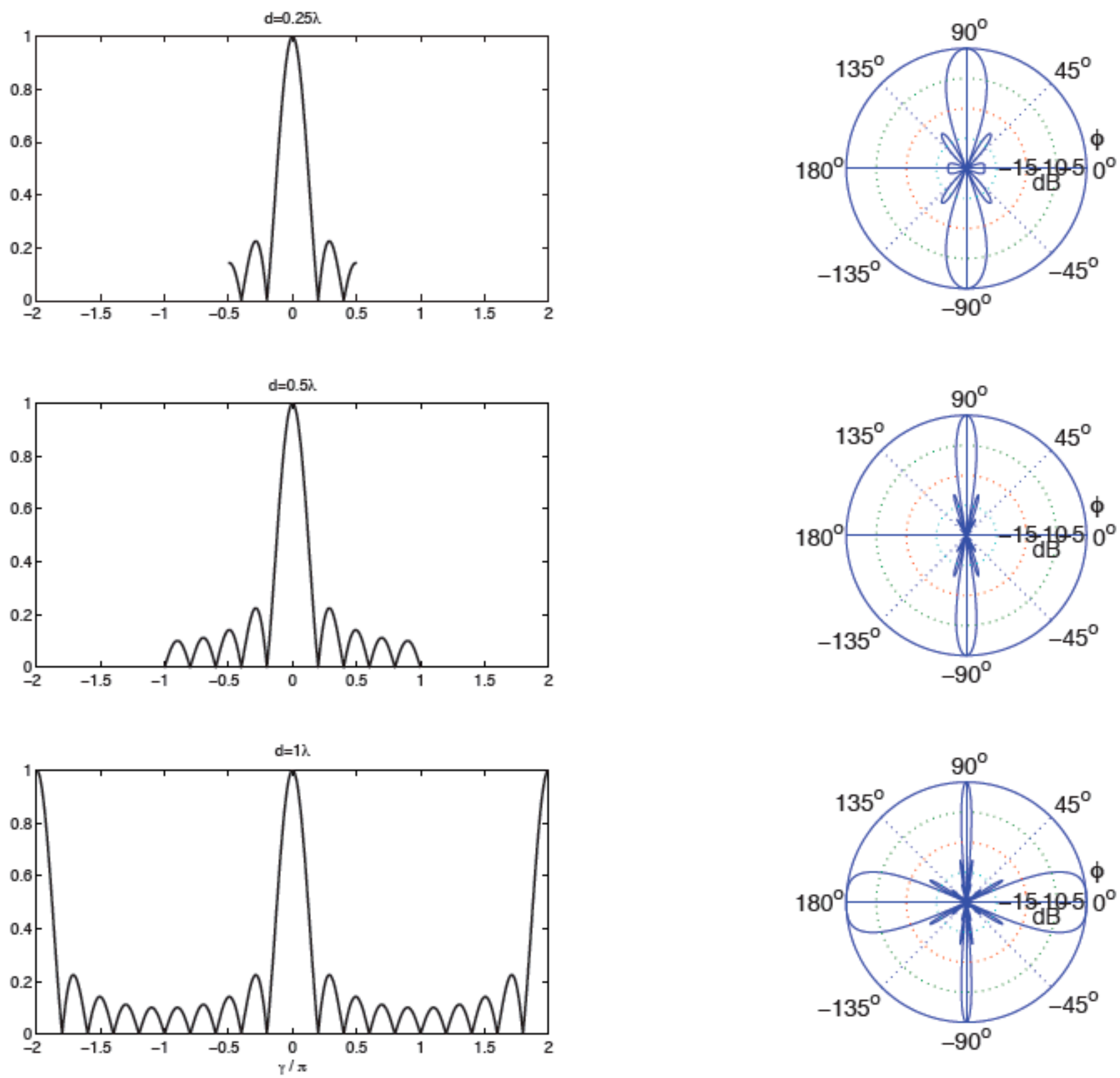
Visible Region

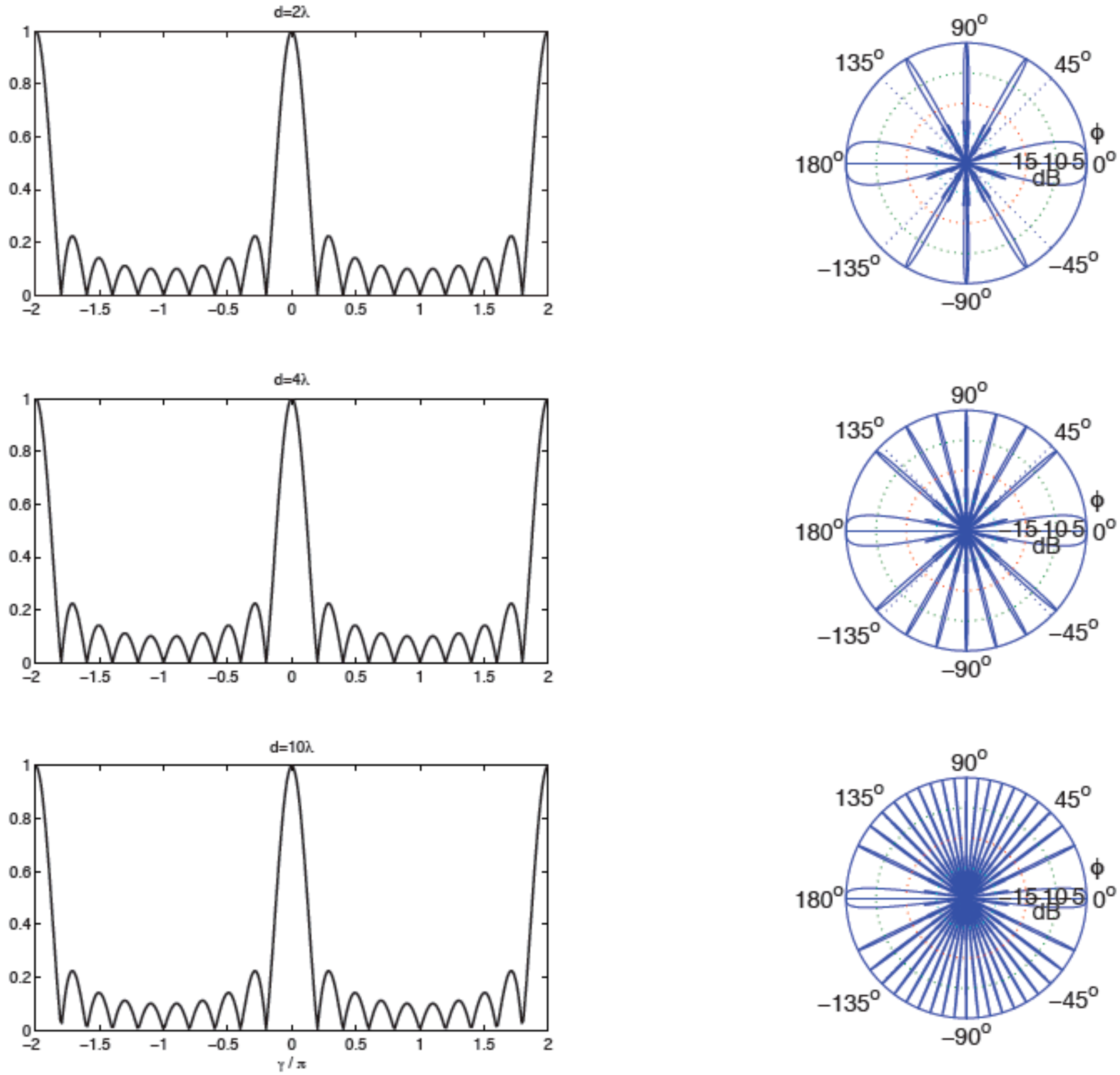
$$d \leq \lambda/2 \rightarrow \gamma \leq 2\pi$$

$$d > \lambda/2 \rightarrow \gamma > 2\pi$$

Values of F_{array} repeat - Grating Lobes

Grating lobes are analogous to classical undersampling (spectral aliasing).


d=0.25λ
d=0.5λ
d=1λ
γ / π
90°
45°
135°
180°
0°
−45°
−135°
−90°
φ
−15 −10 −5
dB


d=2λ
d=4λ
d=10λ
γ / π
90°
135°
45°
180°
0°
φ
dB
−135°
−45°
−90°

Steering and Grating Lobes

For arbitrary steering direction:

$$F_{array} = \sum_m I_m e^{jkdm(\cos\psi_x - \cos\psi_{x0})} = \sum_m I_m e^{jm(\gamma-\gamma_0)}$$

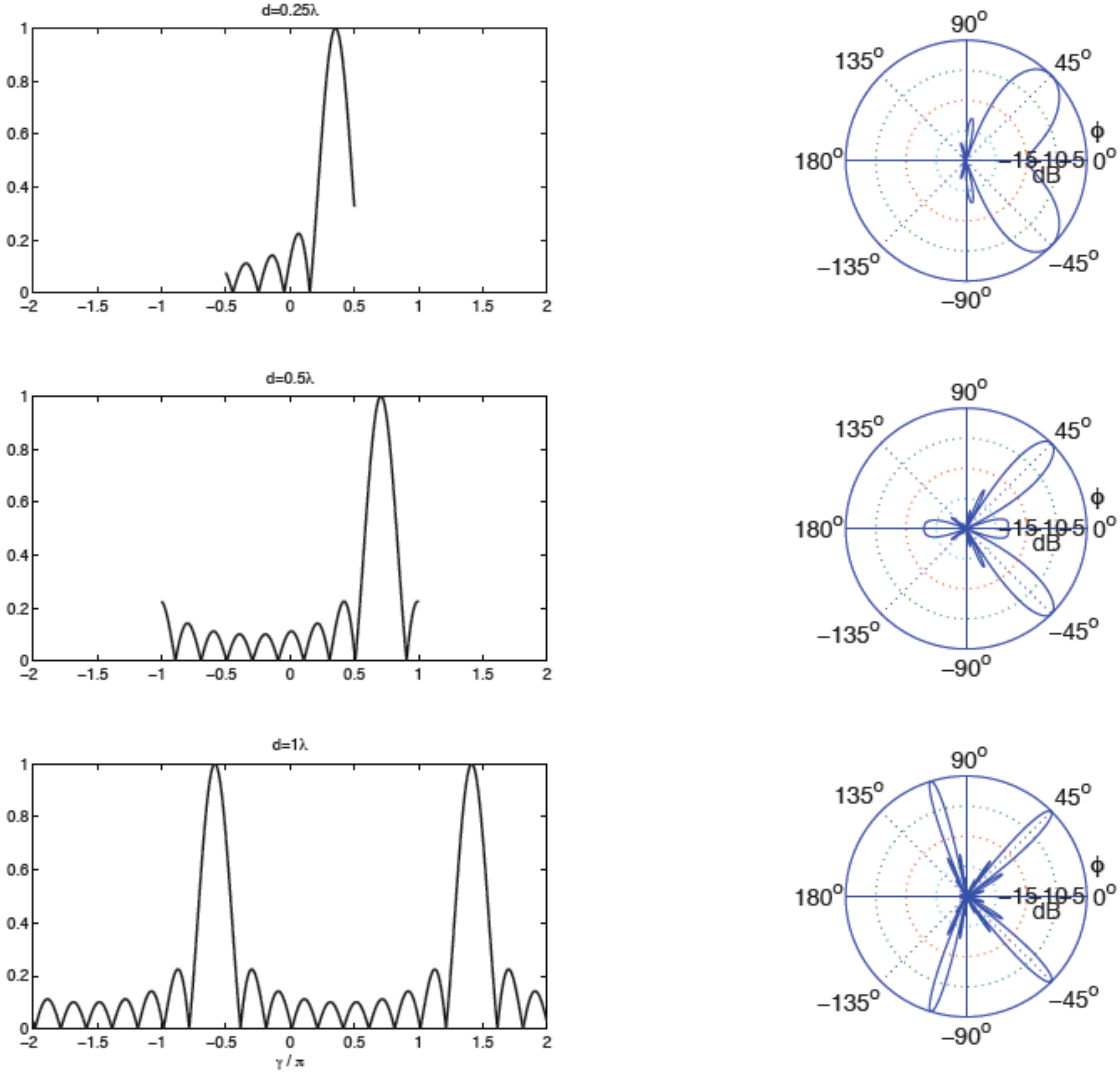
$$-kd(1+\cos\psi_{x0}) \leq \gamma \leq kd(1-\cos\psi_{x0})$$

Modified Visible Region

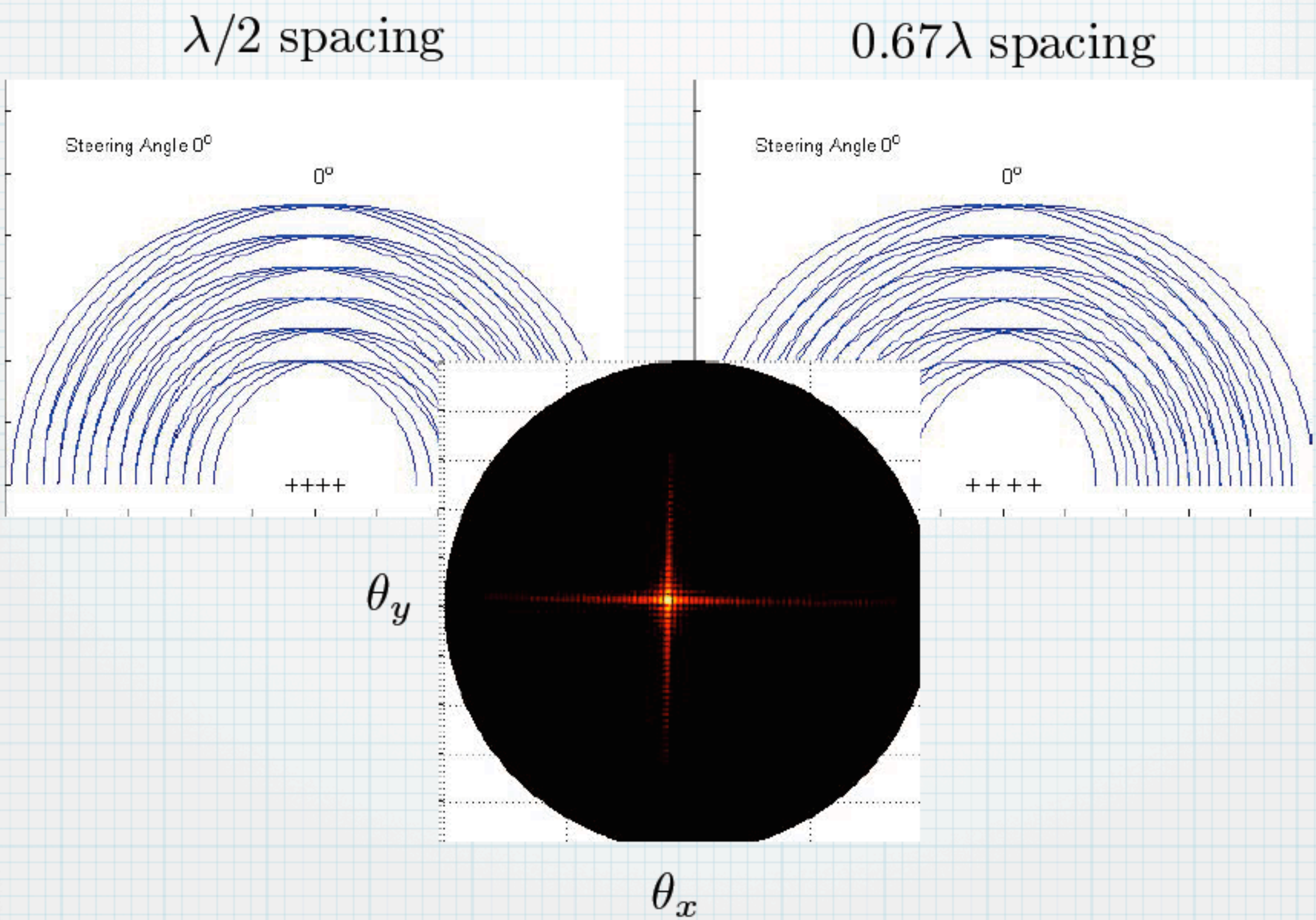
For no grating lobes,

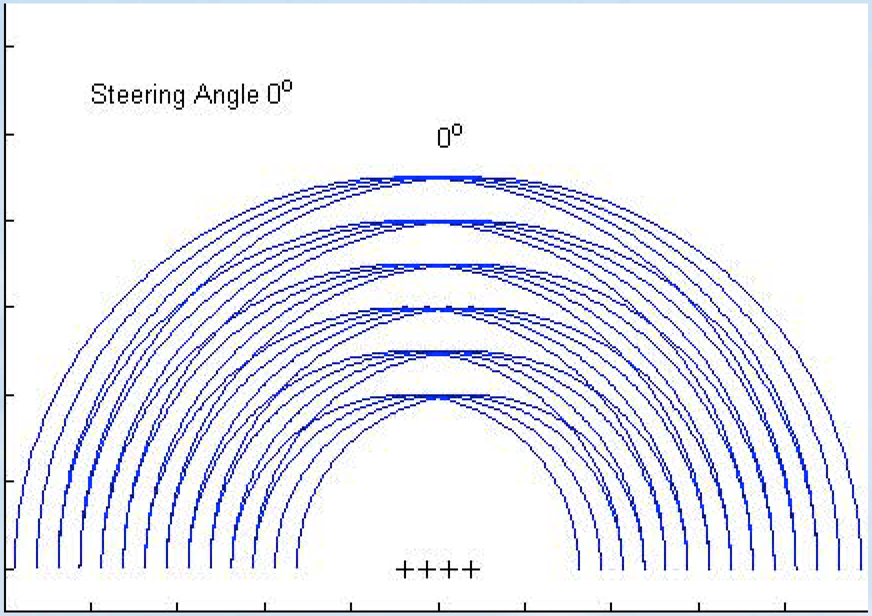
$$\gamma \leq 2\pi \qquad d \leq \frac{\lambda}{1+|\cos\psi_{x0}|}$$

Also note that beam broadens as $\sin\psi_0$ as beam is steered

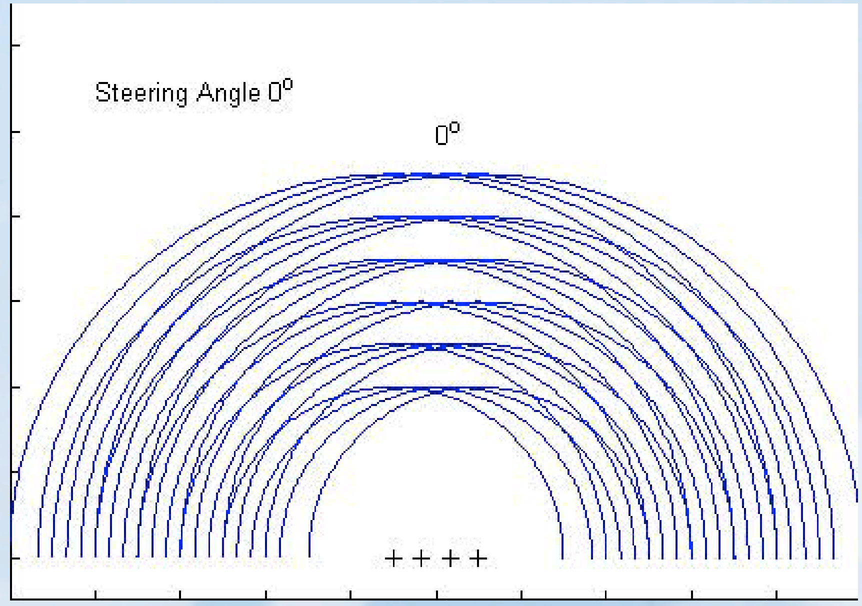

d=0.25λ
d=0.5λ
d=1λ
γ / π
90°
45°
135°
180°
0°
−45°
−135°
−90°
ϕ
dB

Steering and Grating Lobes





Steering Angle 0°
0°
++++



Steering Angle 0°
0°
+ + + +

Mutual Coupling / Impedance

- Array gain - related to gain of individual element.
- Gain of isolated element very different from element gain within array.
- Element pattern will also vary across array.
- **Actual element gain usually not known - must be simulated/measured.**

For an N element array:

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

Mutual impedance Self impedance

- Solve for I
- Compute Poynting vector
- Use this to compute radiation pattern

- Important to minimize mutual coupling -> Can cause problems (standing waves “hot spots”, etc.

