Lag profile inversion and its applications

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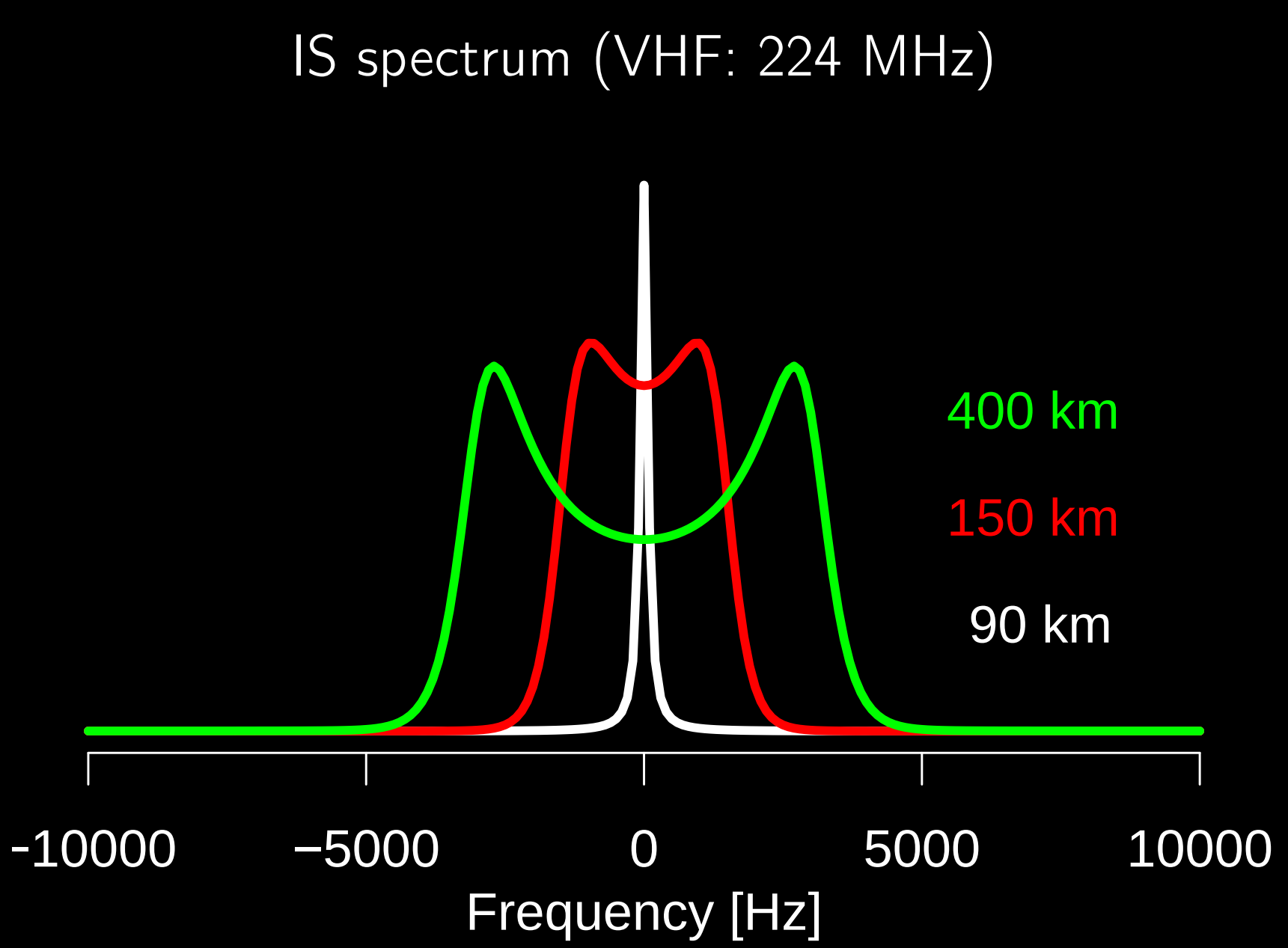
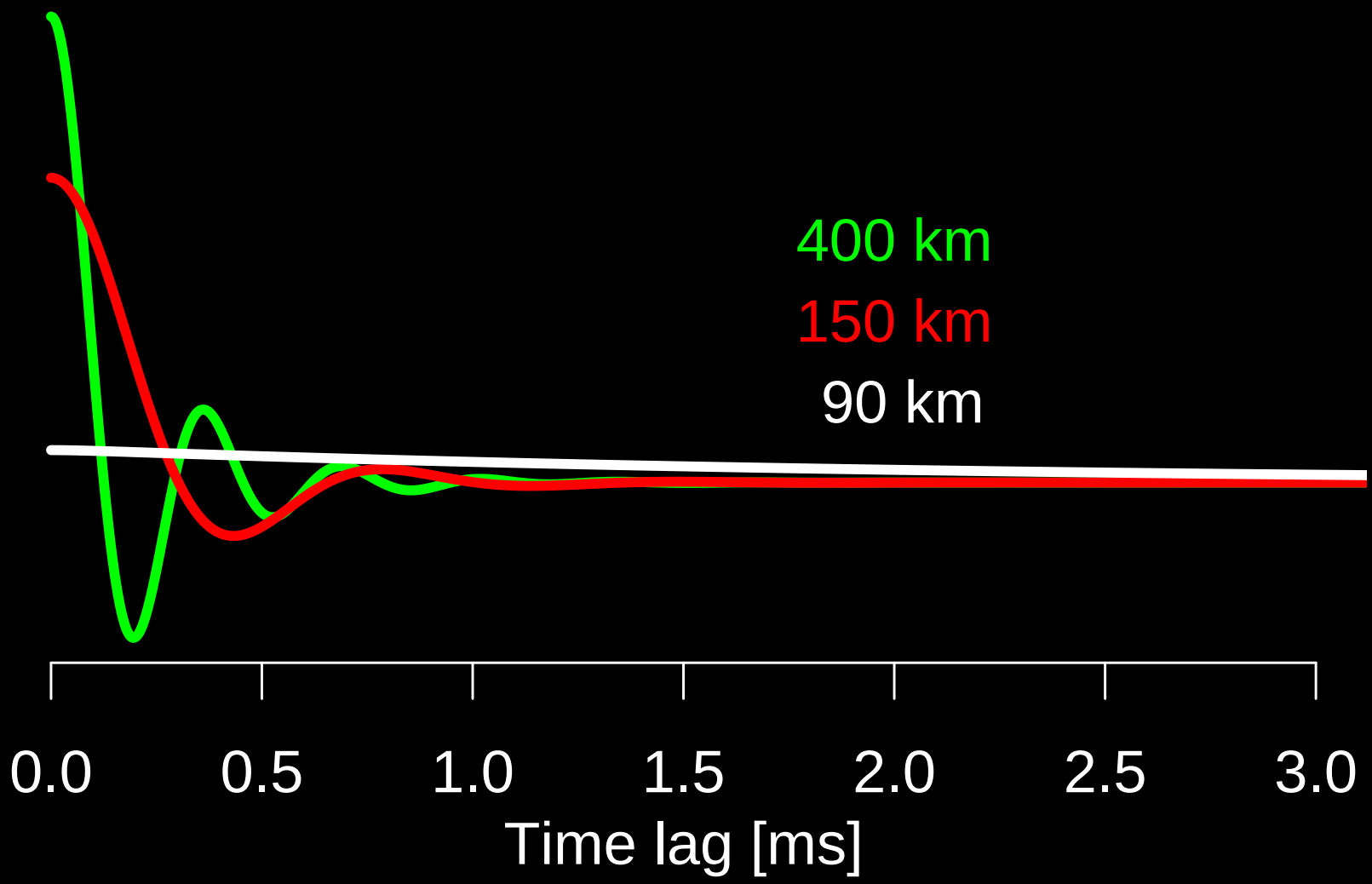
Outline

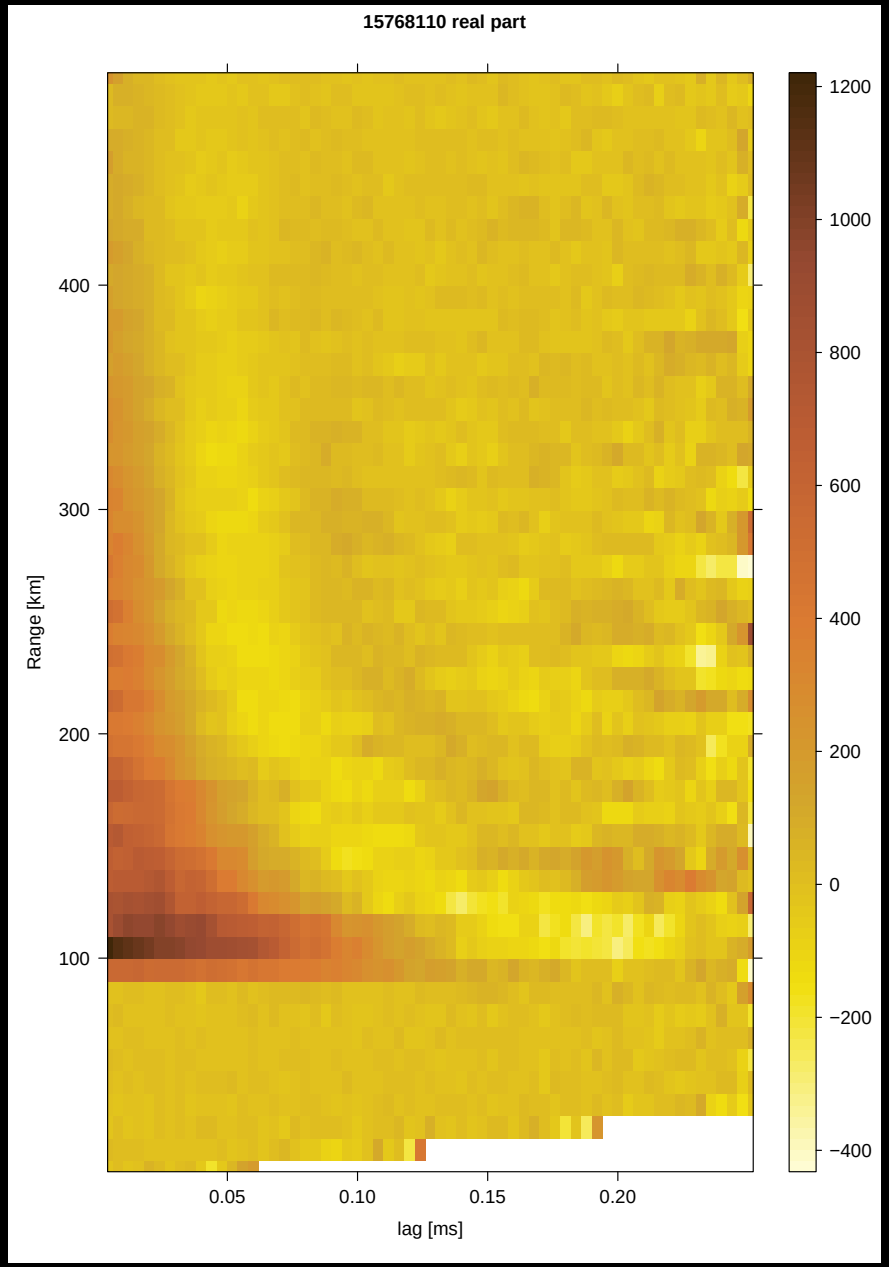
Lag profile inversion and decoding
- What is a lag profile?
- Range ambiguity as an inverse problem
- Decoding vs. Lag Profile Inversion