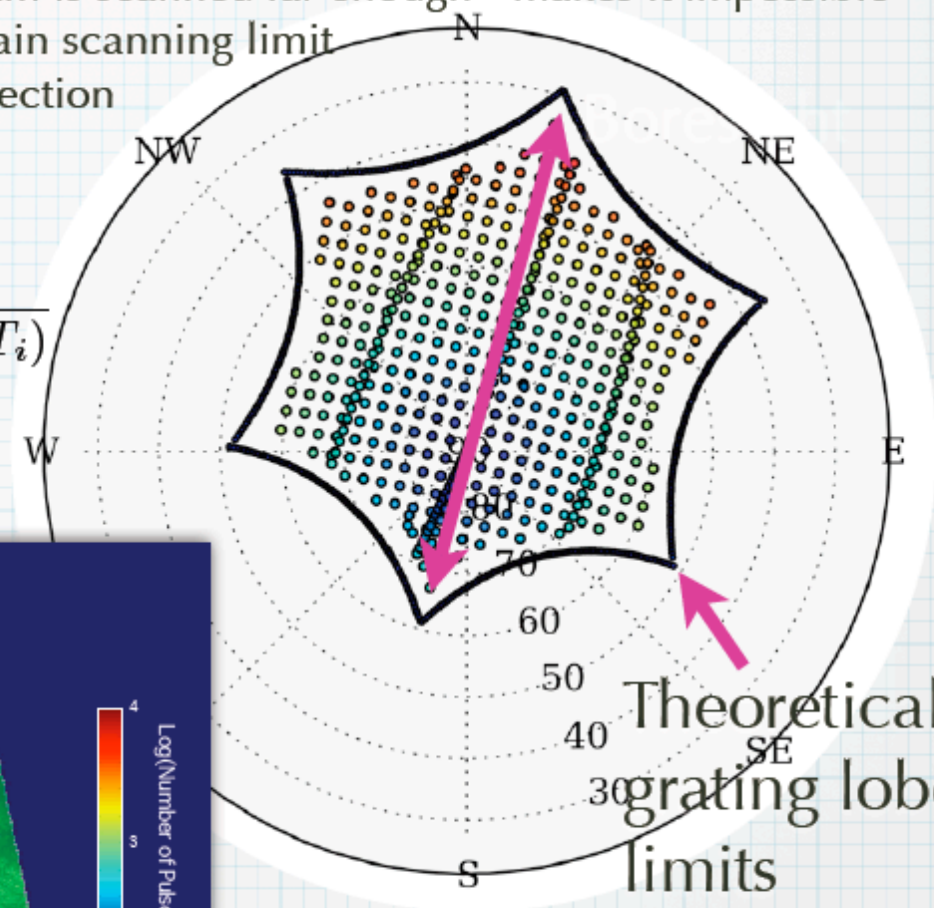
AMISR (3)

- **Recall:** Grating lobes will appear when beam is scanned far enough - makes it impossible to do incoherent scatter science beyond certain scanning limit
- **Recall:** Gain pattern will vary with scan direction

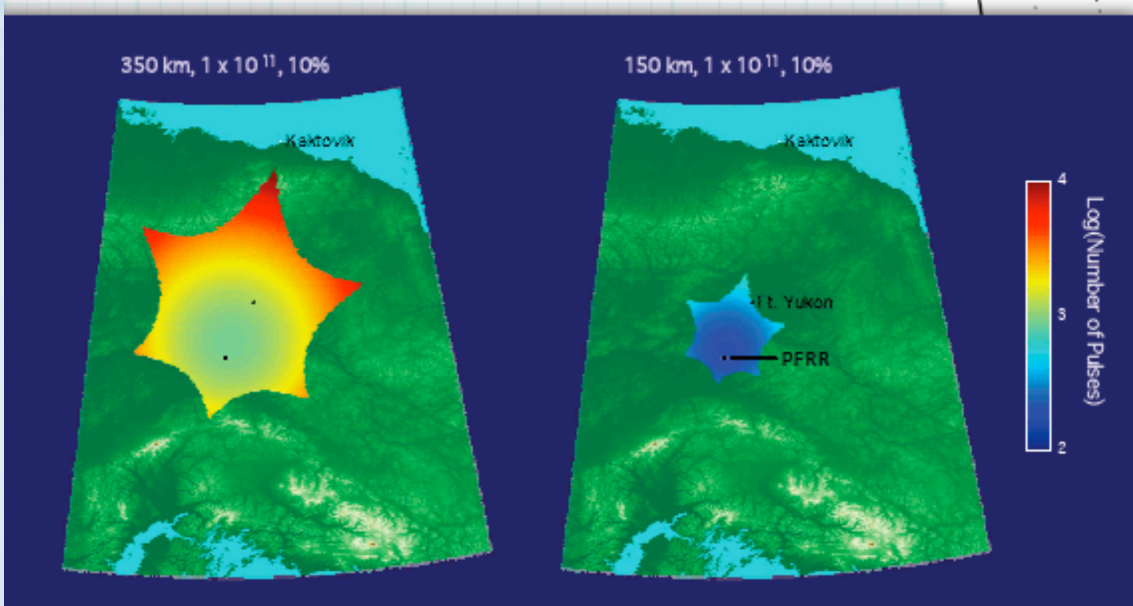
Should have seen an equation like:

$$P_r = \frac{P_t \tau_p K_{sys}}{r^2} \frac{N_e}{(1+k^2\lambda_D^2)(1+k^2\lambda_D^2+T_e/T_i)}$$

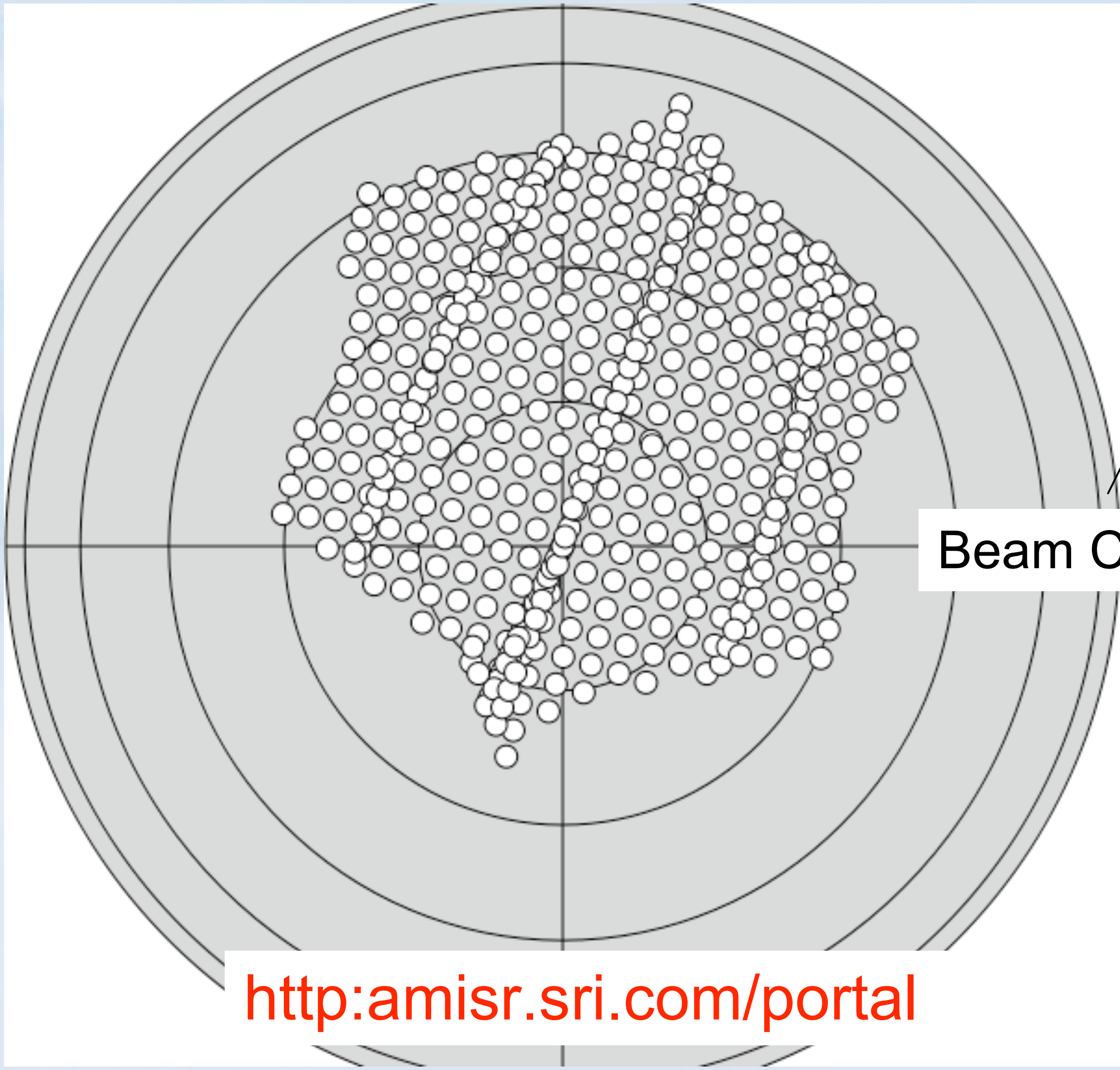
System Constant becomes dependent on look direction



Theoretical grating lobe limits



Available beam positions



http:amisr.sri.com/portal

63221	-21.51	61.46	1.361447e-19
63227	-18.77	60.10	1.346032e-19
63239	-16.23	58.68	1.324891e-19
63245	-13.89	57.20	1.298195e-19
63251	-11.73	55.69	1.266172e-19
63257	-9.72	54.13	1.229106e-19
63263	-7.85	52.53	1.187336e-19
63269	-6.11	50.90	1.141255e-19
63275	-4.48	49.24	1.091302e-19
63281	-2.95	47.55	1.037957e-19
63287	-1.51	45.84	9.817419e-20
63293	-0.15	44.10	9.232058e-20
63299	126.61	69.25	9.356377e-20
63305	121.38	69.66	9.845681e-20
63311	115.98	69.92	1.031070e-19
63317	110.50	70.00	1.074713e-19
63323	105.02	69.92	1.115083e-19
63329	99.62	69.66	1.151791e-19
63335	94.39	69.25	1.184474e-19
63341	89.39	68	
	84.65	67	
	80.22	67	
	76.09	66.19	1.268898e-19
63371	72.27	65.14	1.277260e-19
63377	68.74	63.99	1.280215e-19
63383	65.50	62.76	1.277695e-19
63389	62.51	61.46	1.269685e-19
			3e-19
			3e-19
			3e-19
63413	52.73	55.69	1.184336e-19
63419	50.72	54.13	1.150550e-19
63431	48.85	52.53	1.112323e-19
63437	47.11	50.90	1.070012e-19
63443	45.48	49.24	1.024020e-19
63449	43.95	47.55	9.747898e-20
6345			
6347			
6358			
6377			
63809	-159.50	76.00	9.499506e-20
63905	-159.50	78.00	1.023999e-19

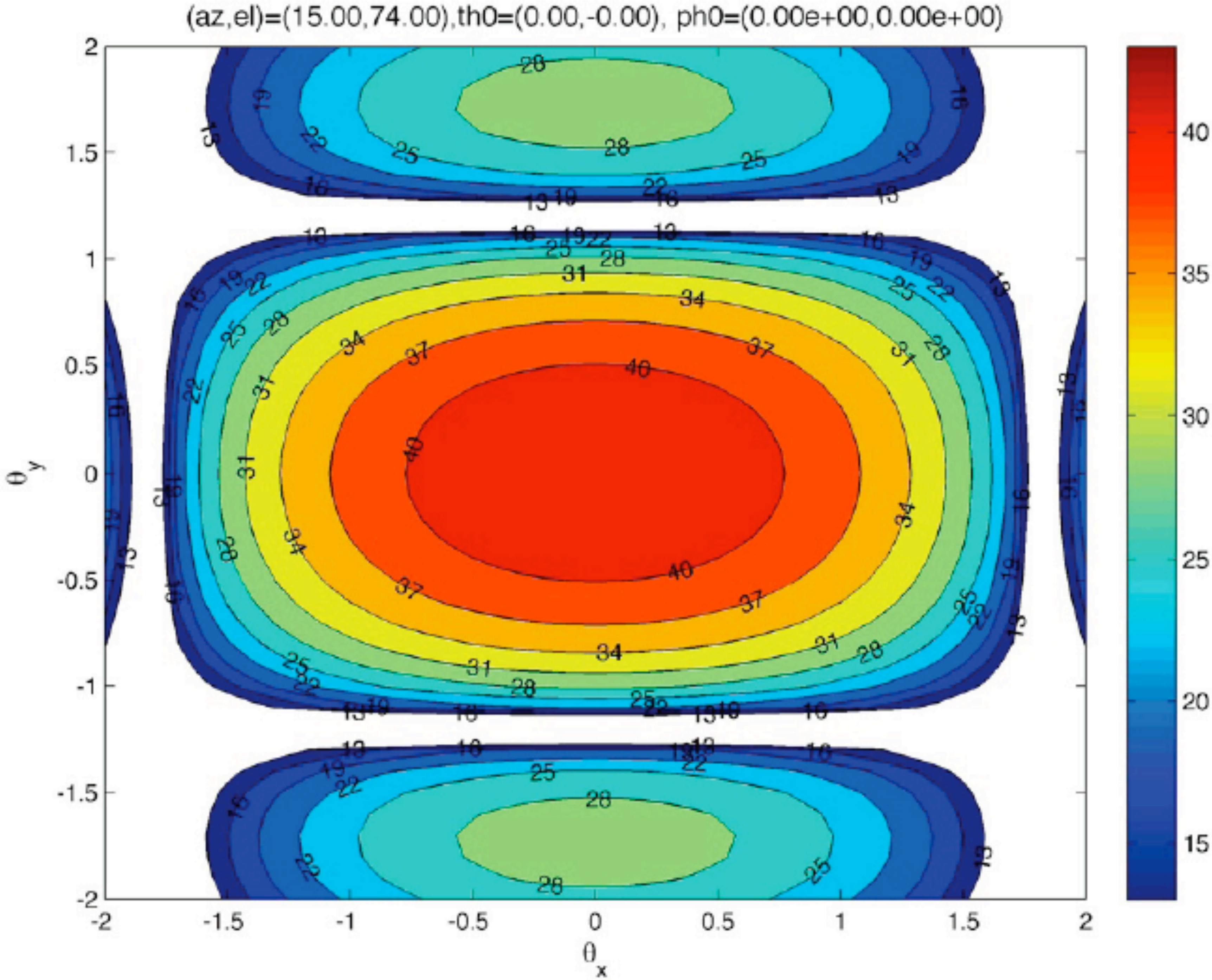
Beam Code

Elevation (El)

Azimuth (AZ)

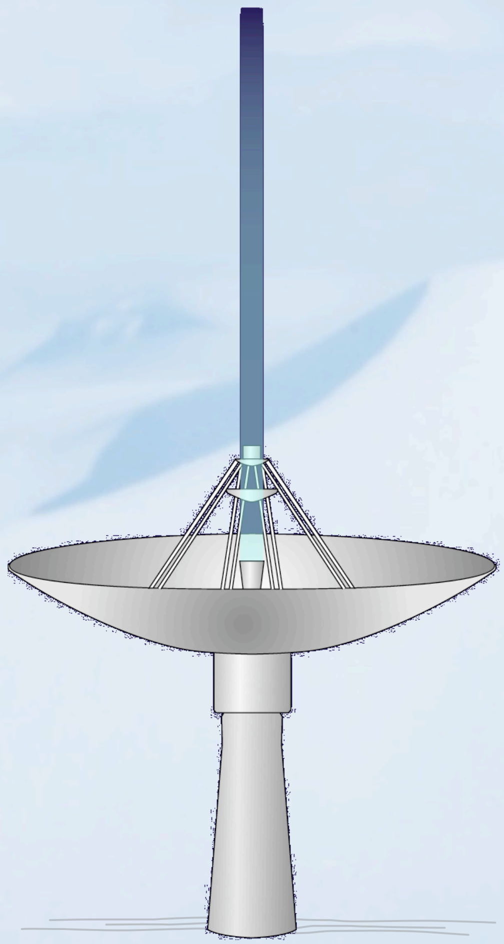
System "constant"

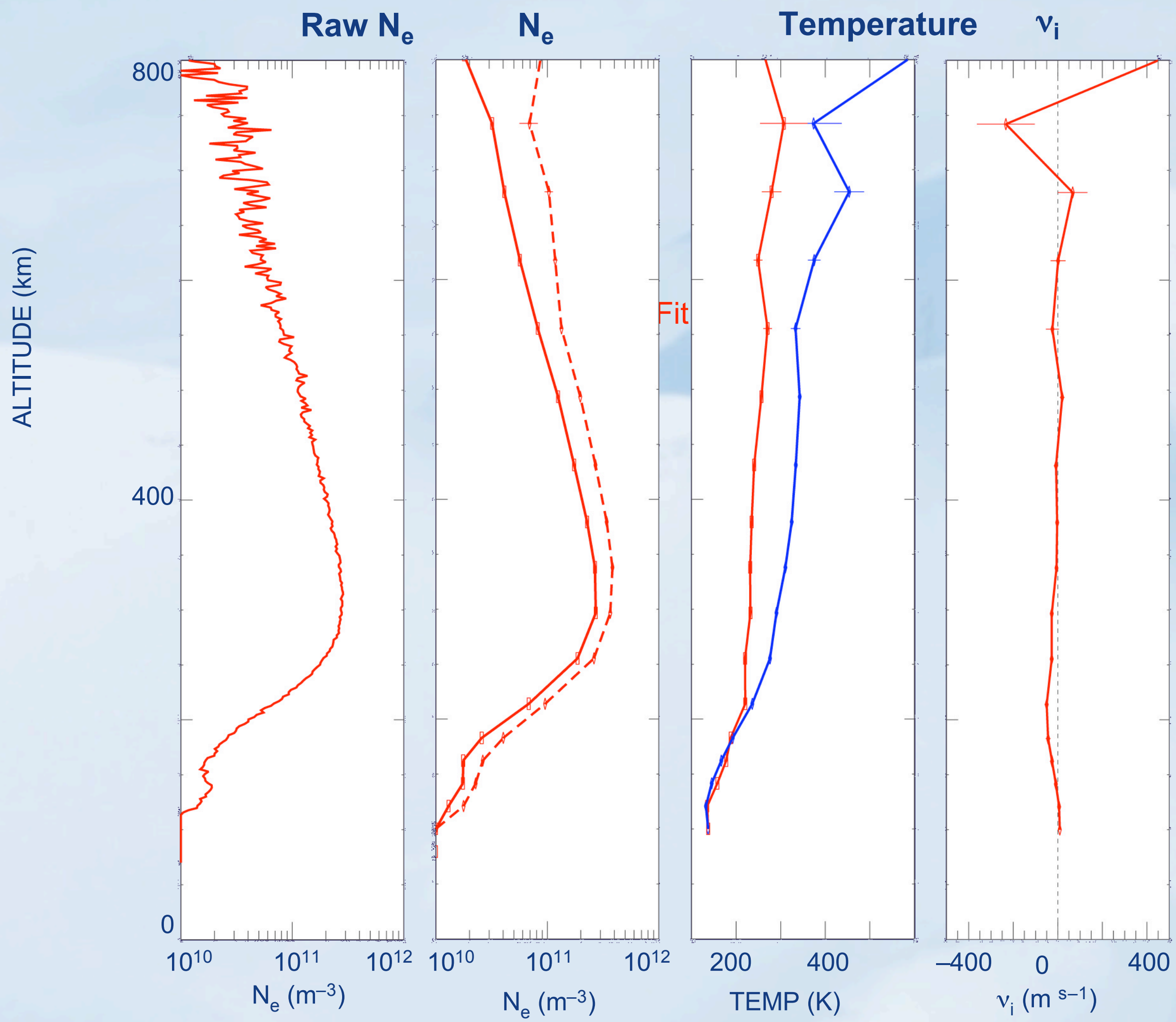
+ ~400 more…


(az,el)=(15.00,74.00),th0=(0.00,-0.00), ph0=(0.00e+00,0.00e+00)
θ_y
θ_x
15
20
25
30
35
40