Applications of lag profile inversion
- Plasma parameters from a special space debris experiment
- Meteor head-echo removal
- Aperiodic modulations


IS spectrum (VHF: 224 MHz)
400 km
150 km
90 km
-10000
−5000
0
5000
10000
Frequency [Hz]

IS autocorrelation function (VHF: 224 MHz)




15768110 real part
Range [km]
100
200
300
400
0.05
0.10
0.15
0.20
lag [ms]
1200
1000
800
600
400
200
0
−200
−400

Range ambiguity as an inverse problem

- True ACF in range gate r_l at time lag τ_k

$$R(r_l, \tau_k) = R_{lk}$$

- Range ambiguity function at time t_n, time lag τ_k, and range gate r_l

$$W(t_n, r_l, \tau_k) = W_{nlk}$$

- Lagged product at time t_n and time lag τ_k

$$z(t_n)z^*(t_n - \tau_k) = m_{nk}$$

-

$$m_{nk} = \sum_l W_{nlk} R_{lk} + \varepsilon_{nk}$$

- During a single integration period we get N lagged products

$$\mathbf{m}_k = (m_{1k}, m_{2k}, \ldots, m_{Nk})^T$$

- with errors

$$\boldsymbol{\varepsilon}_k = (\varepsilon_{1k}, \varepsilon_{2k}, \ldots, \varepsilon_{Nk})^T$$

- The corresponding range ambiguity functions can be collected into a matrix

$$\mathbf{W}_k = \begin{pmatrix} W_{11k} & \ldots & W_{1Lk} \\ W_{21k} & \ldots & W_{2Lk} \\ \vdots & \ddots & \vdots \\ W_{N1k} & \ldots & W_NLk \end{pmatrix}$$

- Similarly, the true lag profile is a vector

$$\mathbf{R}_k = (R_{1k}, R_{2k}, \ldots, R_{Lk})^T$$

- Now we can write down a set of equations

$$\mathbf{m}_k = \mathbf{W}_k \mathbf{R}_k + \varepsilon_k$$

- When $\mathbf{m}_k$, $\mathbf{R}_k$ and ε_k are treated as normally distributed random variables, with error covariance $< \varepsilon_k \varepsilon_k^T >= \mathbf{\Sigma}_k$

$$\mathbf{R}_k = \left(\mathbf{W}_k^H \mathbf{\Sigma}_k^{-1} \mathbf{W}_k \right)^{-1} \mathbf{W}_k^H \mathbf{\Sigma}_k^{-1} \mathbf{m}_k.$$

- Simplifying assumptions are needed in order to avoid inverting the full covariance matrix

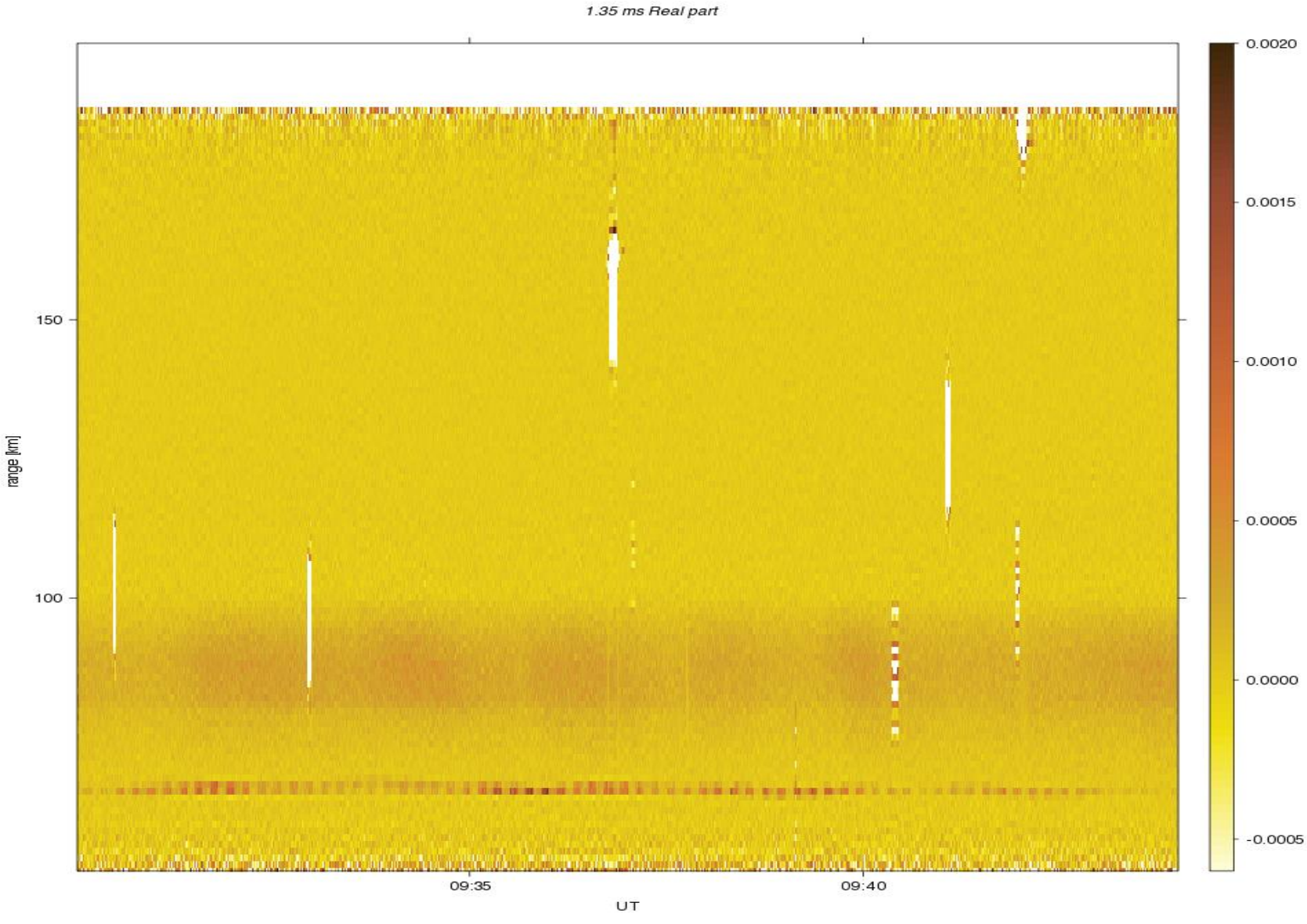
Decoding vs. Lag Profile Inversion

- $\mathbf{\Sigma}_k = \sigma I$ and $\mathbf{W}_k^H \mathbf{W}_k$ is diagonal $\Rightarrow$ matched filter decoding
  - Simple and fast
  - $\mathbf{W}_k^H \mathbf{W}_k$ is diagonal for alternating codes
  - Long sequences of pulses
  - Limited possibilities for combined pulse coding
- $\mathbf{\Sigma}_k$ is diagonal $\Rightarrow$ the present version of lag profile inversion
  - Compute-intensive
  - Enables use of arbitrary modulations

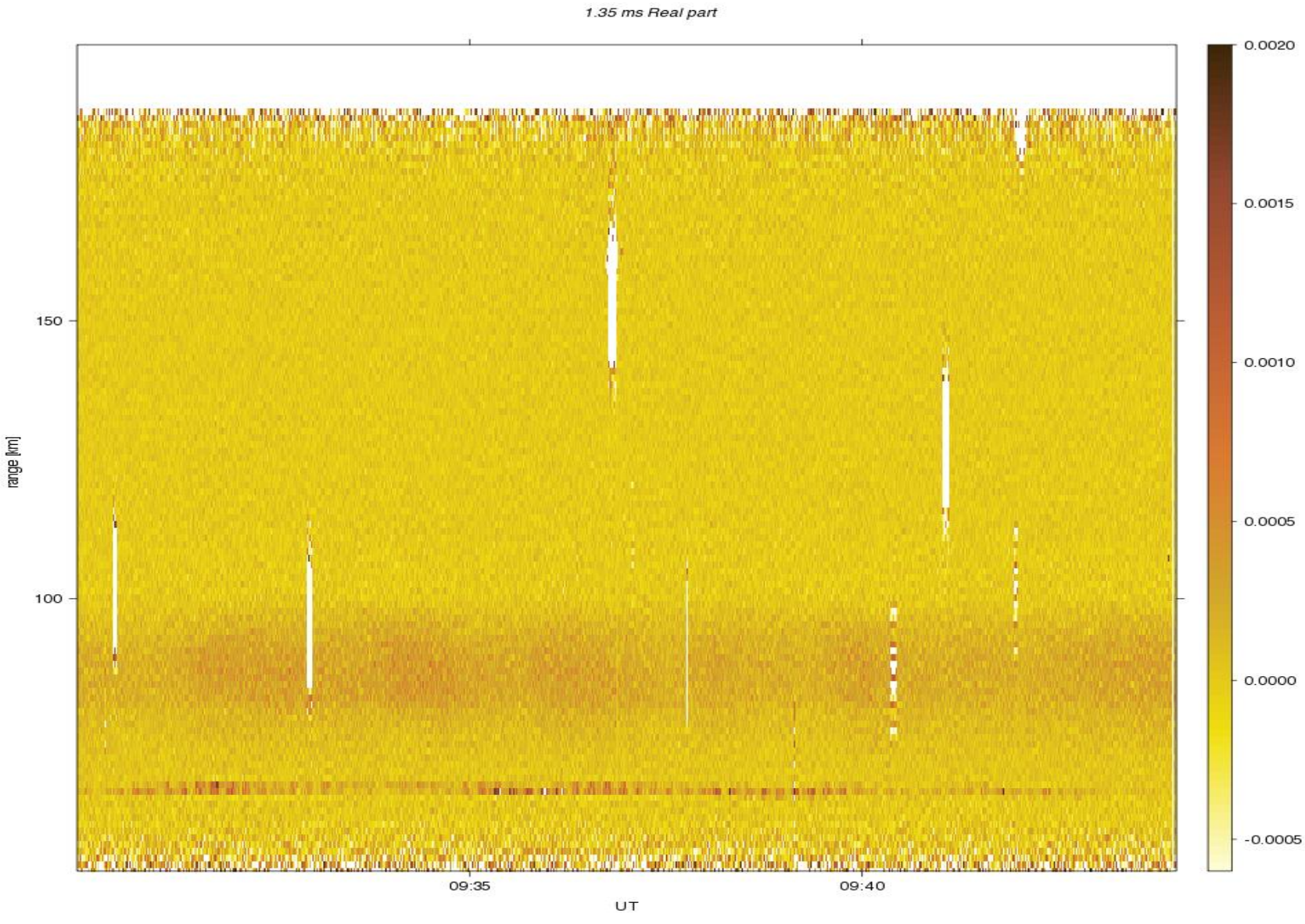
Applications of lag profile inversion

- Re-analysis with (almost) arbitrary resolutions
- Analysis of near-perfect modulations, which cannot be decoded
- Possibility to exclude arbitrary data points from analysis
- Aperiodic modulations (multi-purpose experiments)
- ...