Some benefits of phased arrays…

Plasma Parameter Profile





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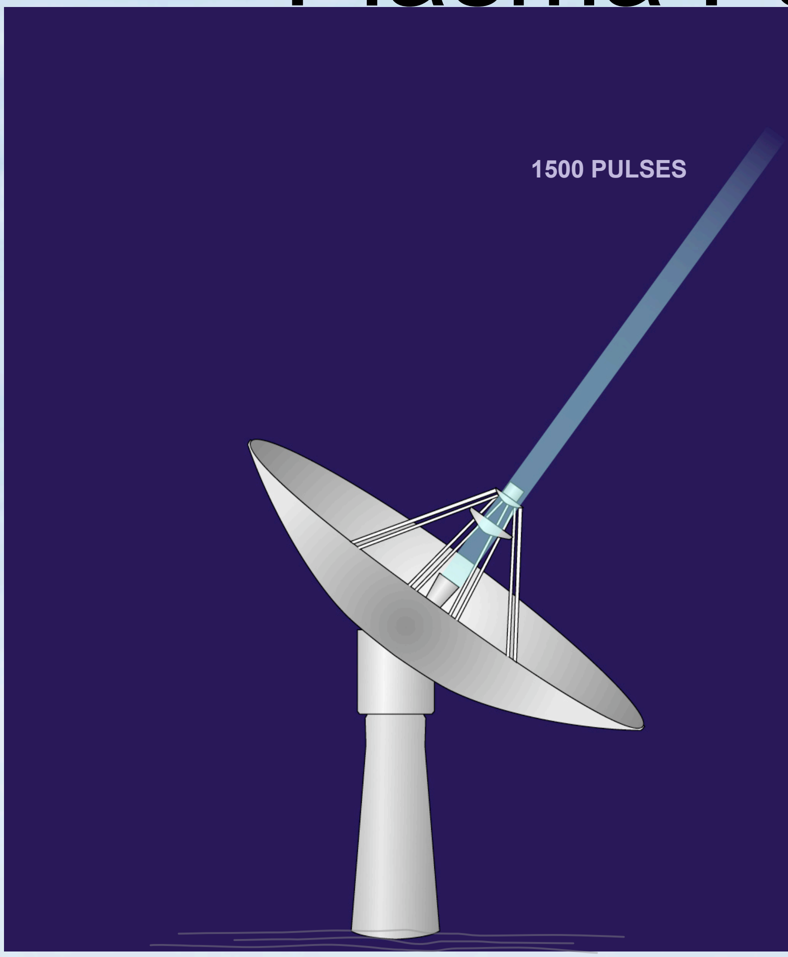
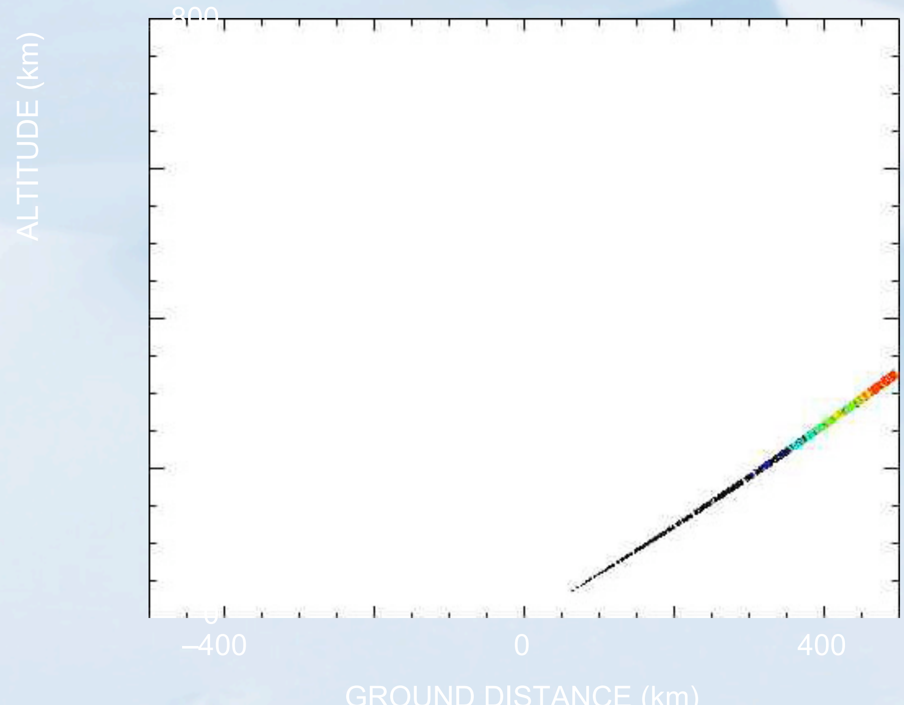
Changing Pointing Direction

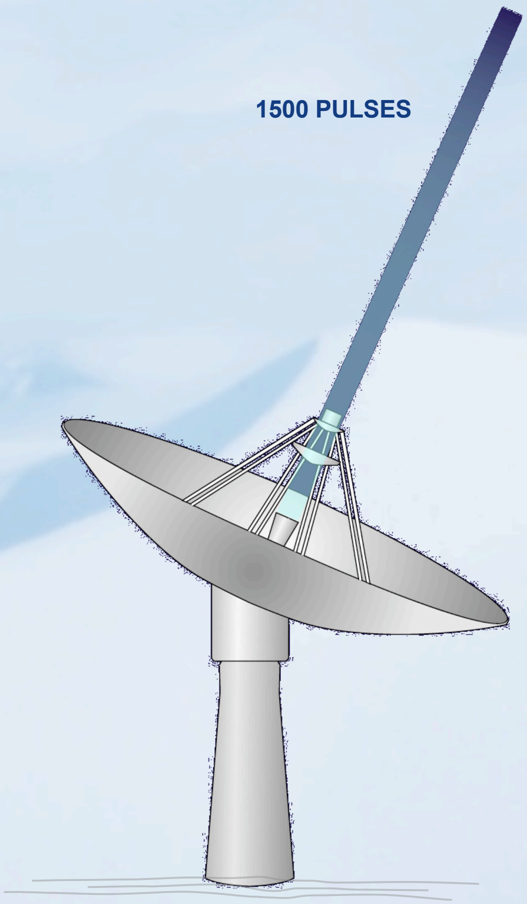
Photo: Mike Lockwood

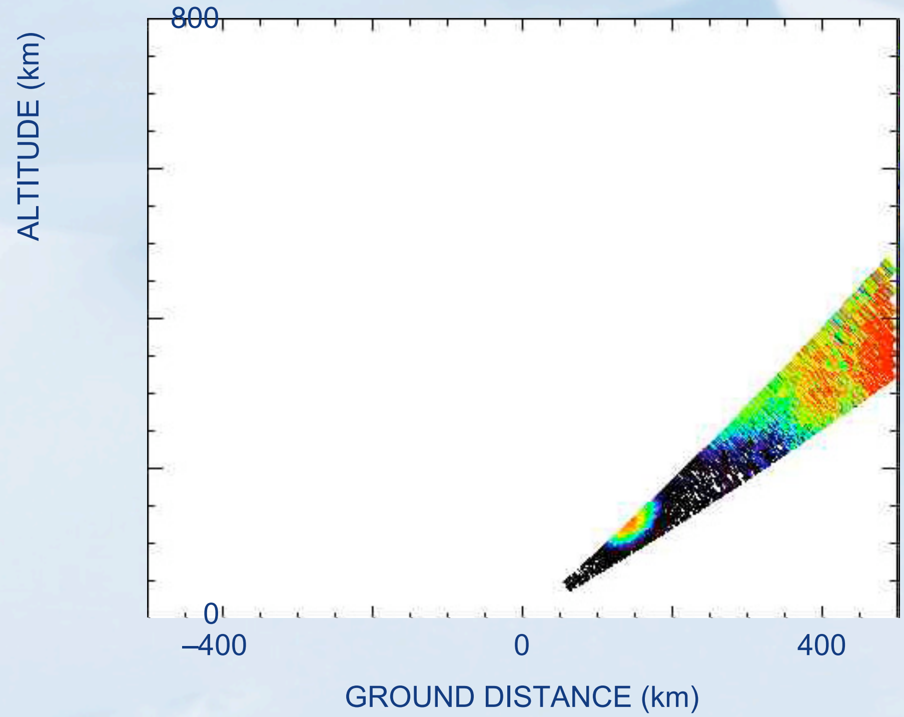
Plasma Parameter Maps



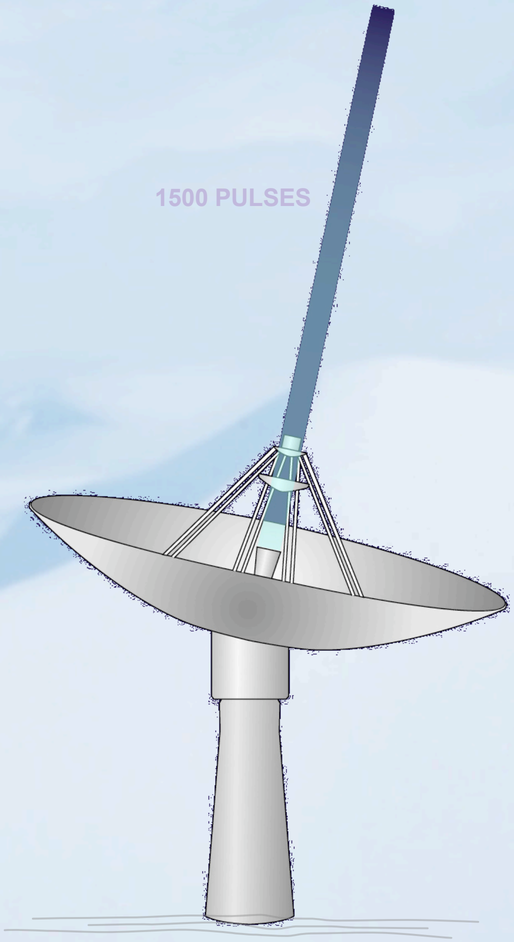


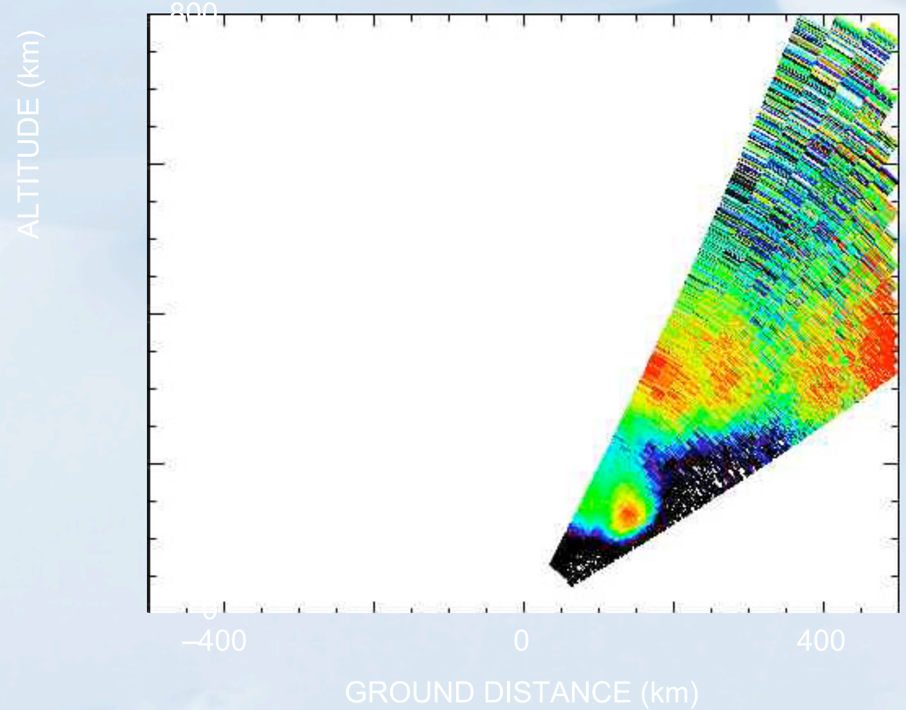
Plasma Parameter Maps



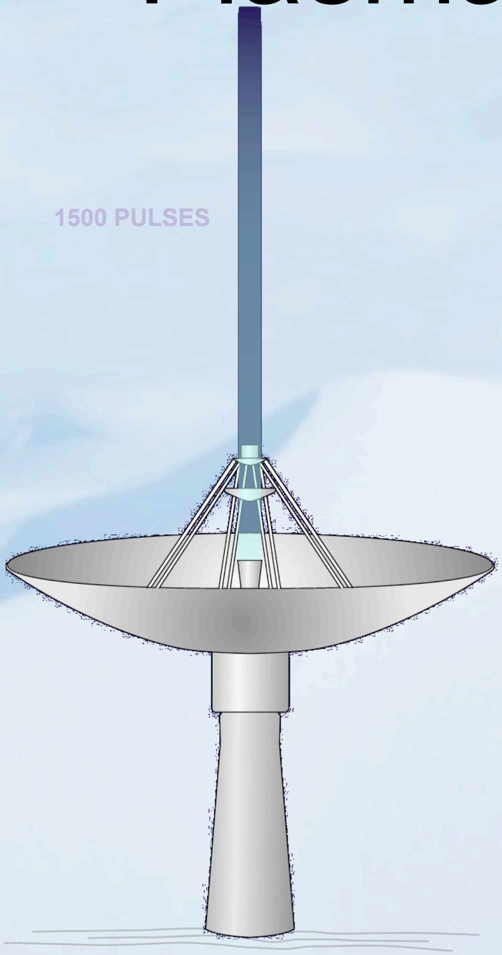


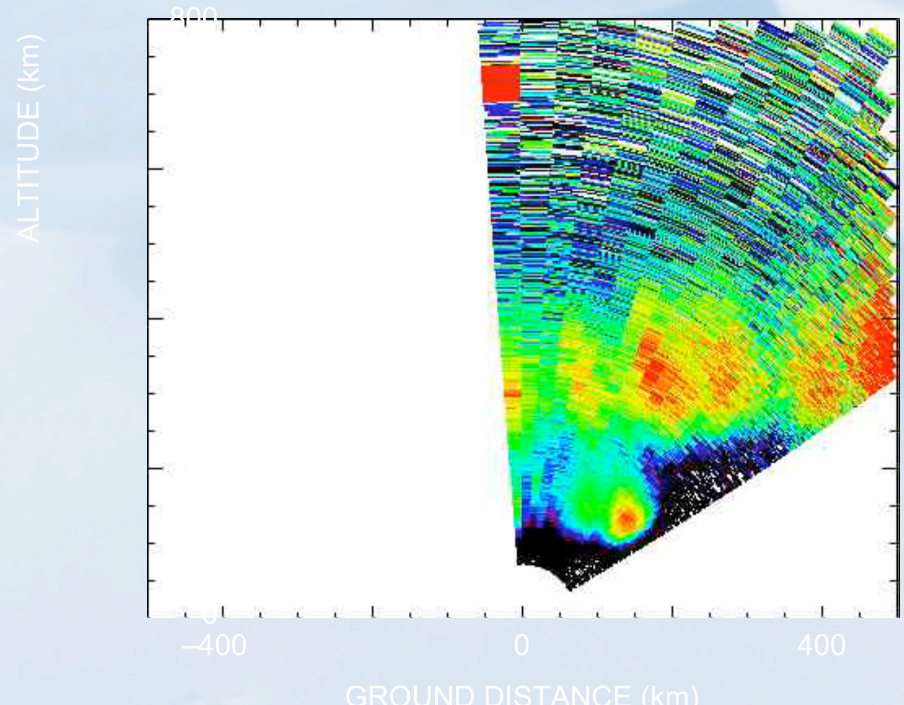
Plasma Parameter Maps



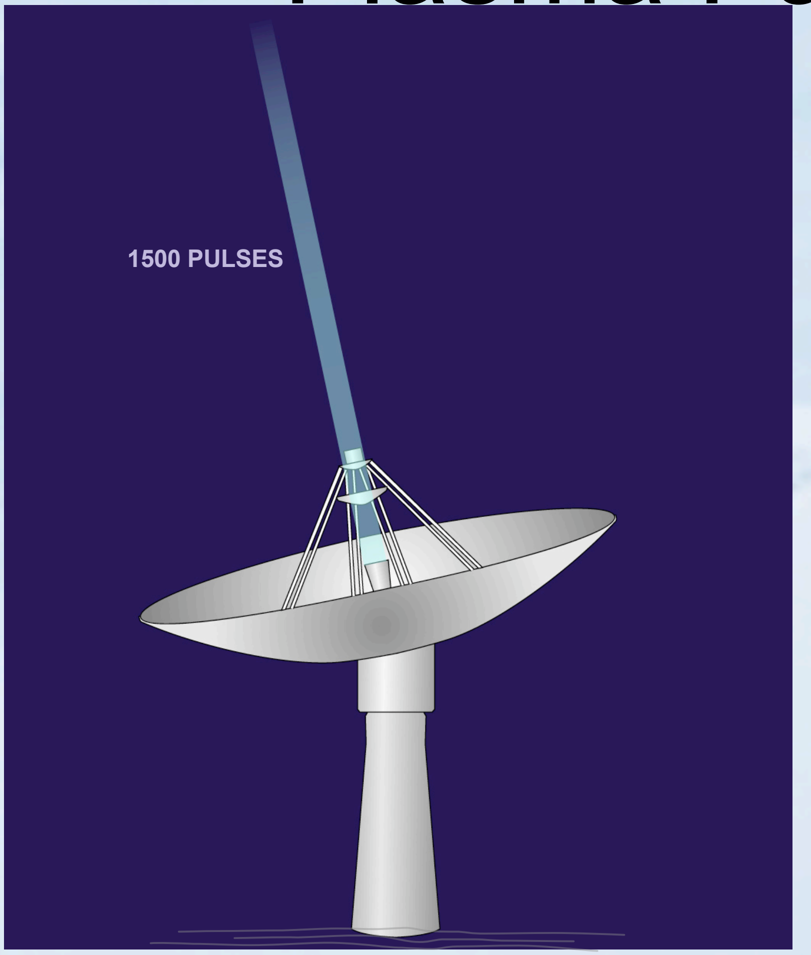


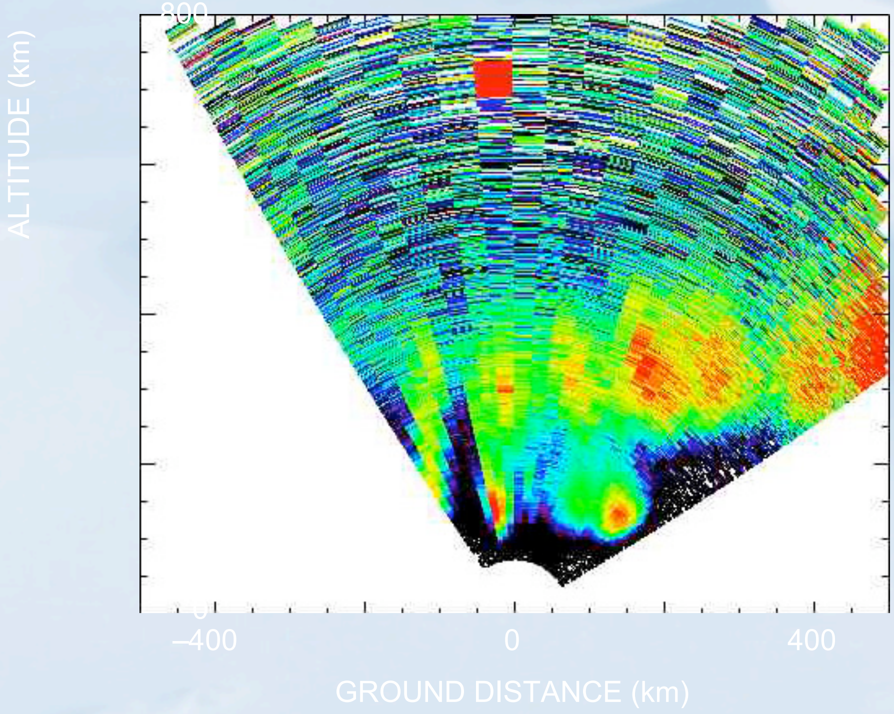
Plasma Parameter Maps



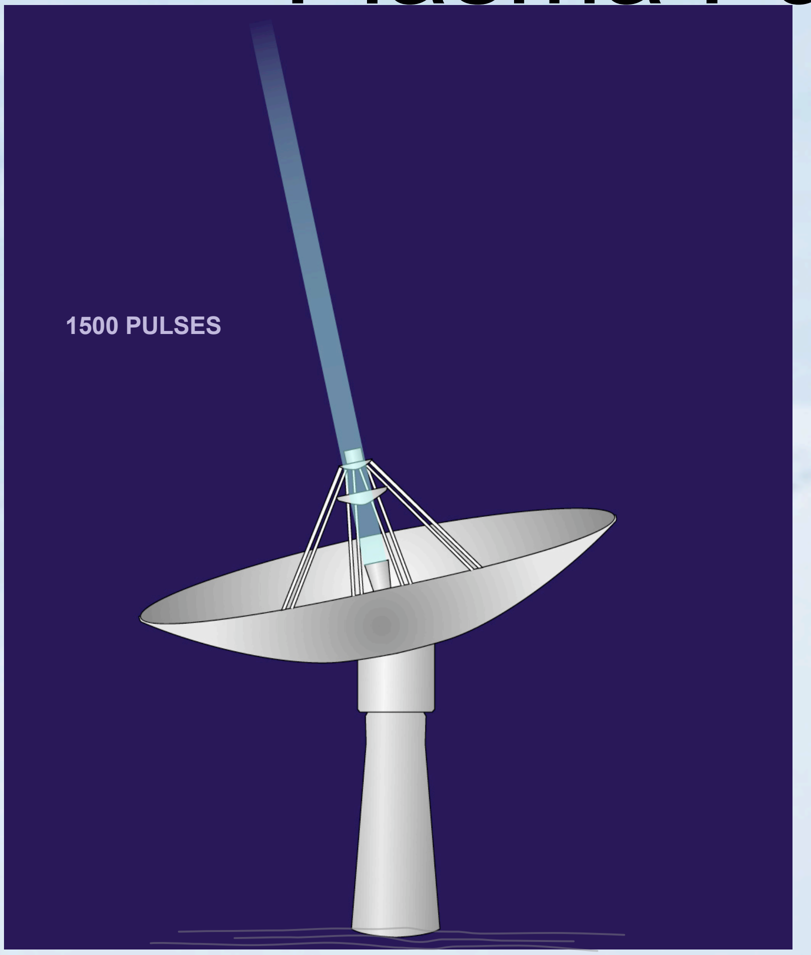


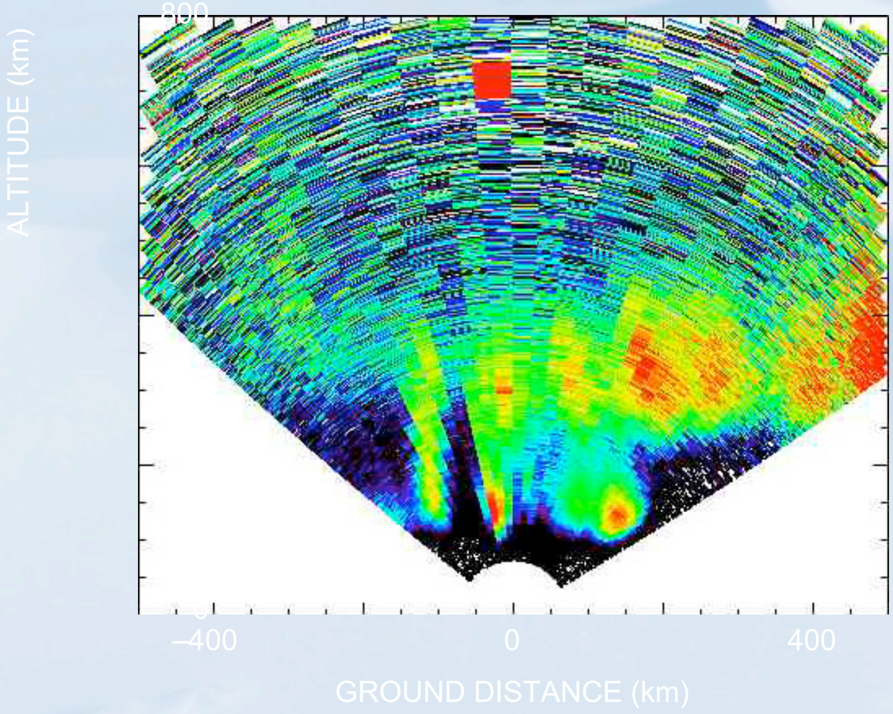
Plasma Parameter Maps



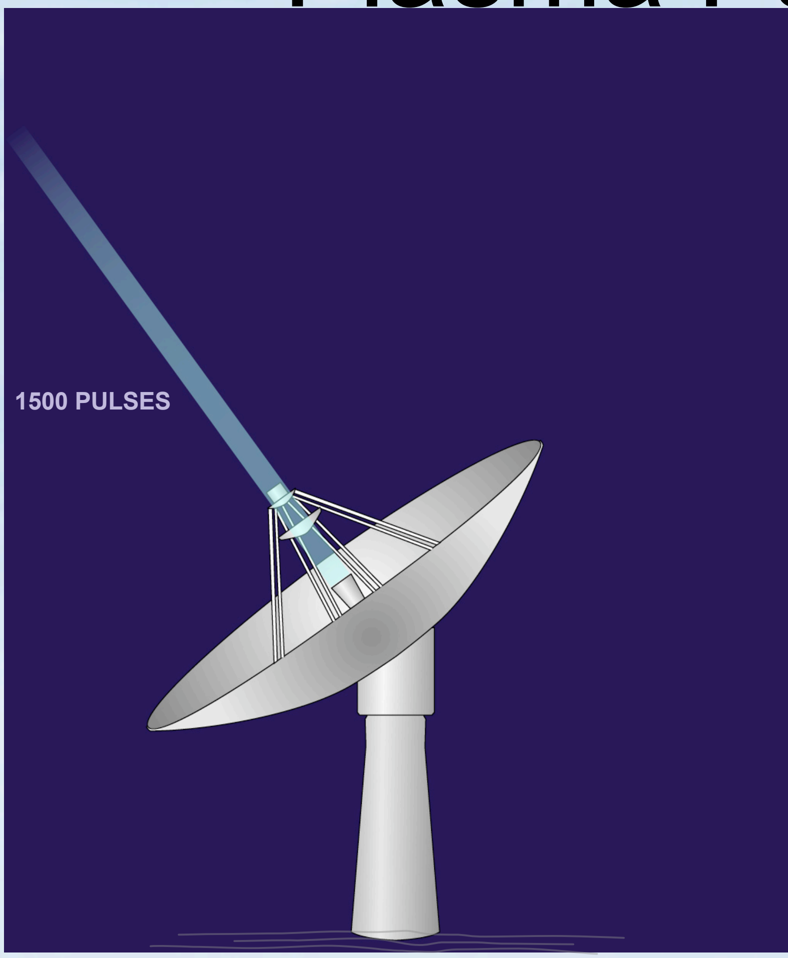


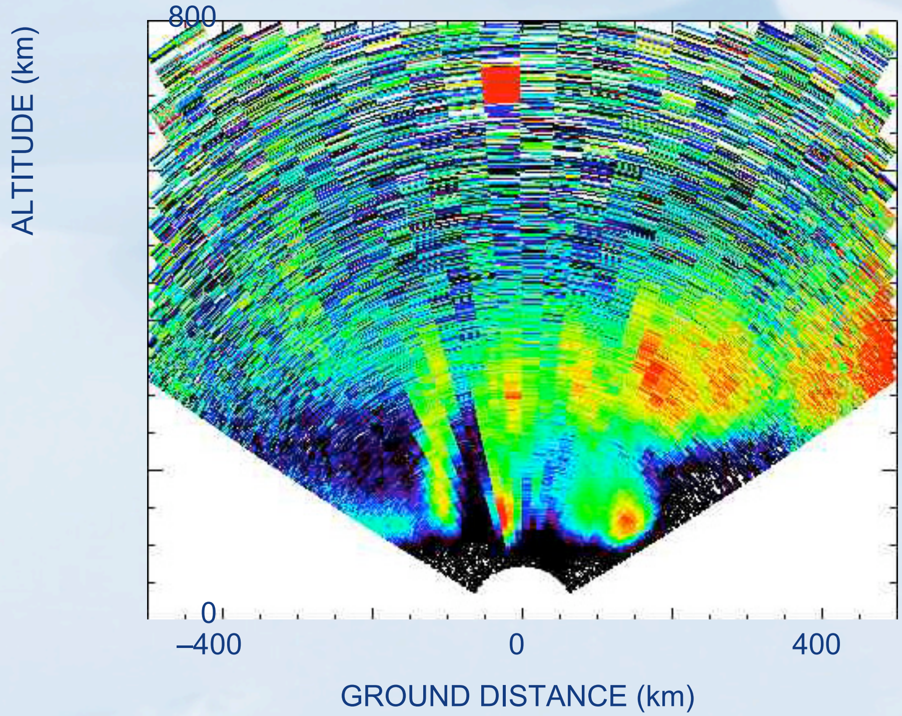
Plasma Parameter Maps





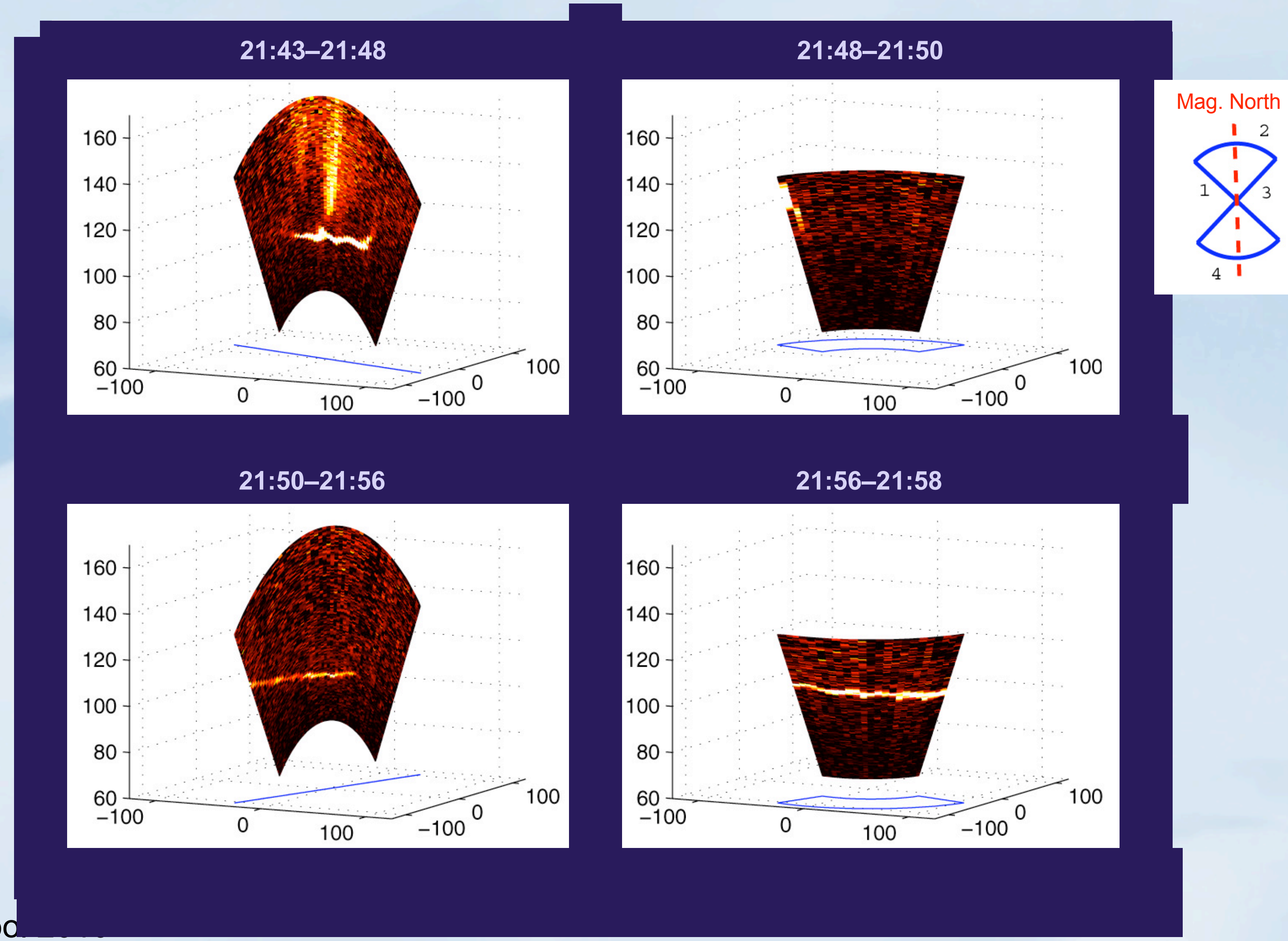
Plasma Parameter Maps



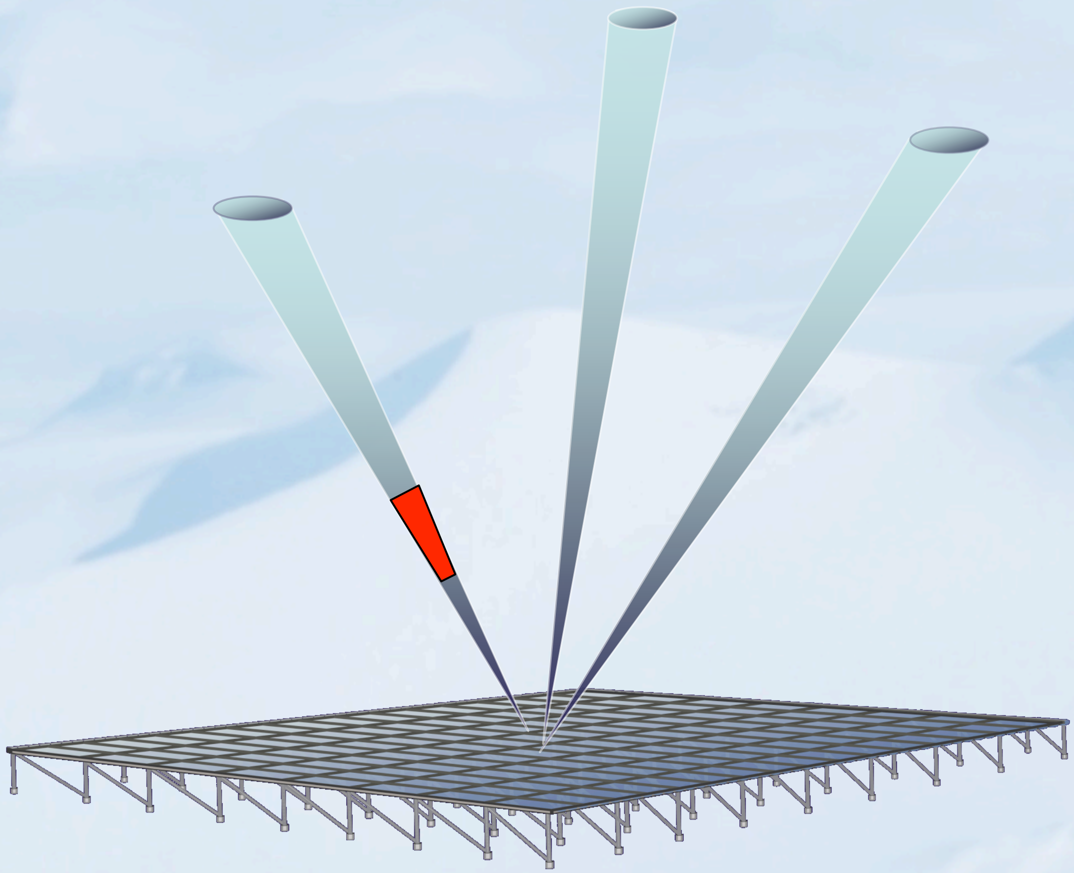


Imaging - the traditional way...