PMWE with 1.0 s time resolution

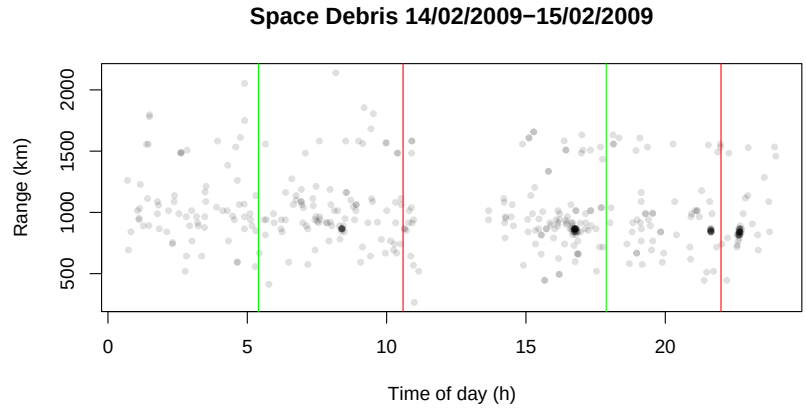


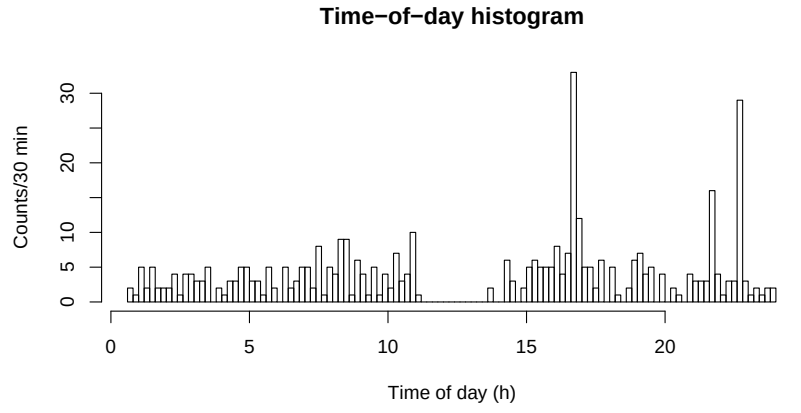
PMWE with 0.2 s time resolution

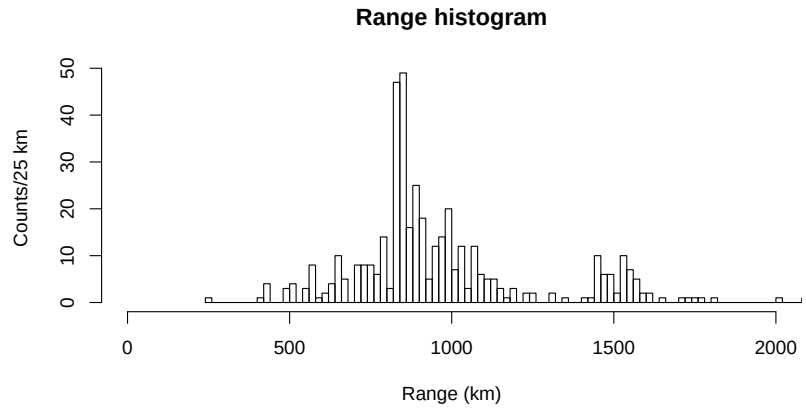


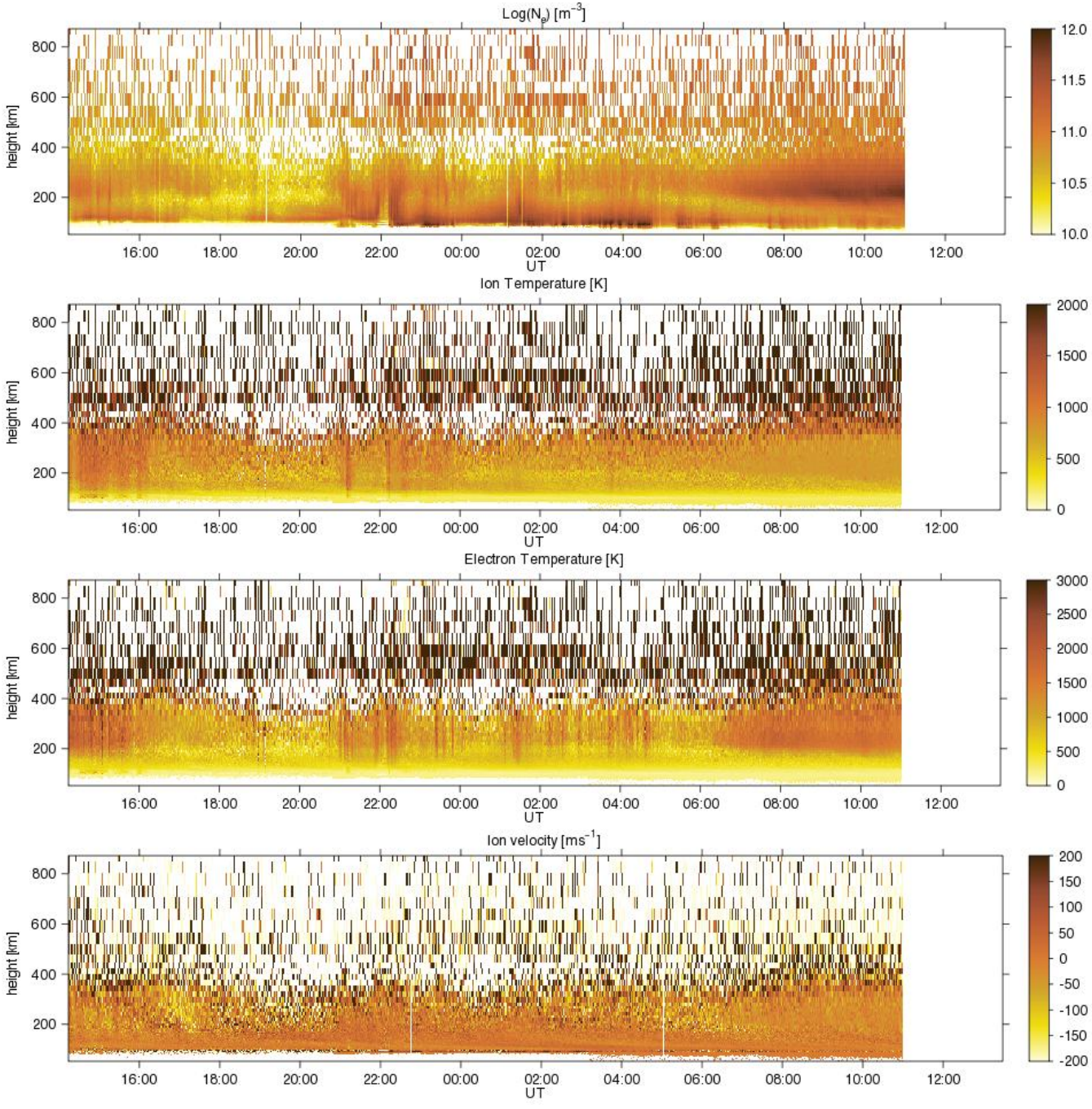
Plasma parameters from a special space debris experiment

- Ramped IPP and phase-coded pulses optimised for space-debris measurements (Iridium-Cosmos collision)
- Lag profile inversion was used for plasma parameter estimation from the same data

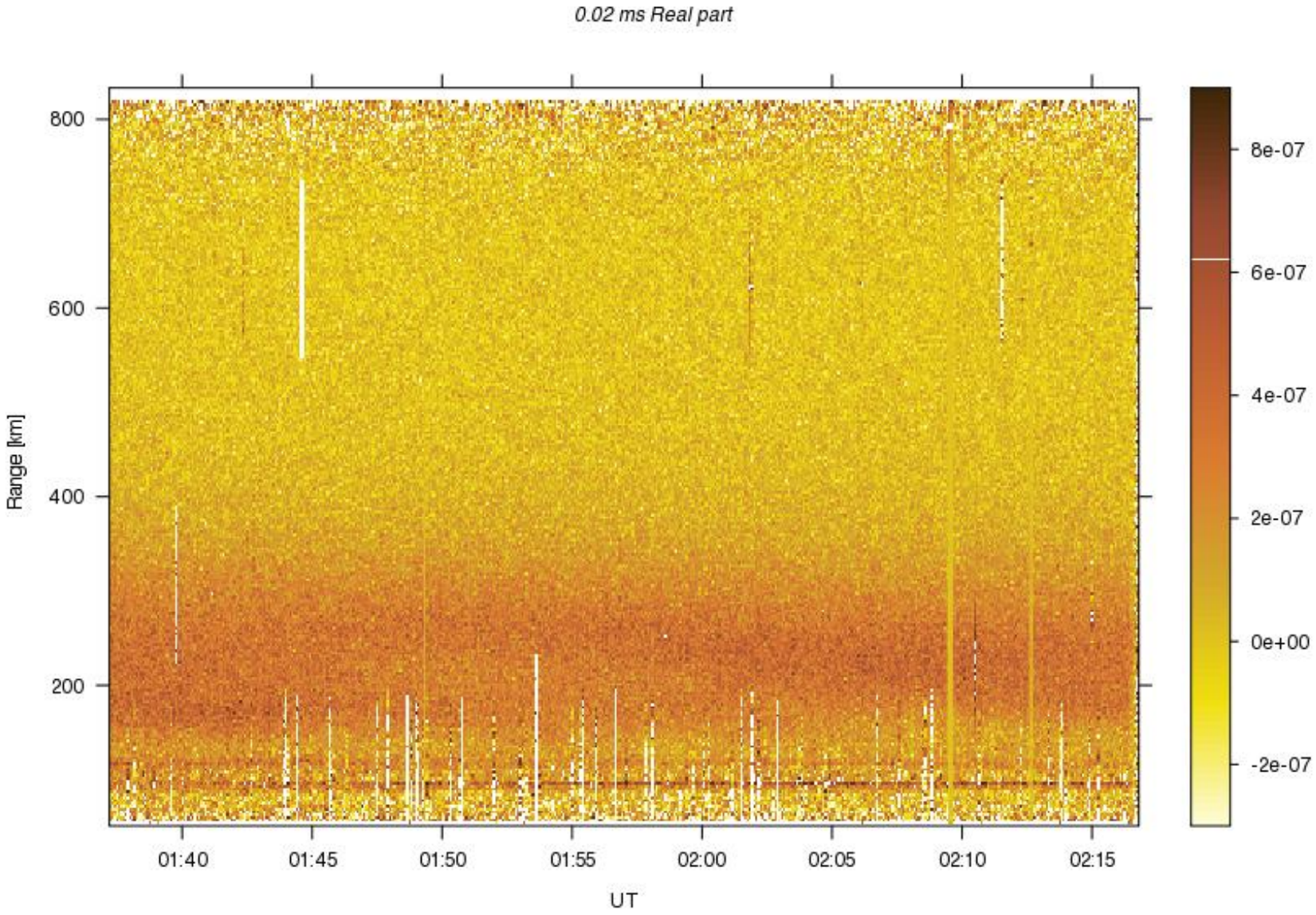

Space Debris 14/02/2009−15/02/2009
Range (km)
500
1000
1500
2000
0
5
10
15
20
Time of day (h)


Time−of−day histogram
Counts/30 min
0
5
10
20
30
0
5
10
15
20
Time of day (h)