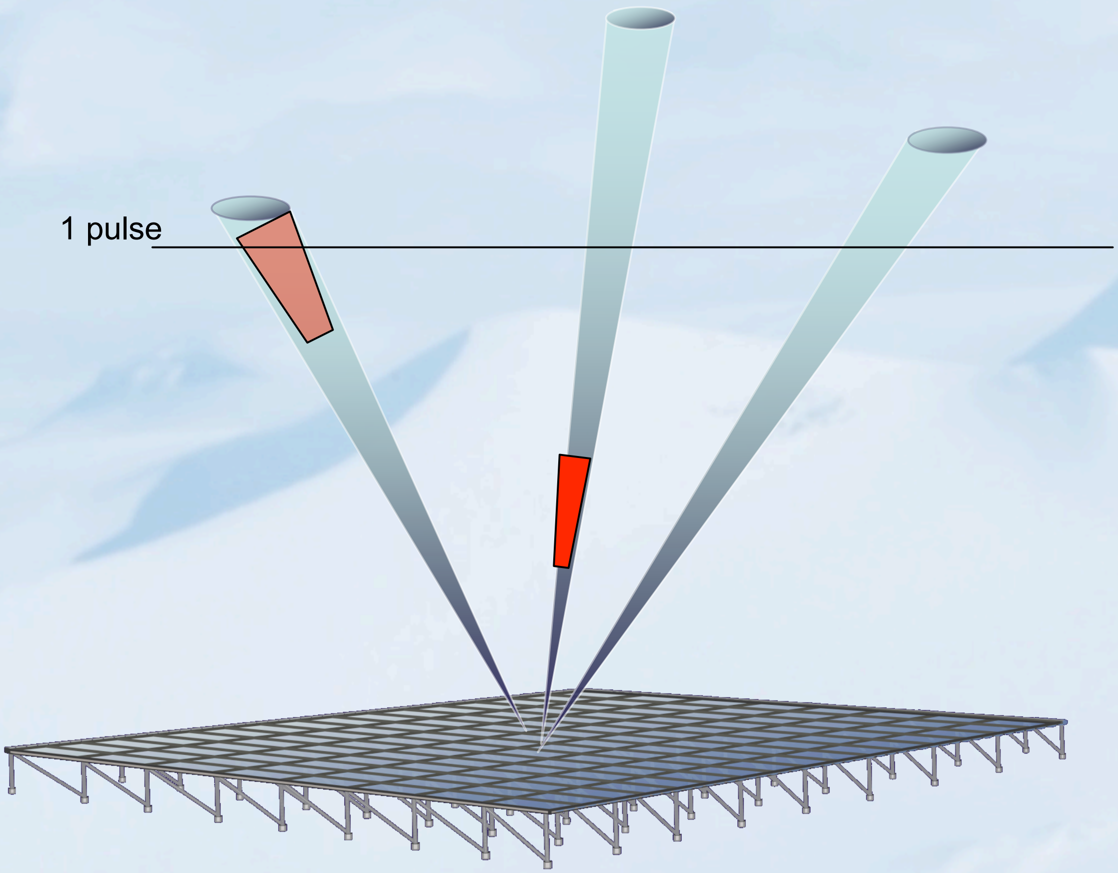
Sporadic E Evolution



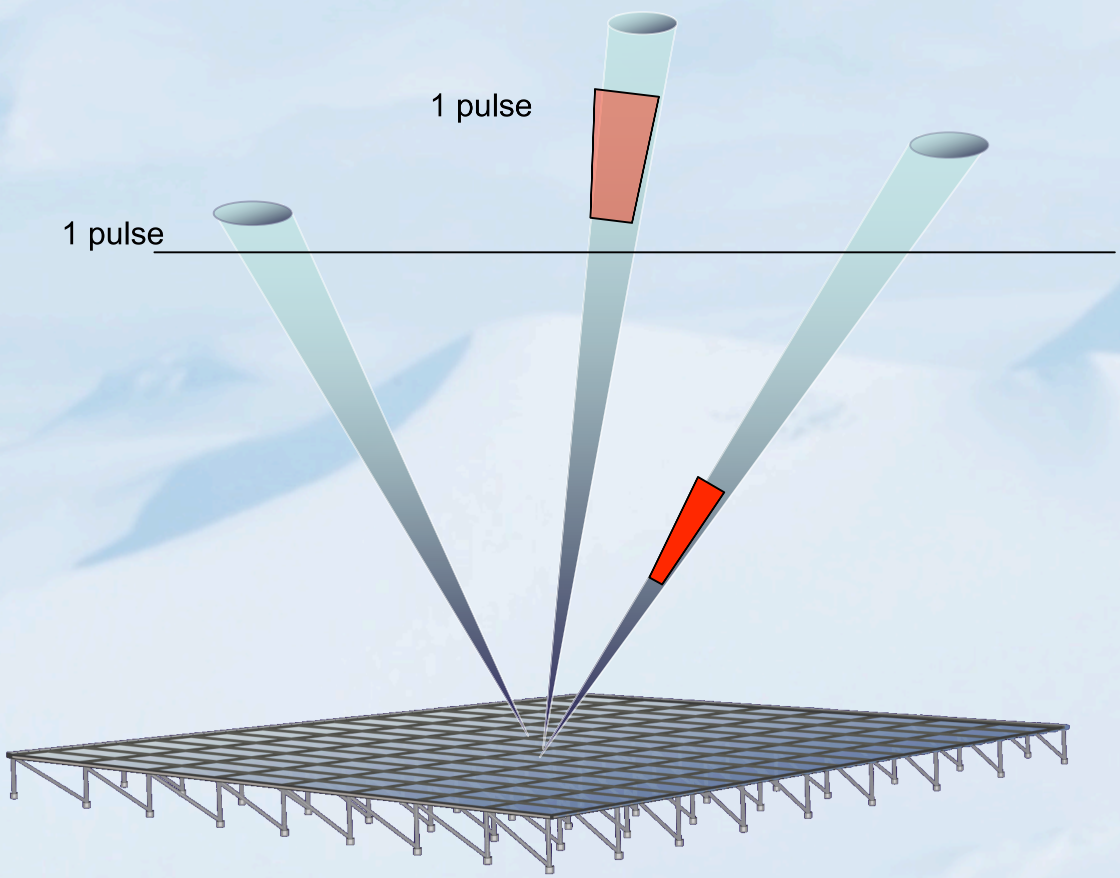
Imaging with AMISR



Imaging with AMISR



Imaging with AMISR



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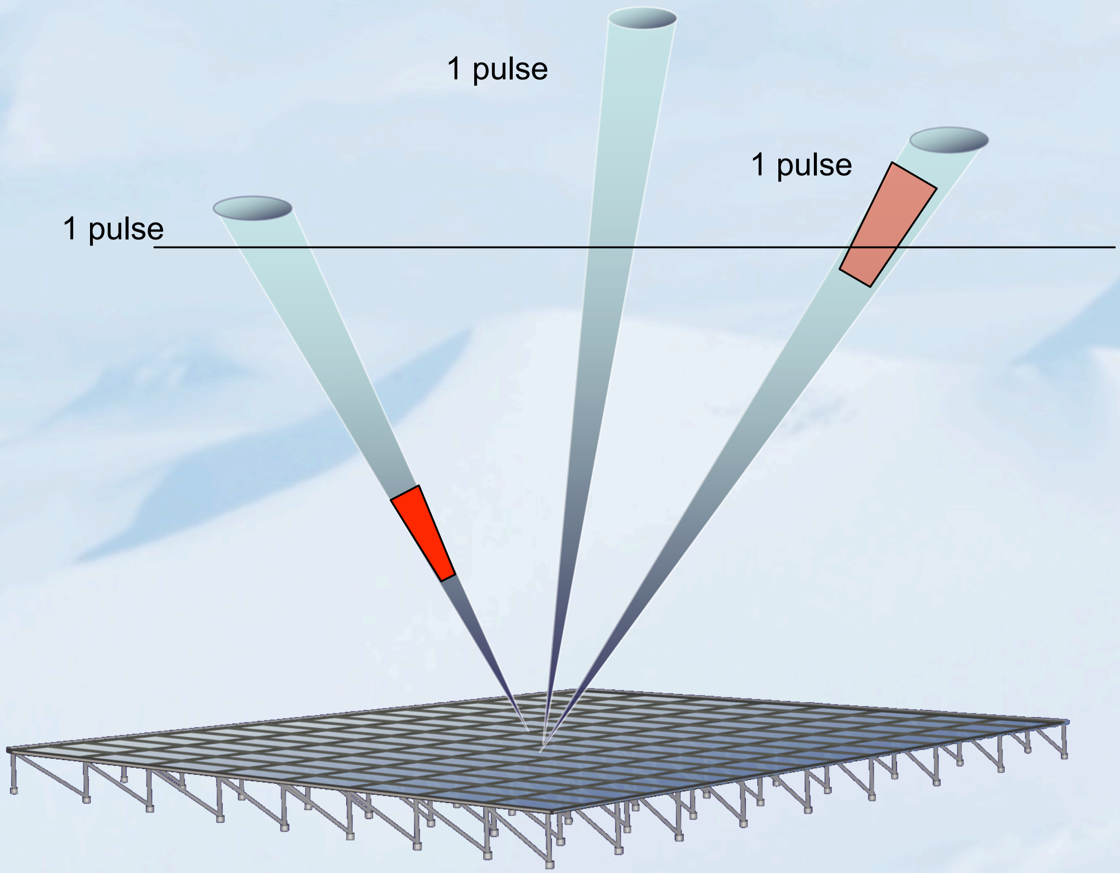

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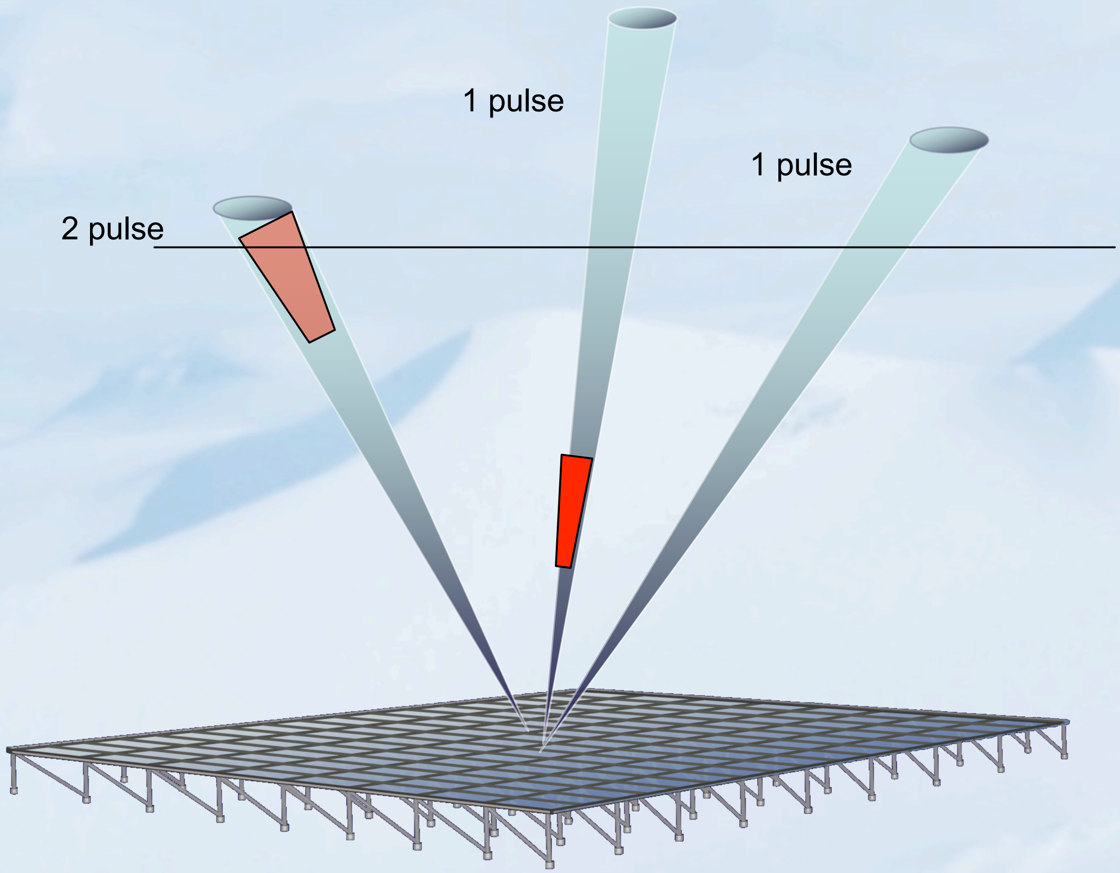

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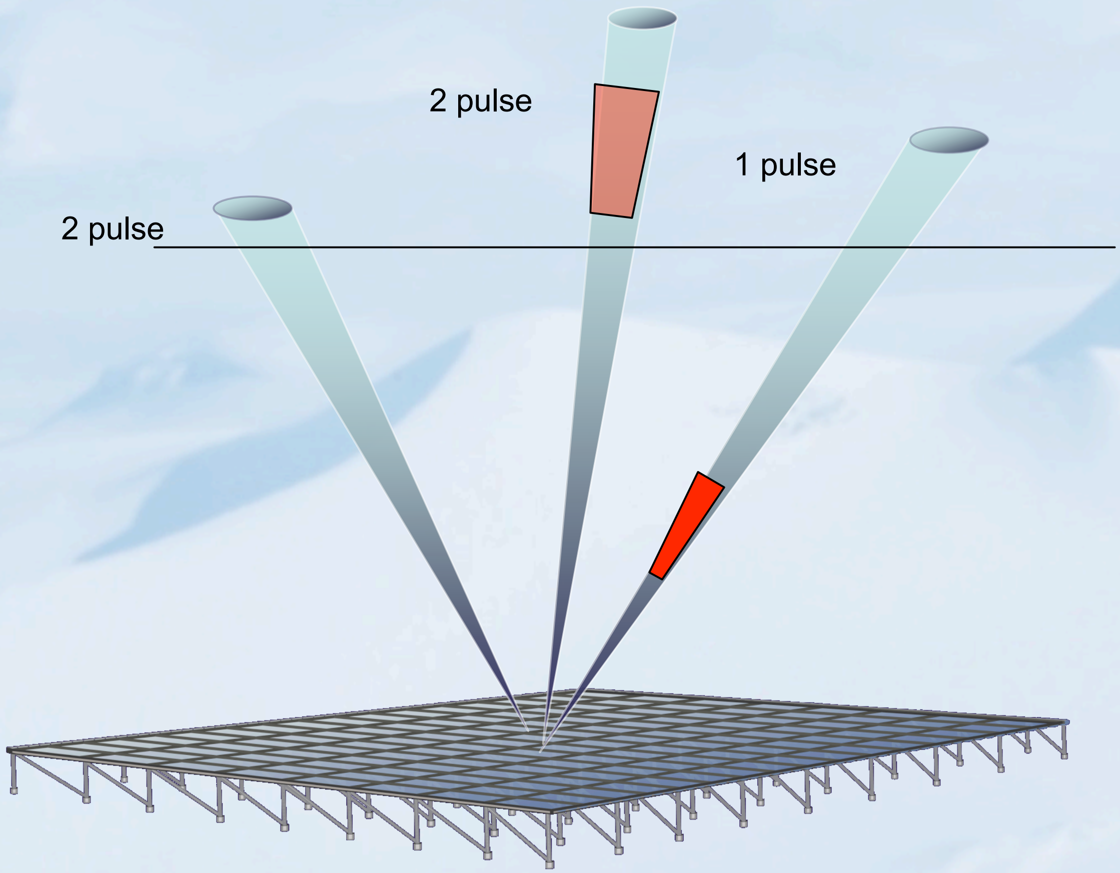

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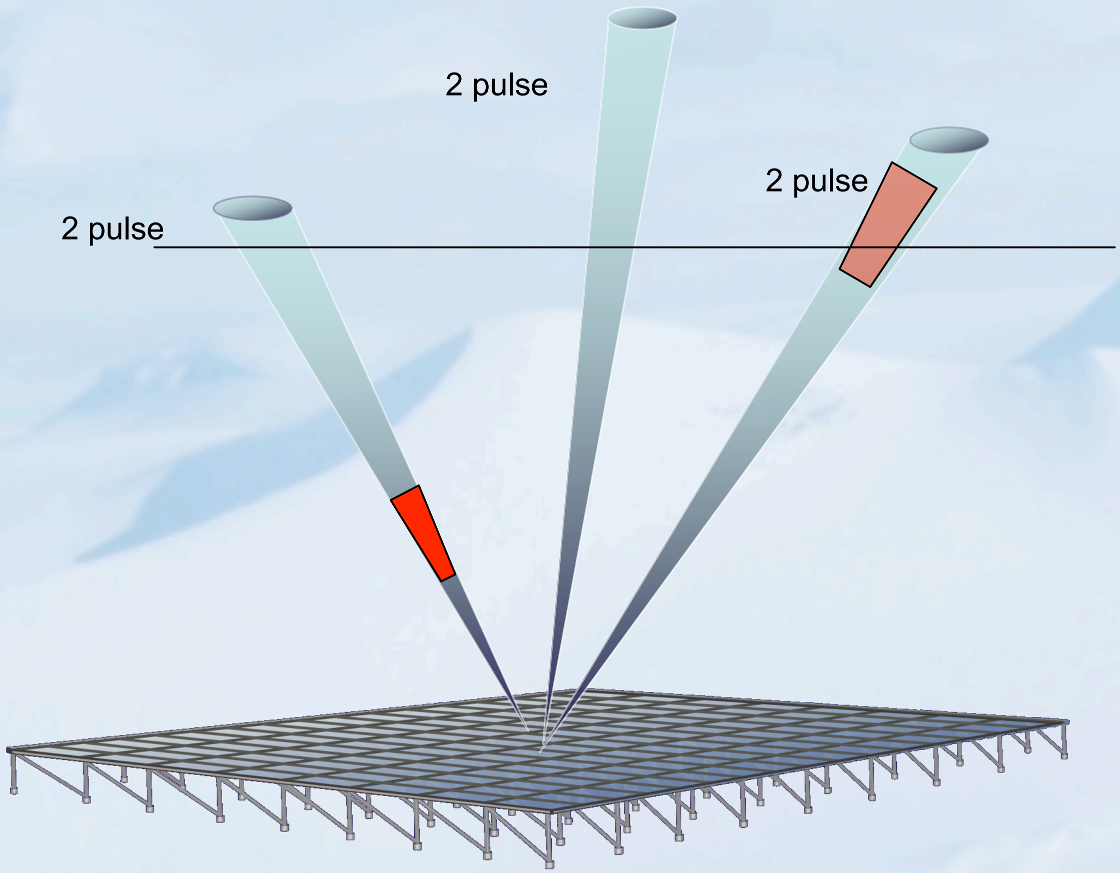

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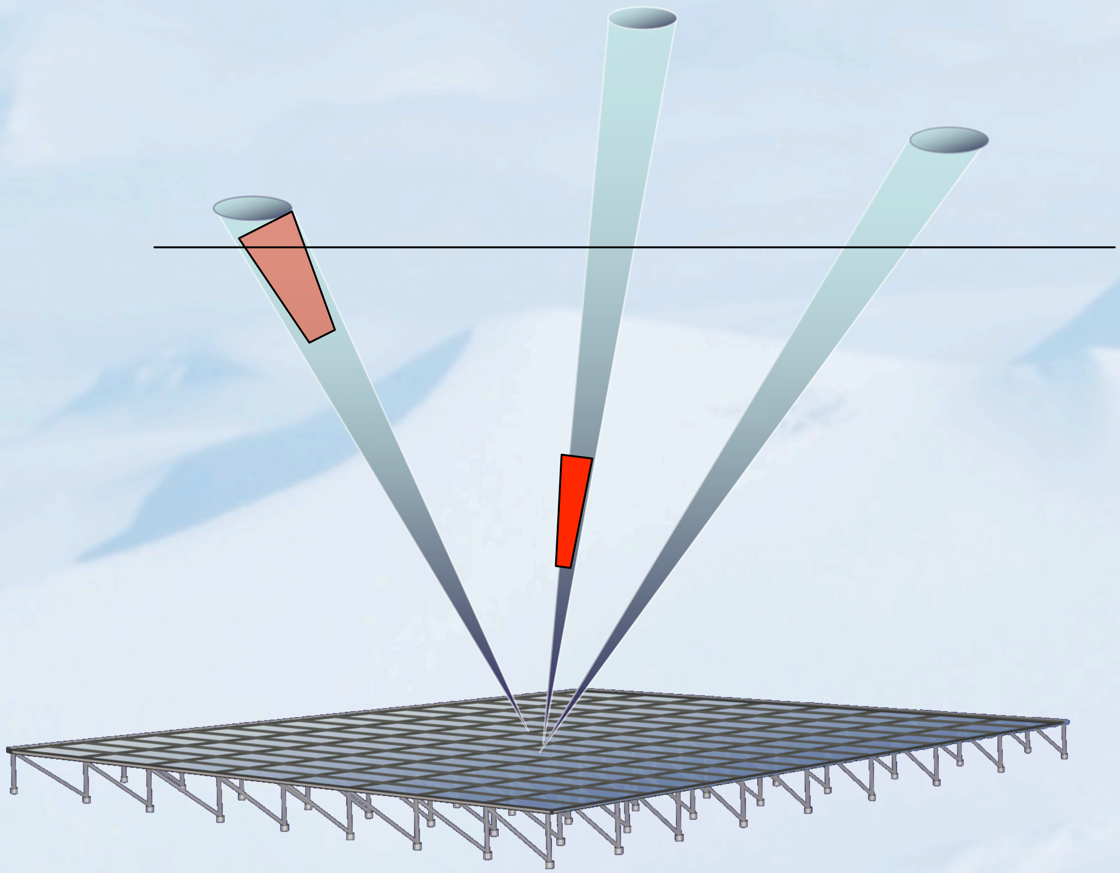
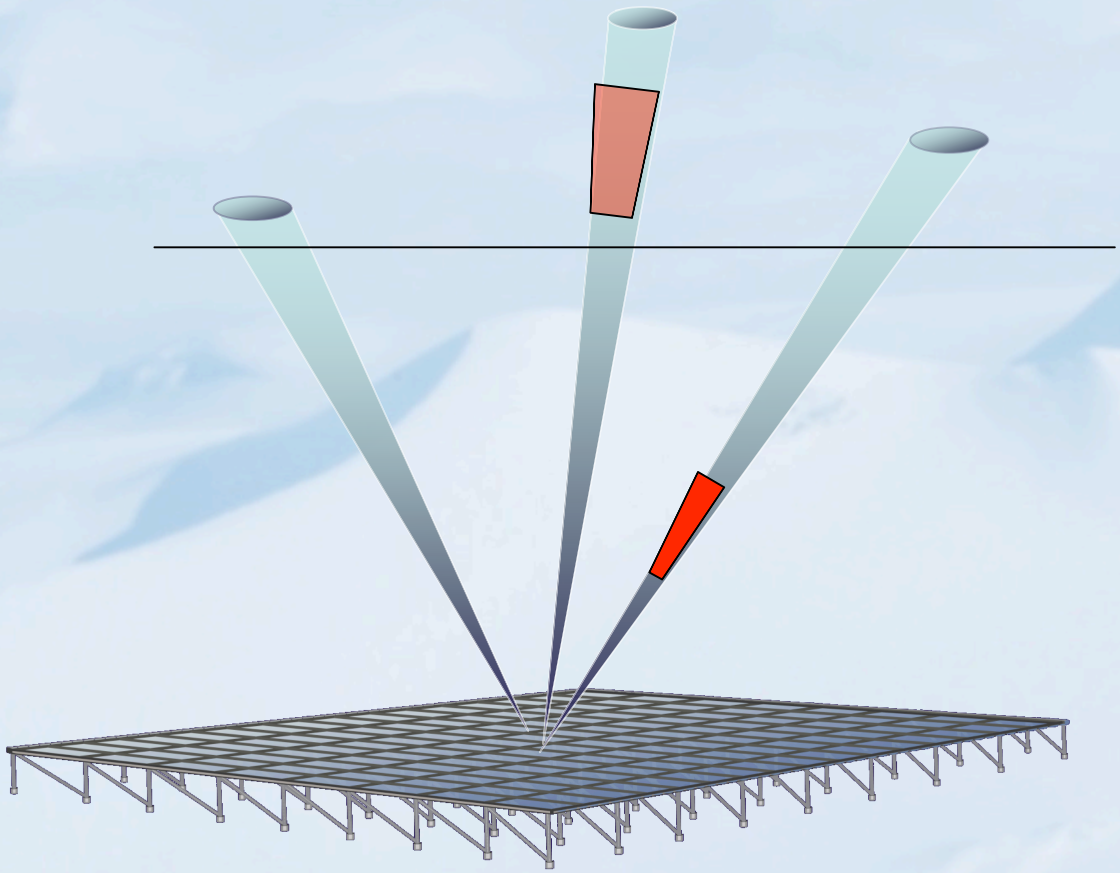


…until “enough” pulses in each direction

Imaging with AMISR



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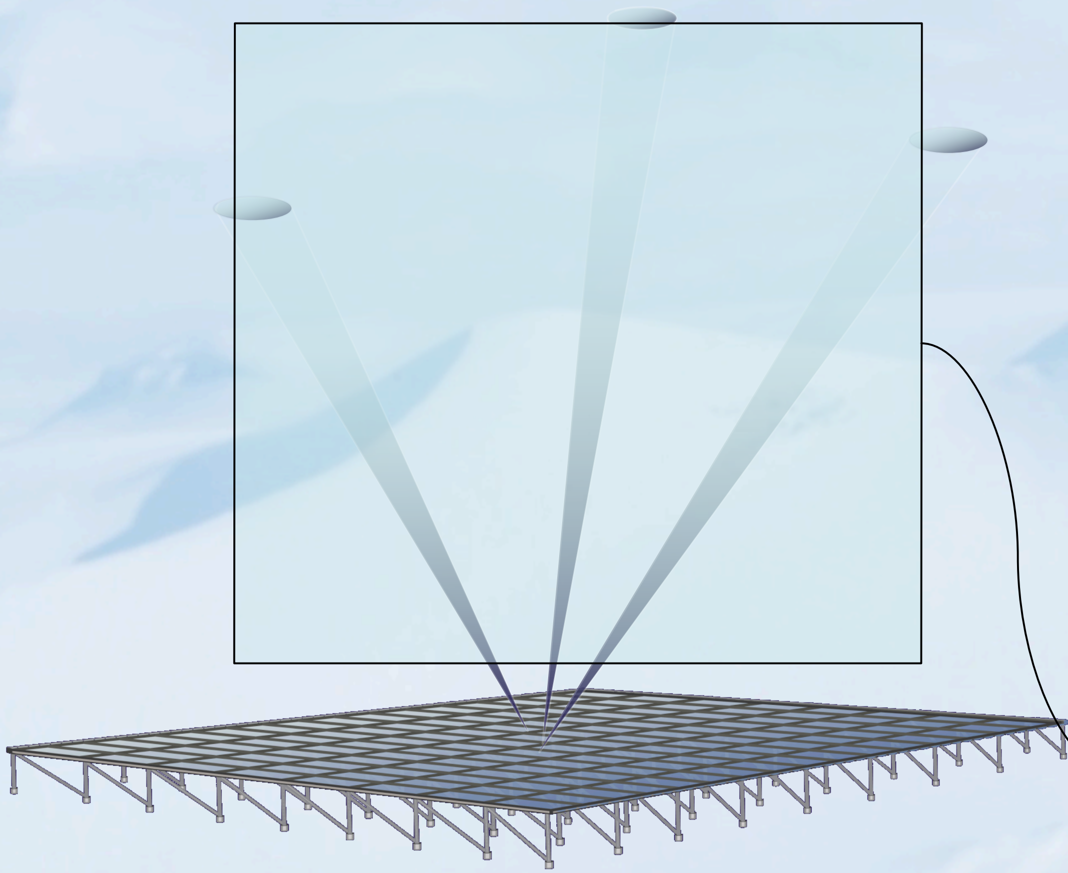