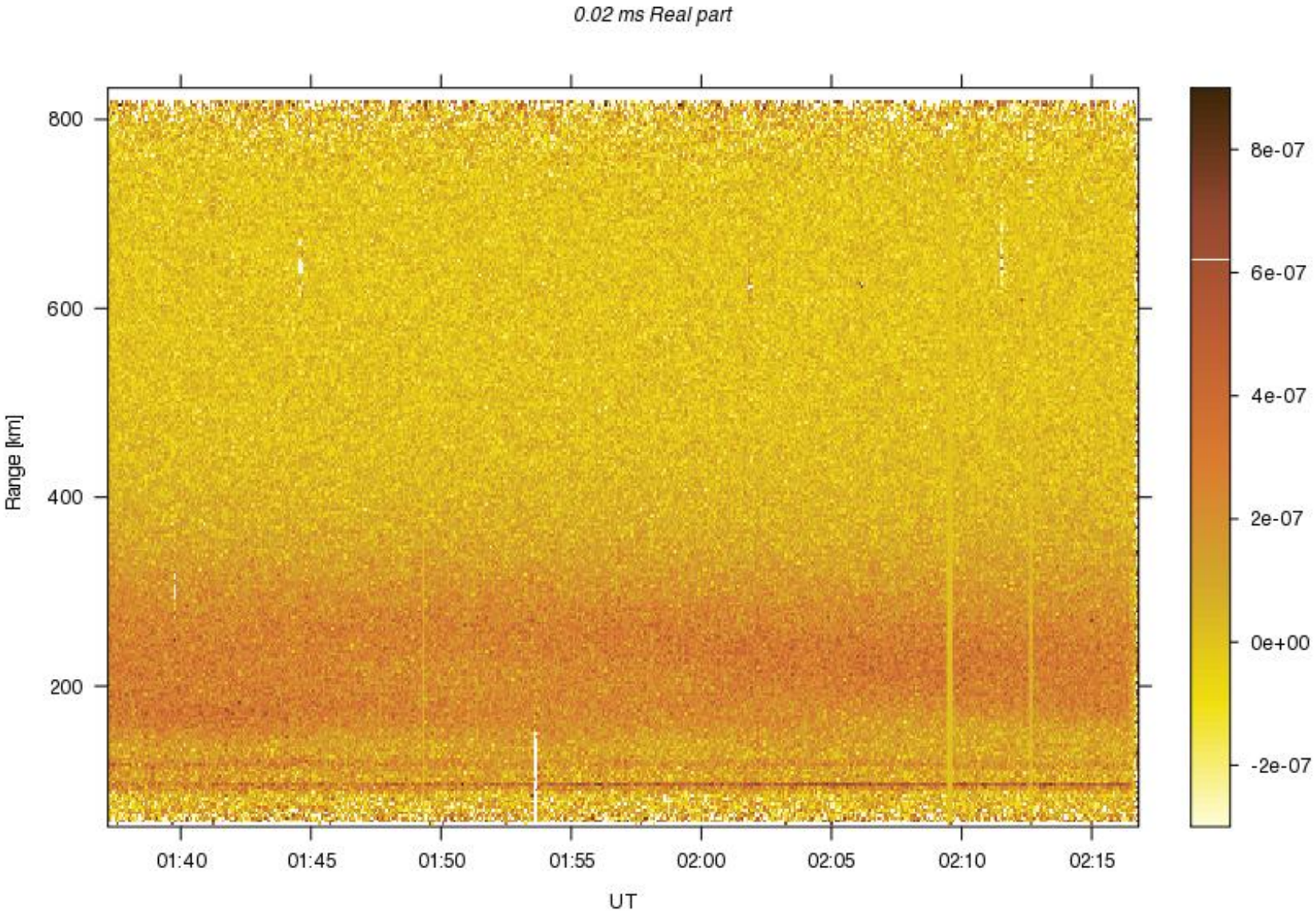

Range histogram
Counts/25 km
0
10
20
30
40
50
0
500
1000
1500
2000
Range (km)


Log(N$_e$) [m^{-3}]
height [km]
800
600
400
200
16:00
18:00
20:00
22:00
00:00
02:00
04:00
06:00
08:00
10:00
12:00
UT
12.0
11.5
11.0
10.5
10.0
Ion Temperature [K]
2000
1500
1000
500
0
Electron Temperature [K]
3000
2500
2000
1500
1000
500
0
Ion velocity [ms^{-1}]
200
150
100
50
0
-50
-100
-150
-200

Decoded lag profiles



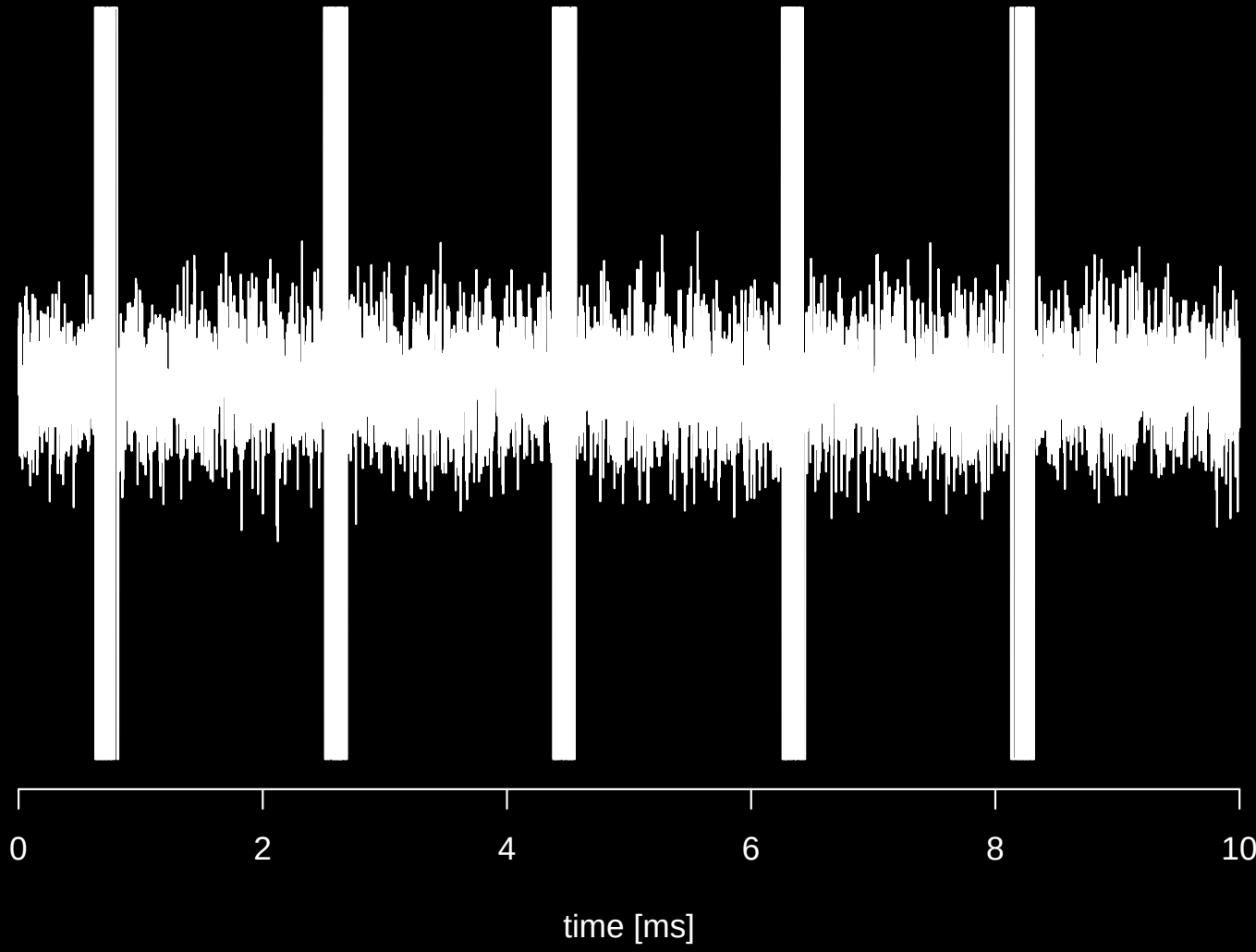
Lag profile inversion result, with meteor removal