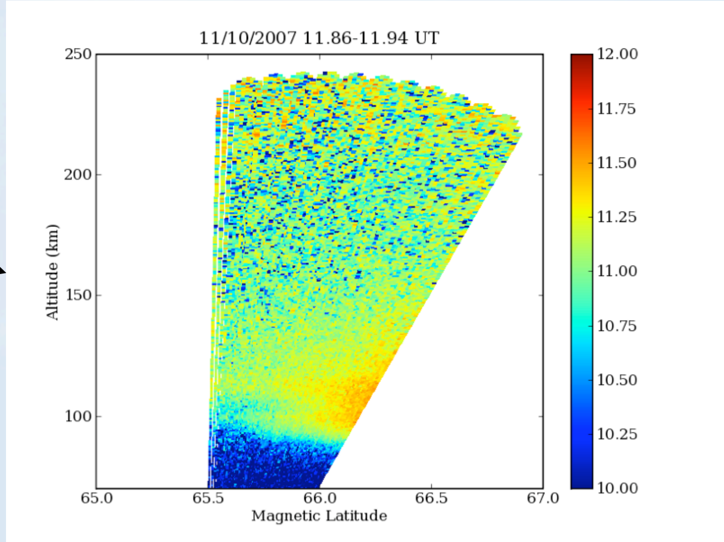

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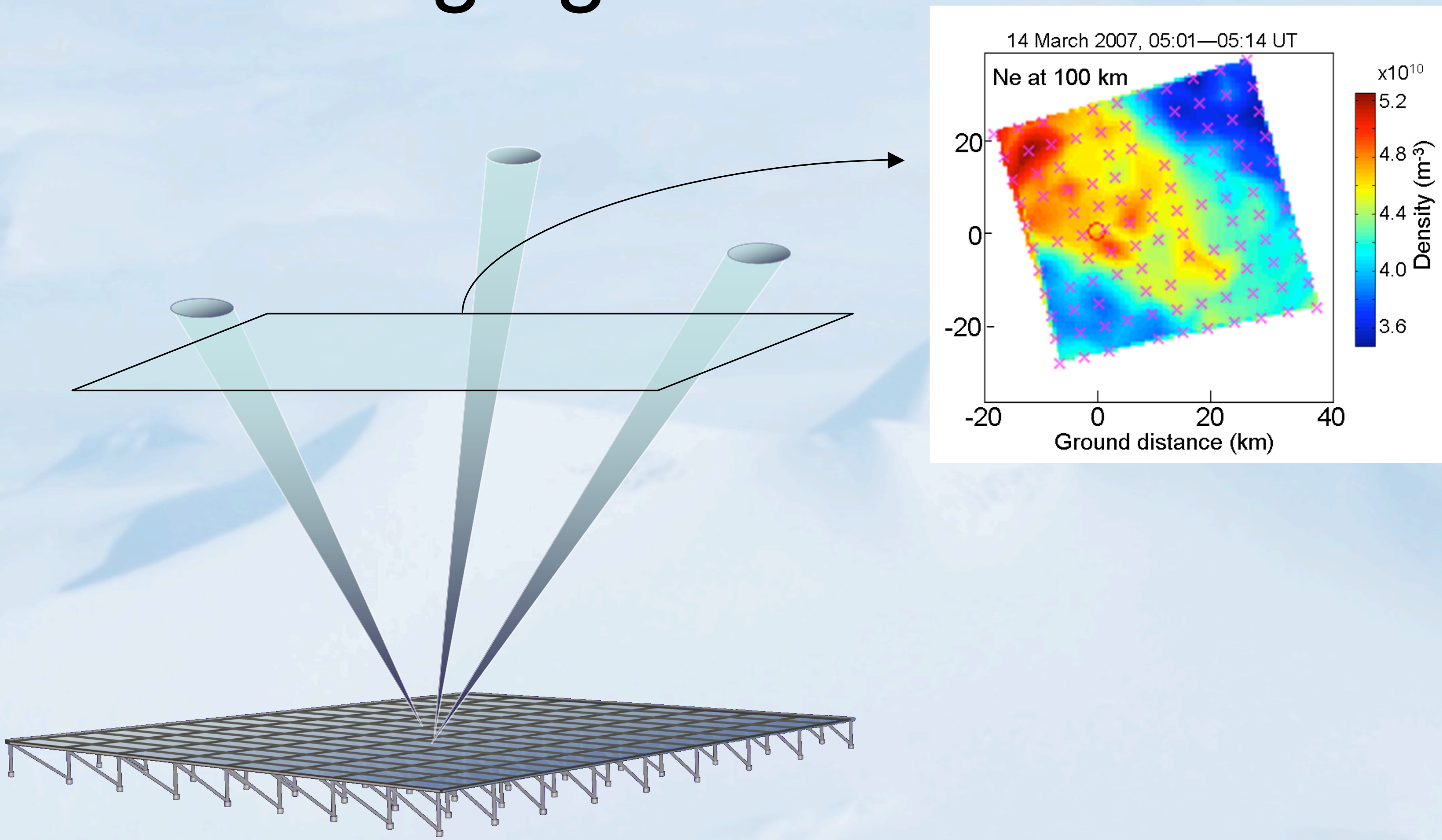

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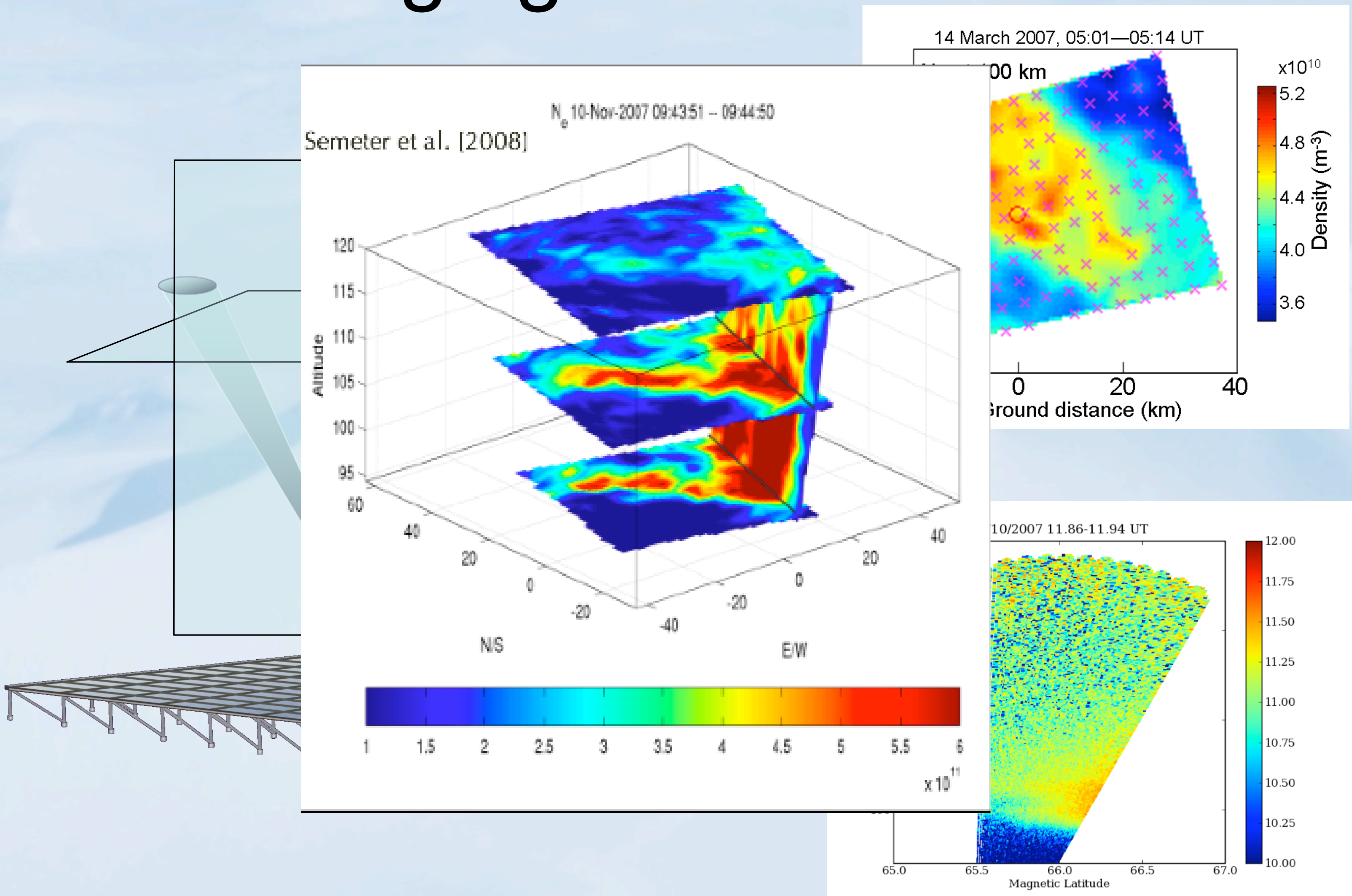

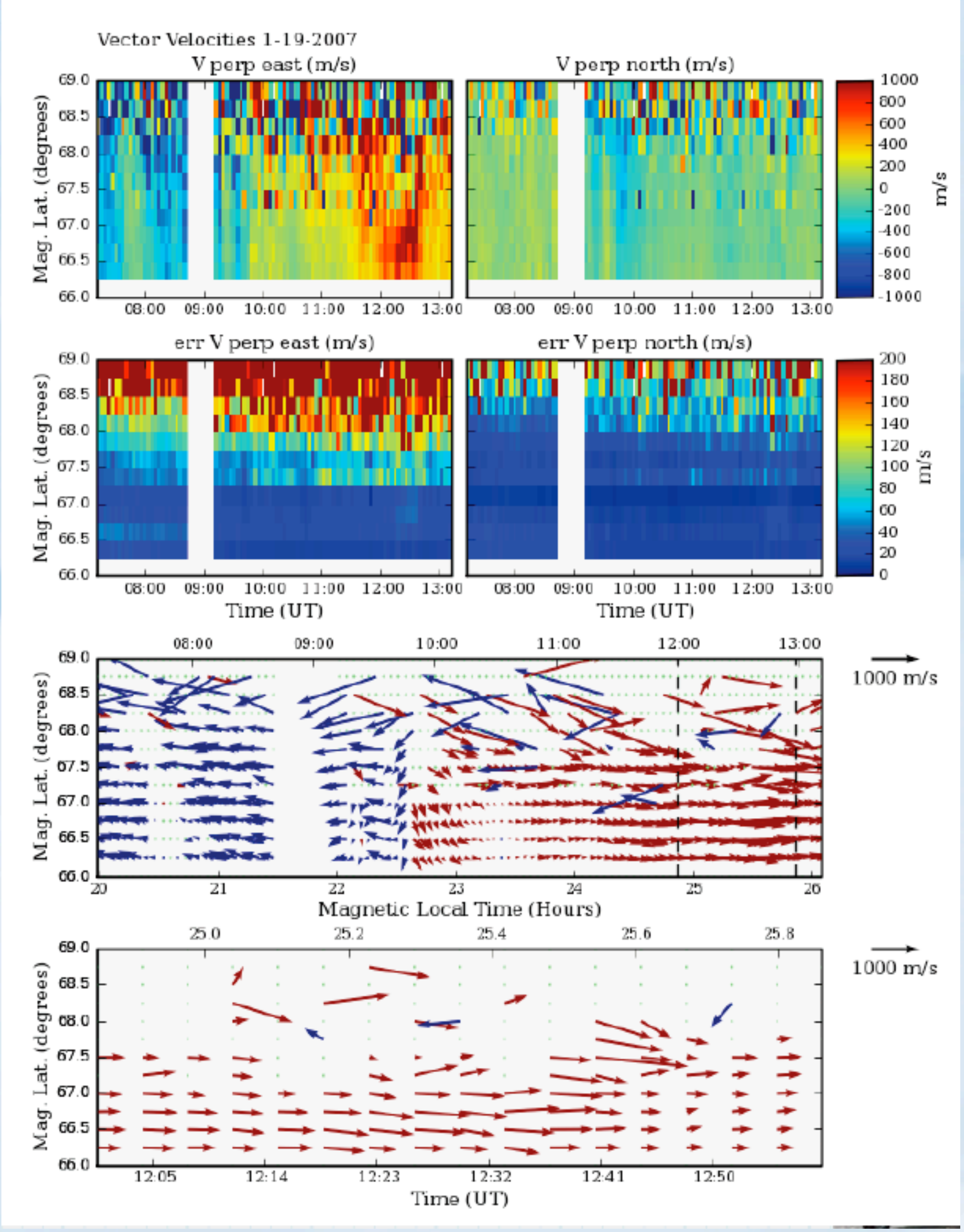

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Monostatic E Field Estimation

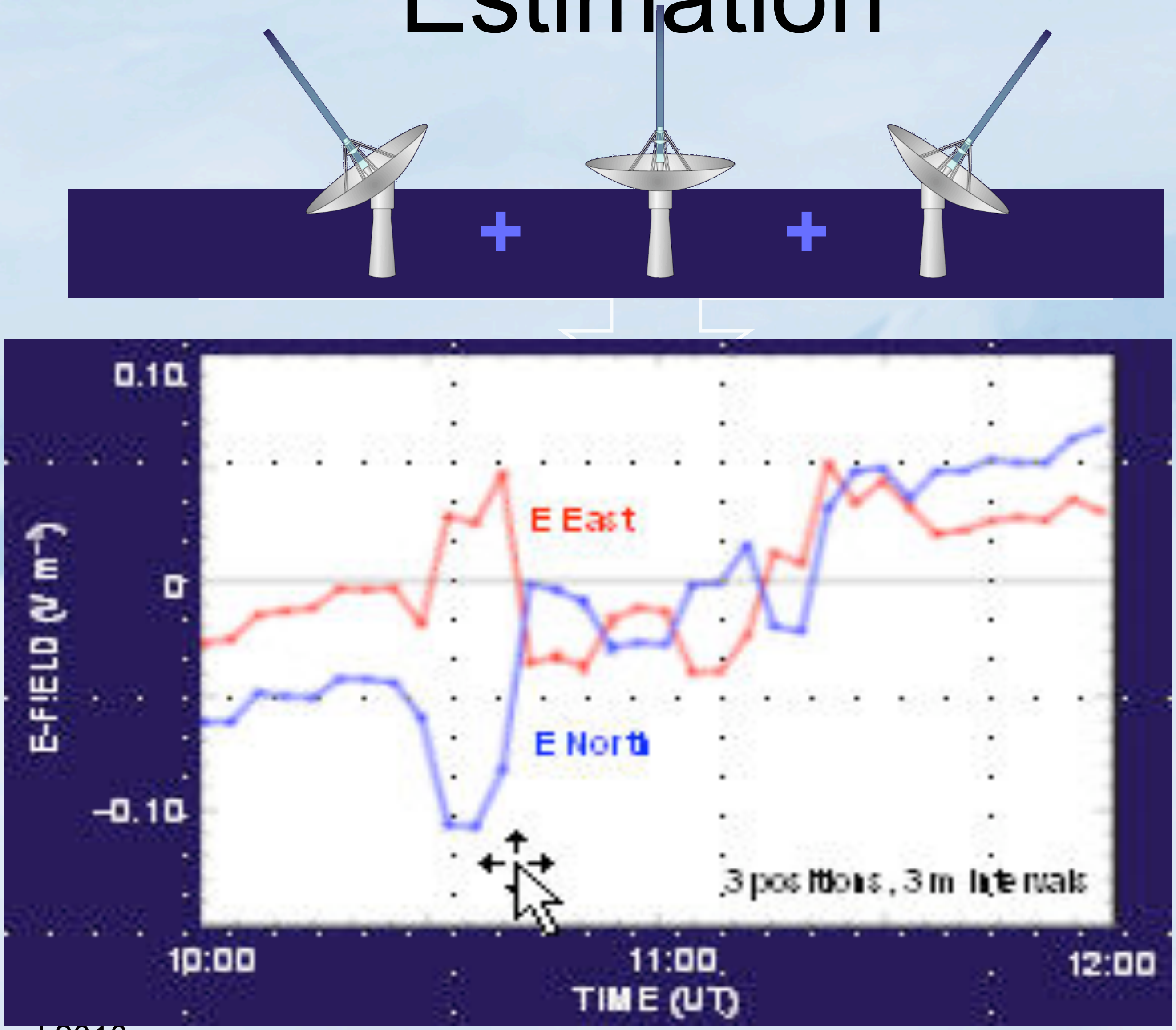
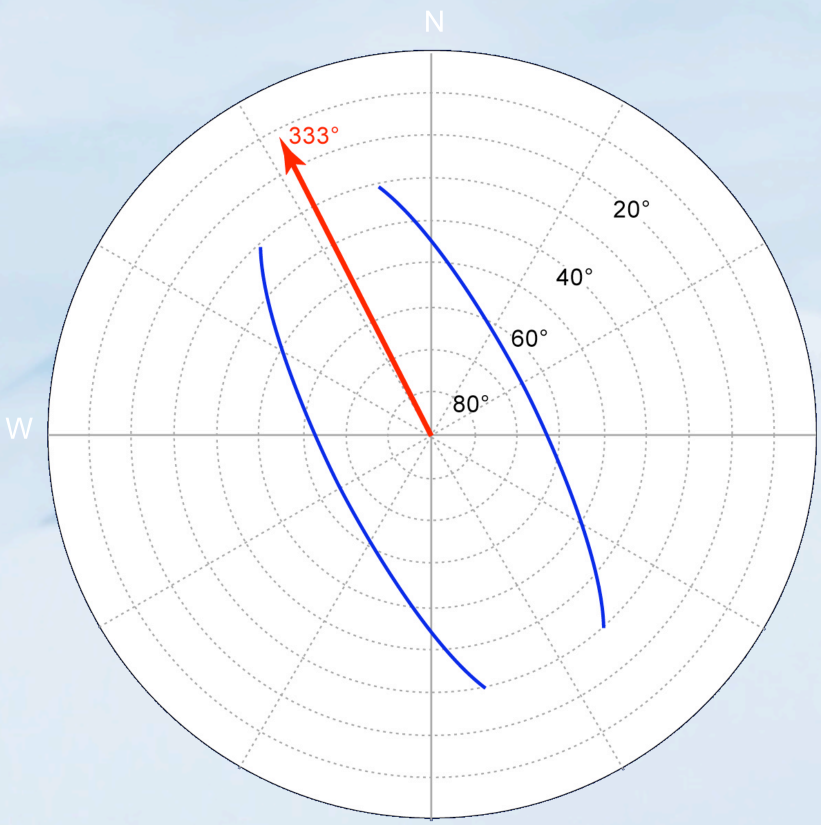


Photo: Craig Heinselman

Ion Velocity (E-field) Maps



Composite Scans for
E-field Estimation

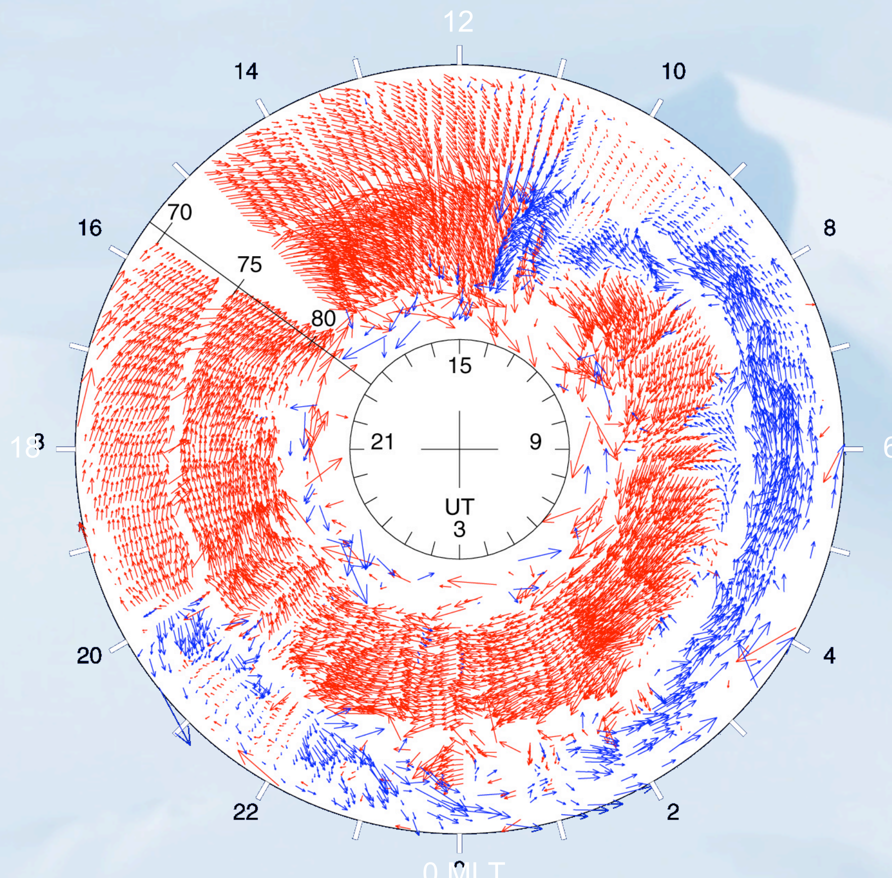


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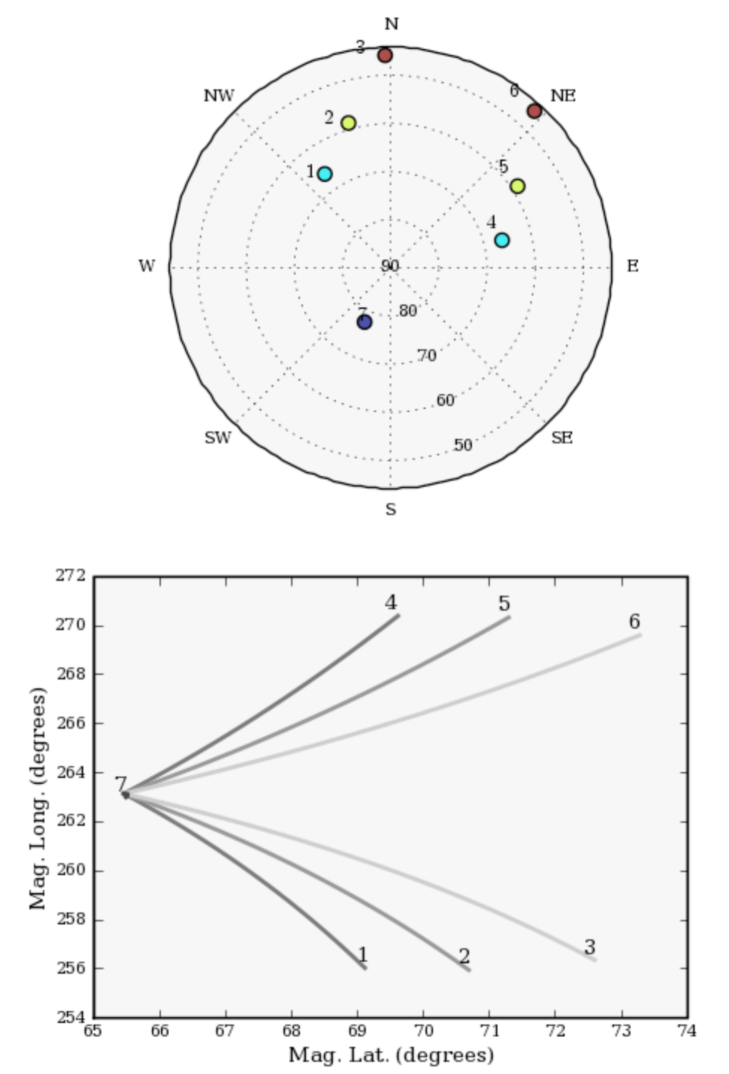
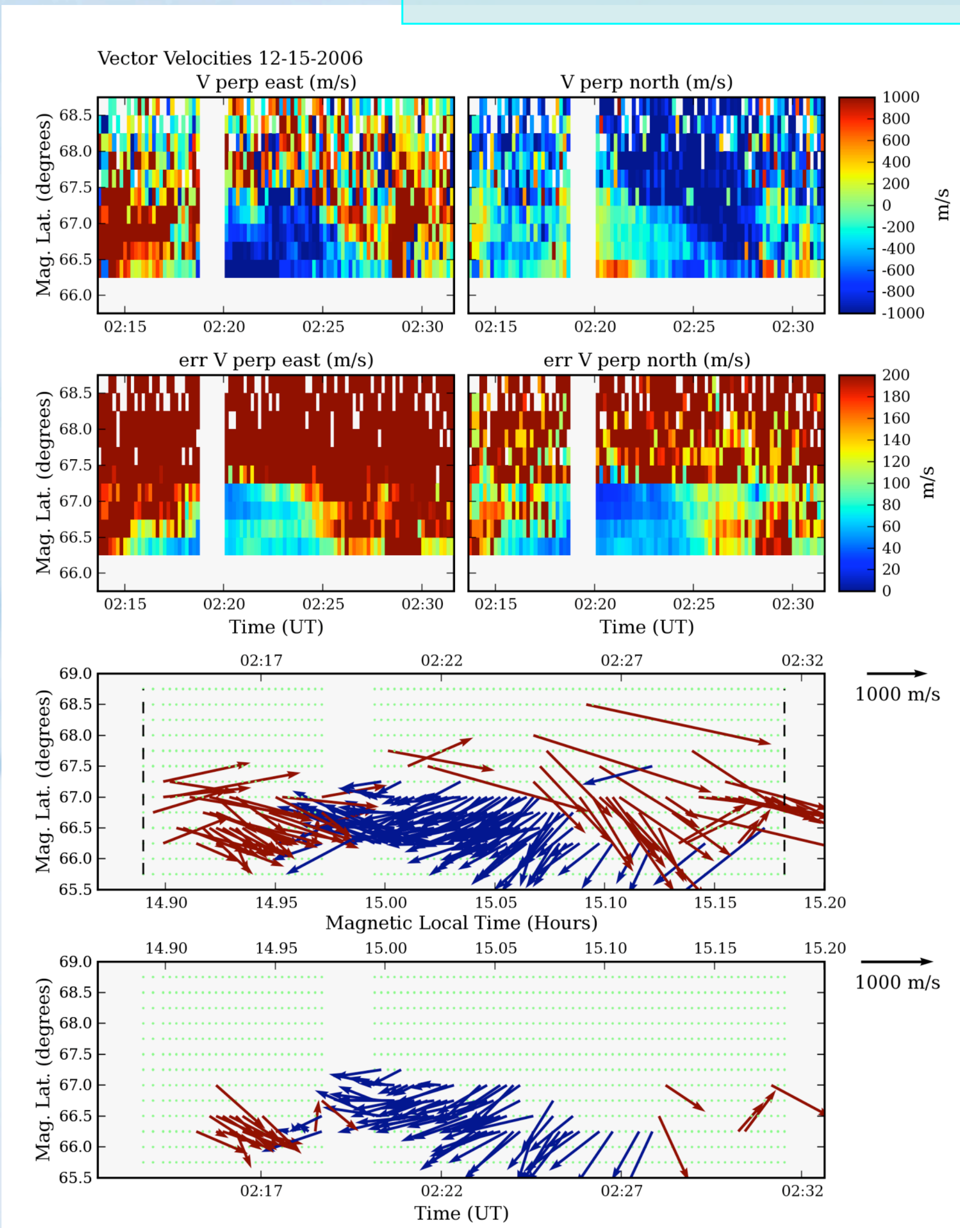
Photo: Craig Heinselman

E-fields with AMISR



Combined velocities



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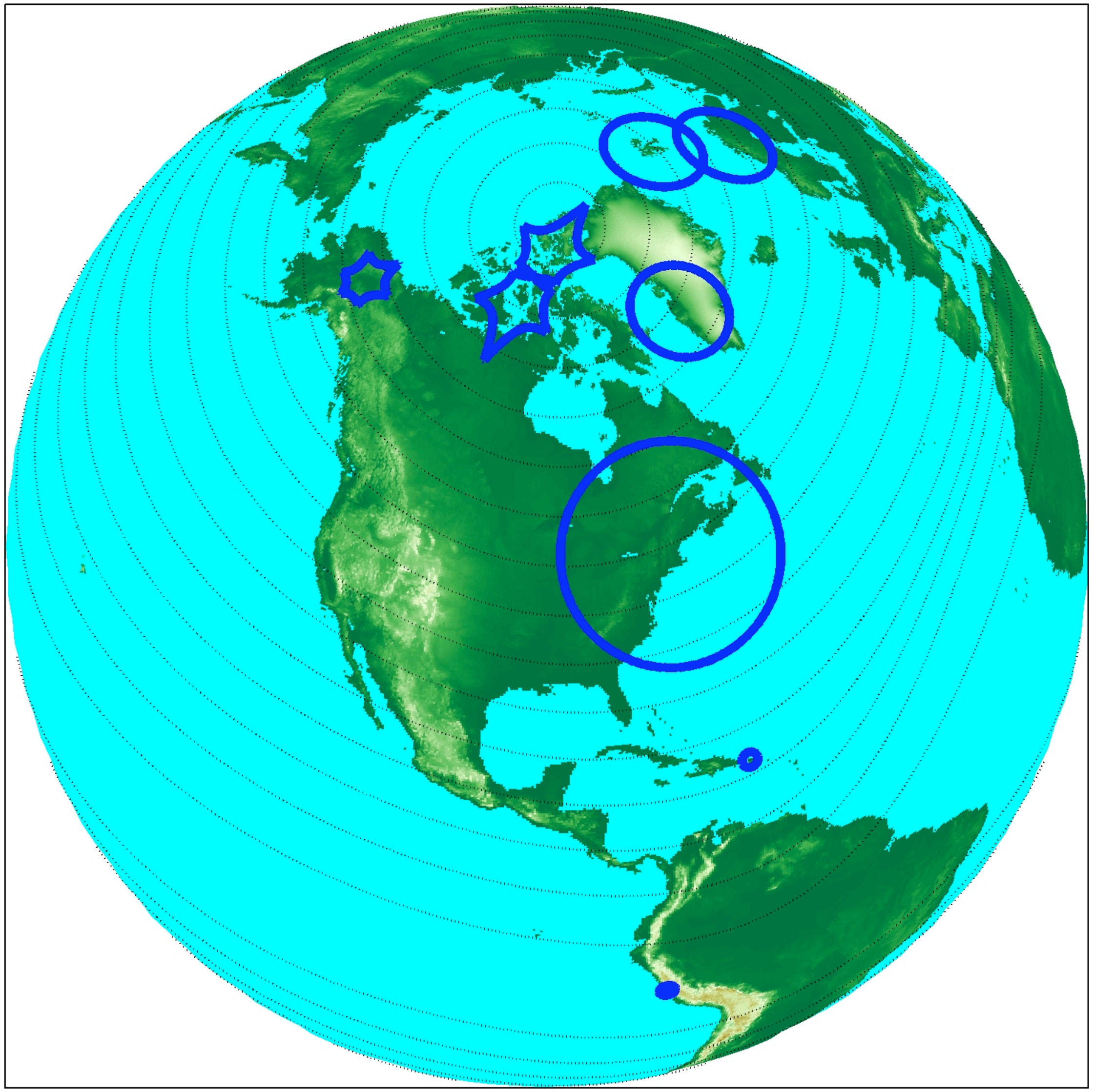
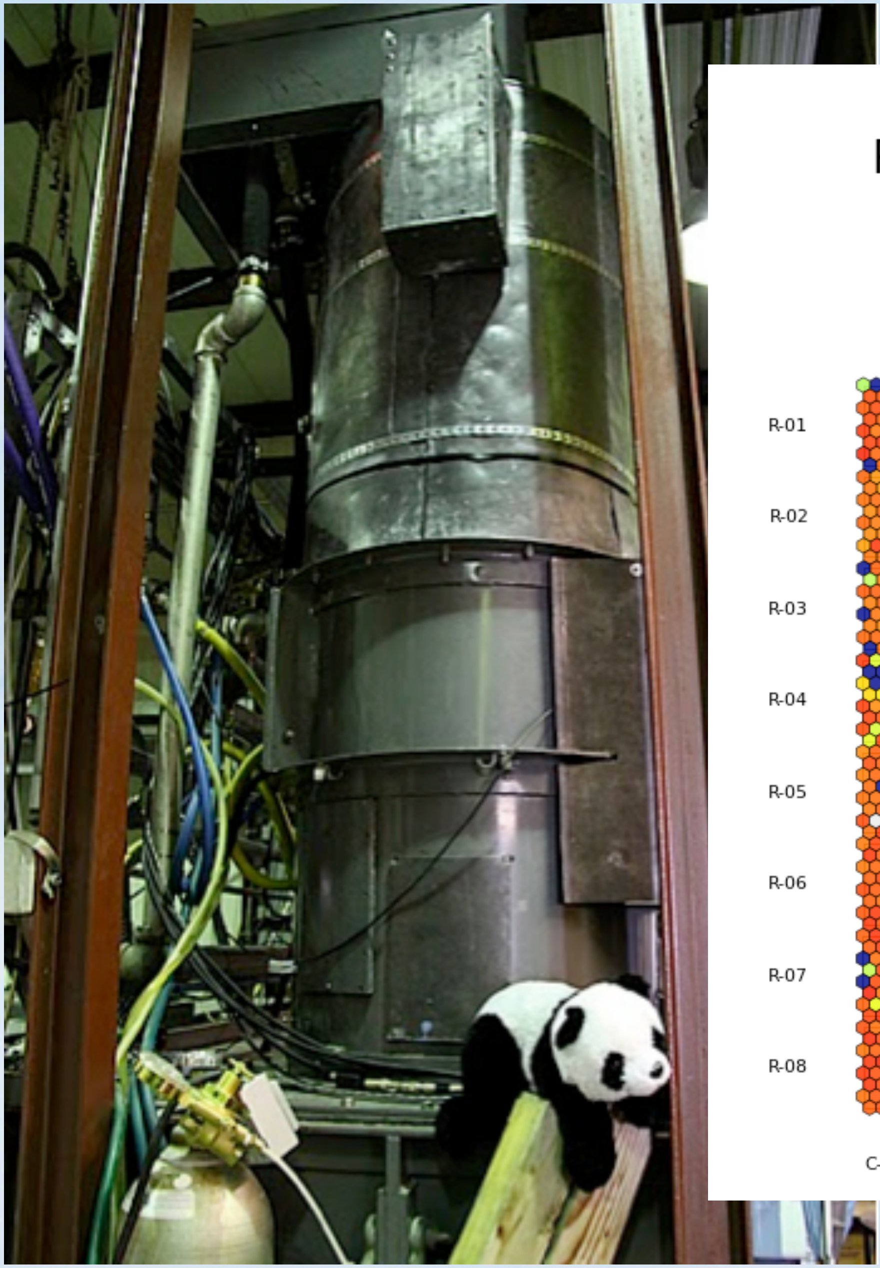
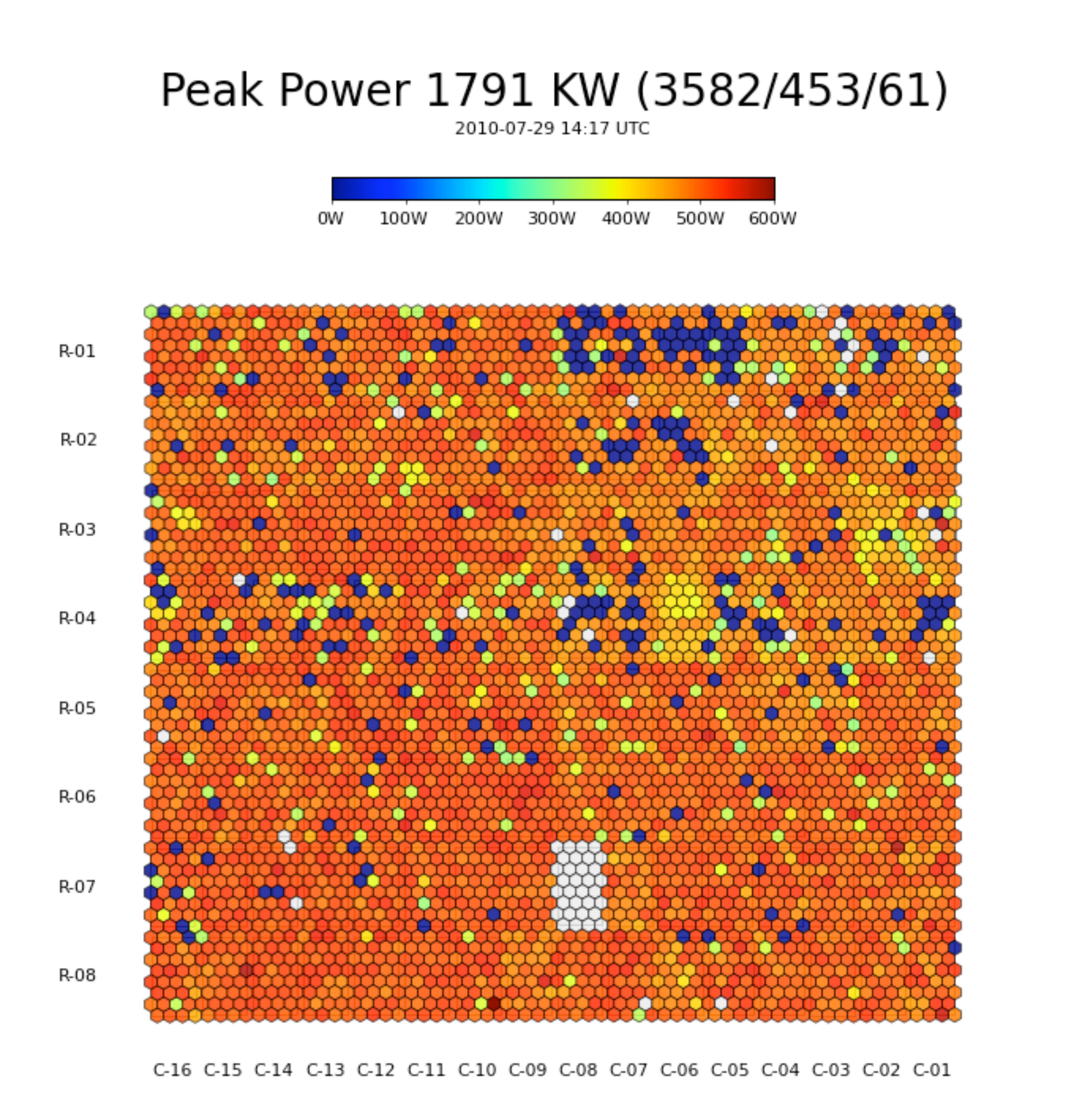


Courtesy: Craig Heinselman

Some other benefits of phased arrays…

No




Peak Power 1791 KW (3582/453/61)
2010-07-29 14:17 UTC
0W 100W 200W 300W 400W 500W 600W
R-01
R-02
R-03
R-04
R-05
R-06
R-07
R-08
C-16 C-15 C-14 C-13 C-12 C-11 C-10 C-09 C-08 C-07 C-06 C-05 C-04 C-03 C-02 C-01

No moving parts

Photo: Mike Lockwood

Photo: Craig Heinselman

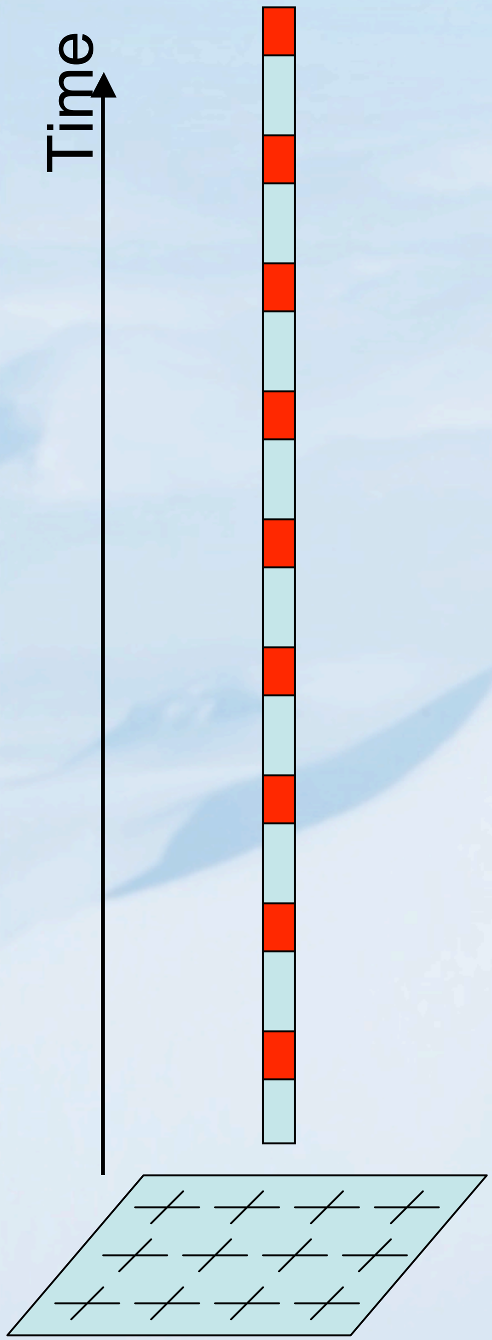
Photo: Craig Heinselman

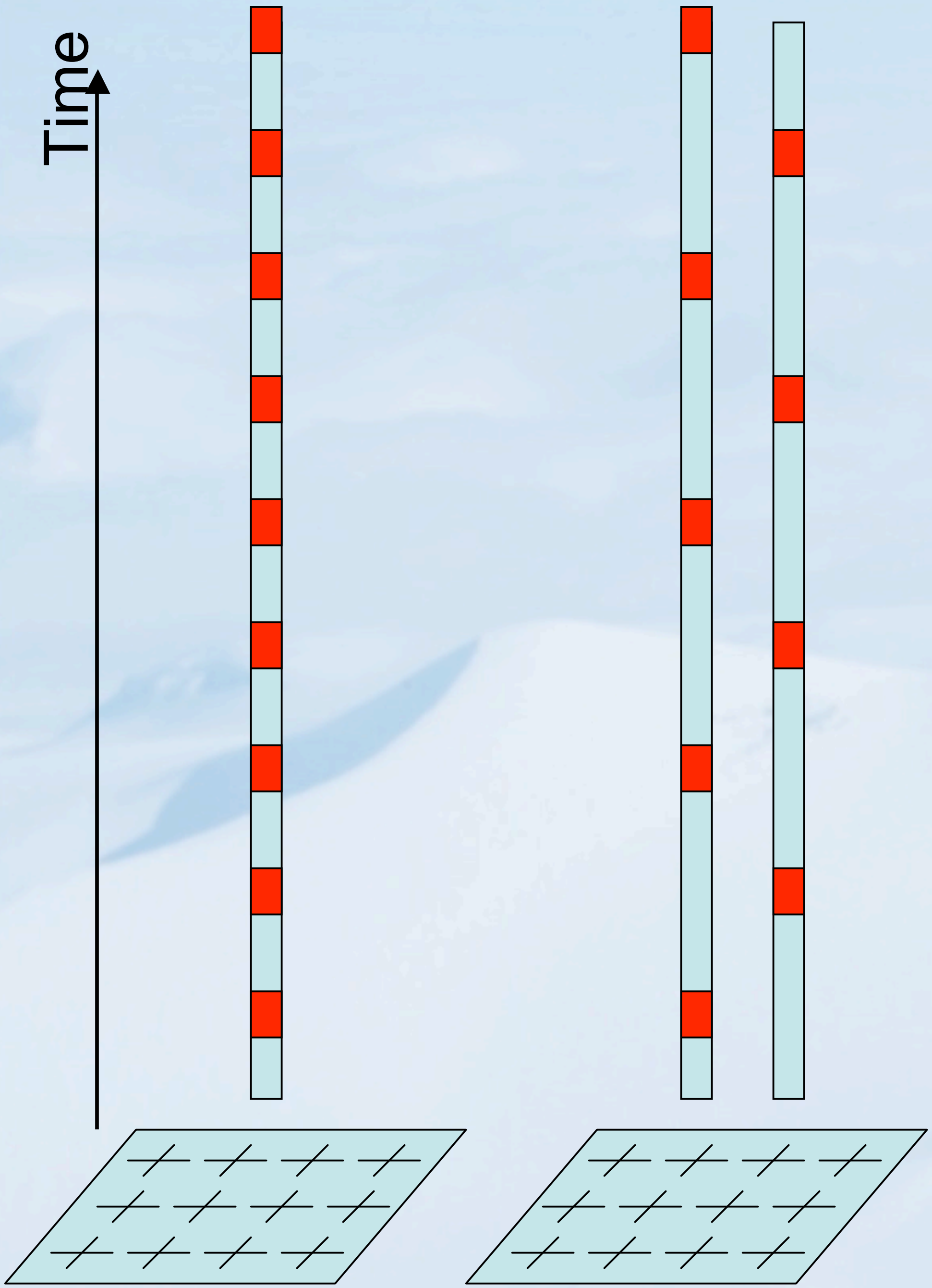
perties...


Photo: Craig Heinselman

can be remotely
perated

Word of warning...


Time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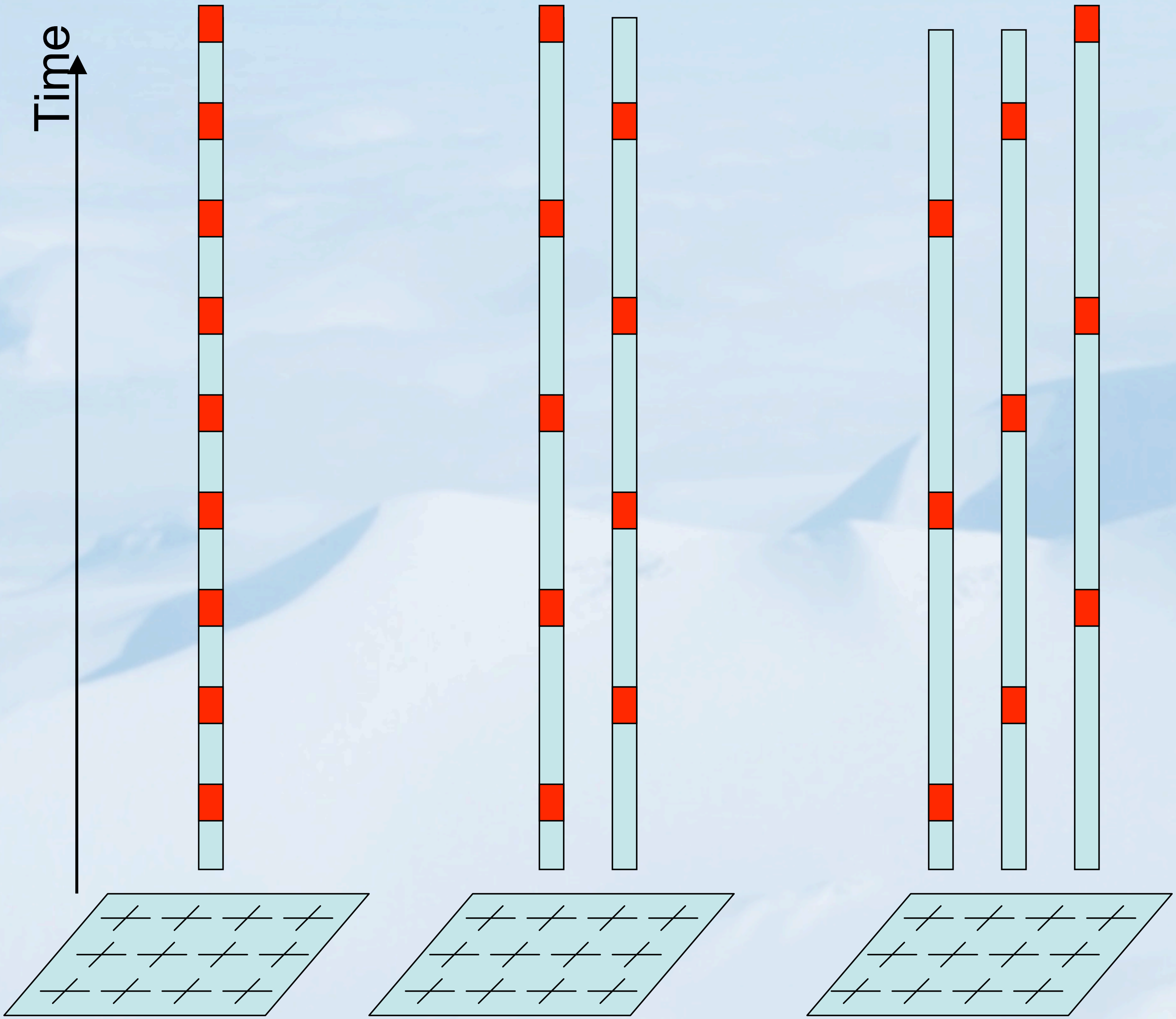
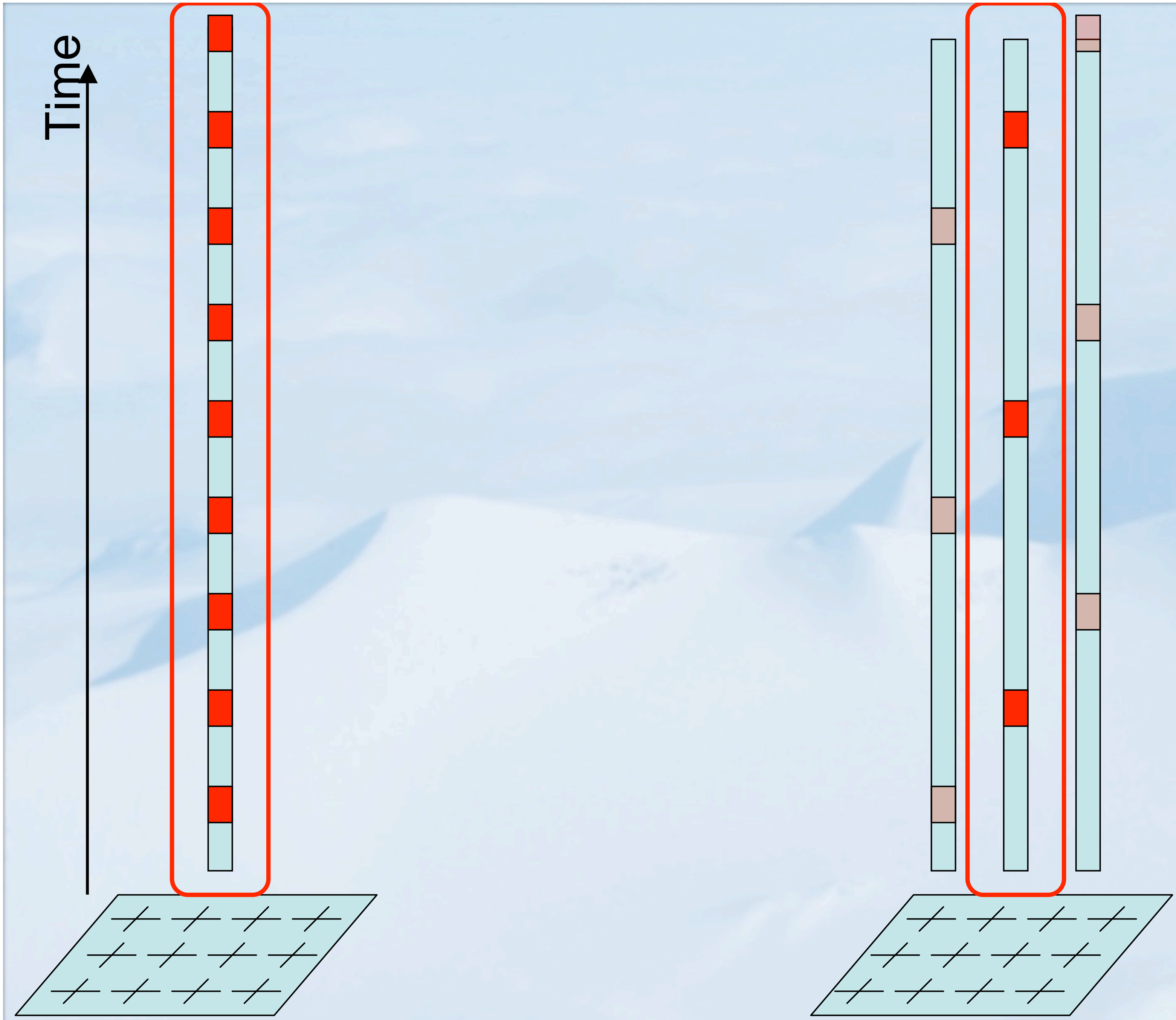
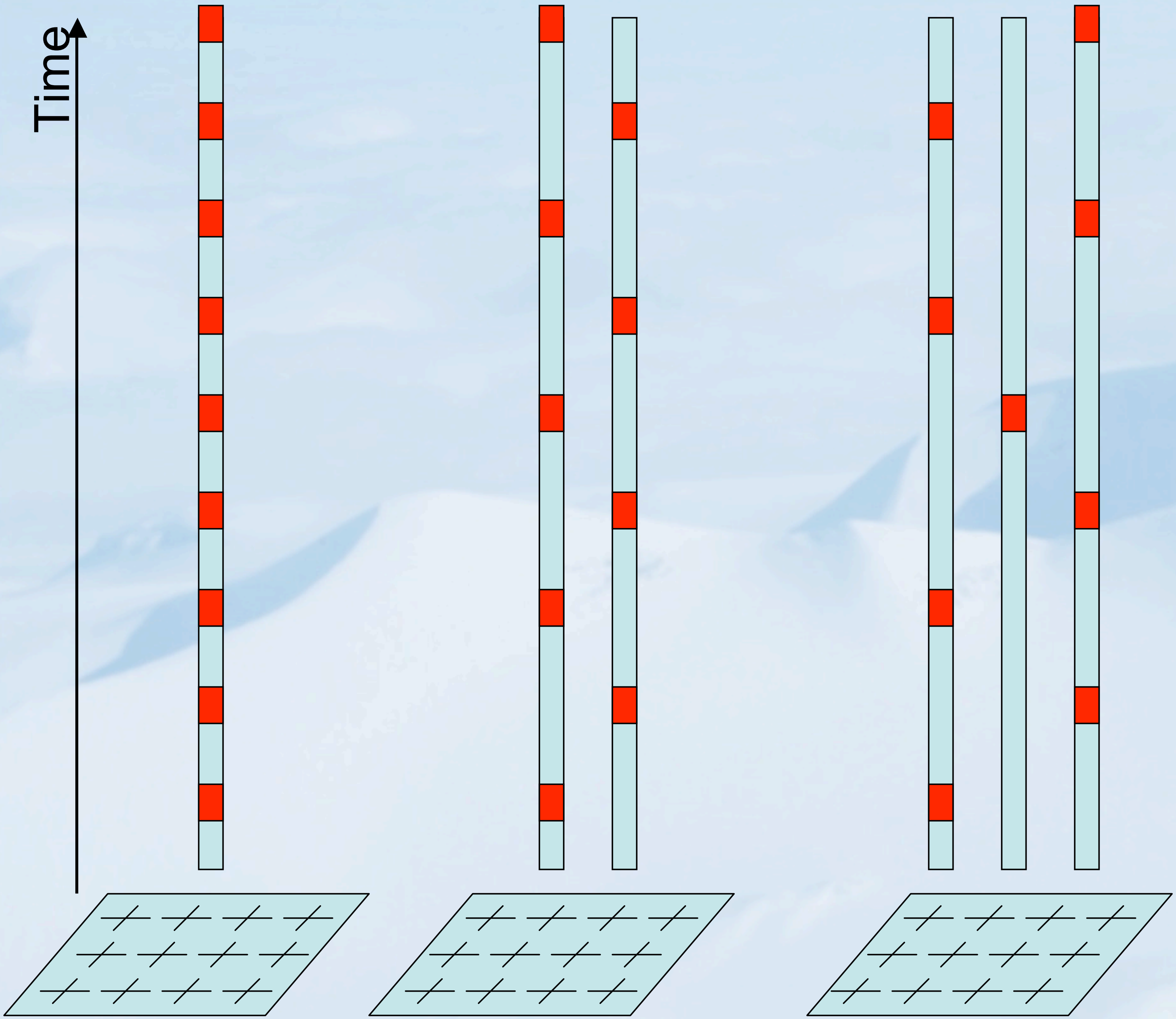
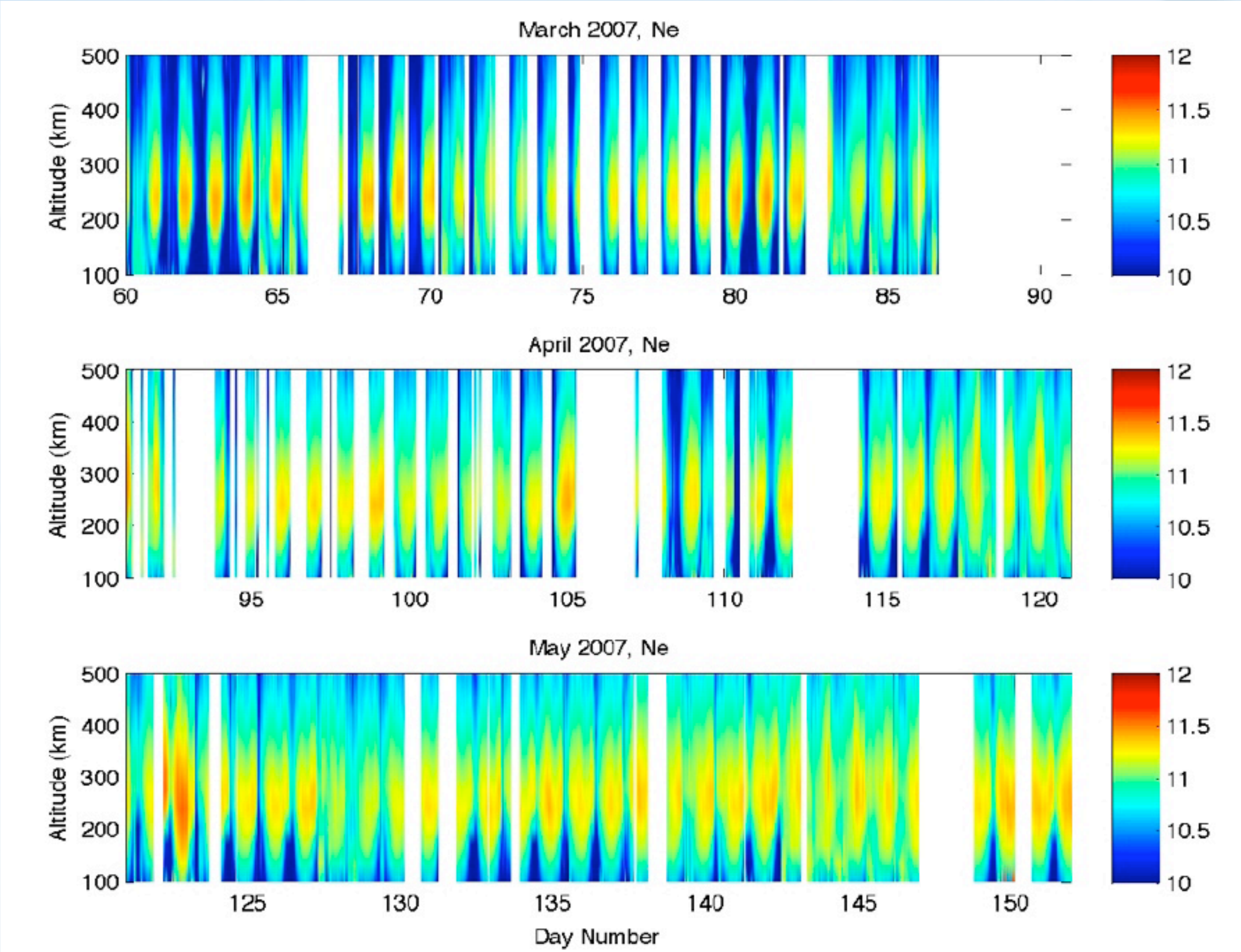

Time


Time


Time


Time


March 2007, Ne
Altitude (km)
500
400
300
200
100
60
65
70
75
80
85
90
12
11.5
11
10.5
10
April 2007, Ne
Altitude (km)
95
100
105
110
115
120
May 2007, Ne
Altitude (km)
125
130
135
140
145
150
Day Number