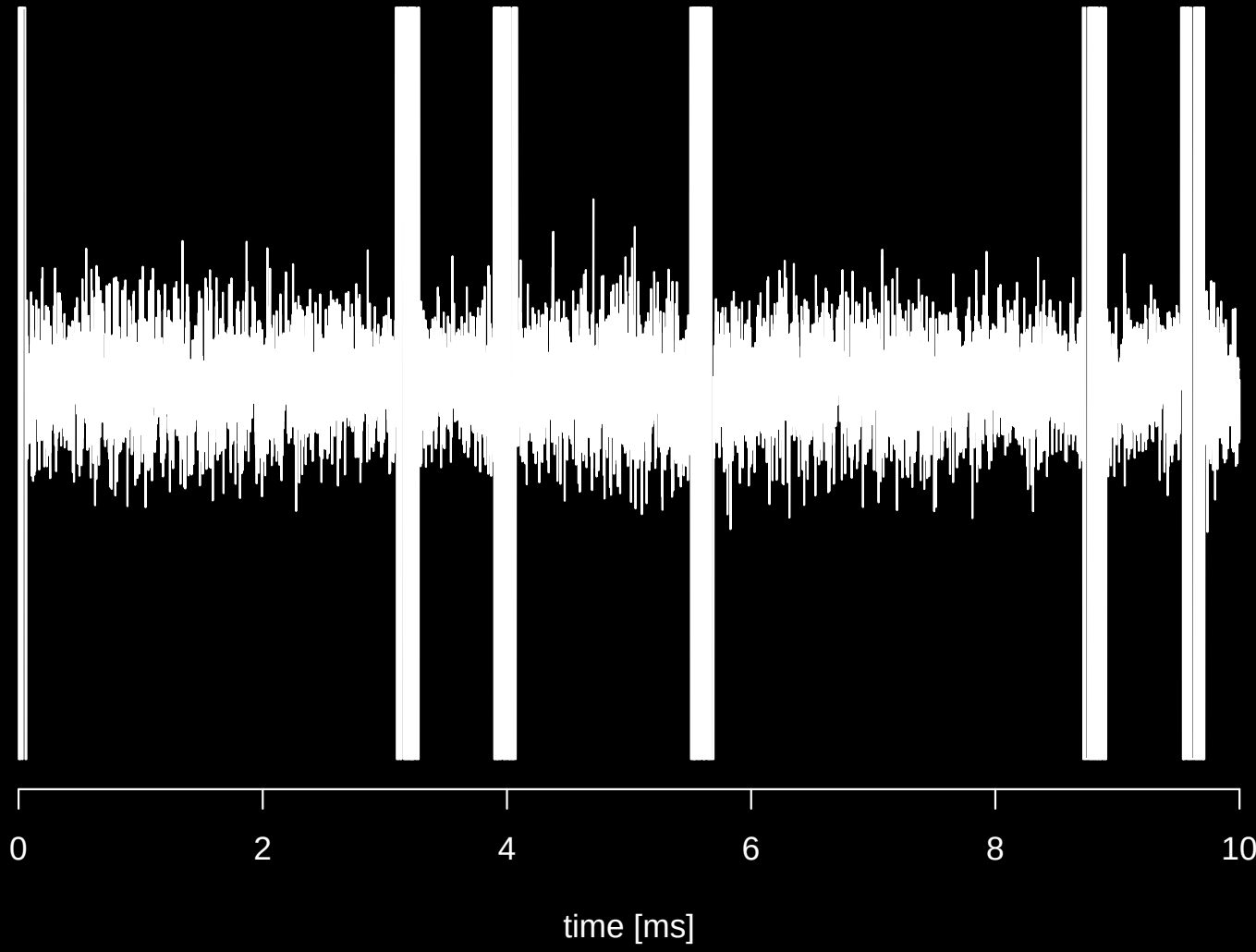
Aperiodic modulations

- Alternating codes require uniform inter-pulse periods
- Aperiodic transmissions would provide improved coverage in range and time-lag
- The data analysis is possible with lag profile inversion

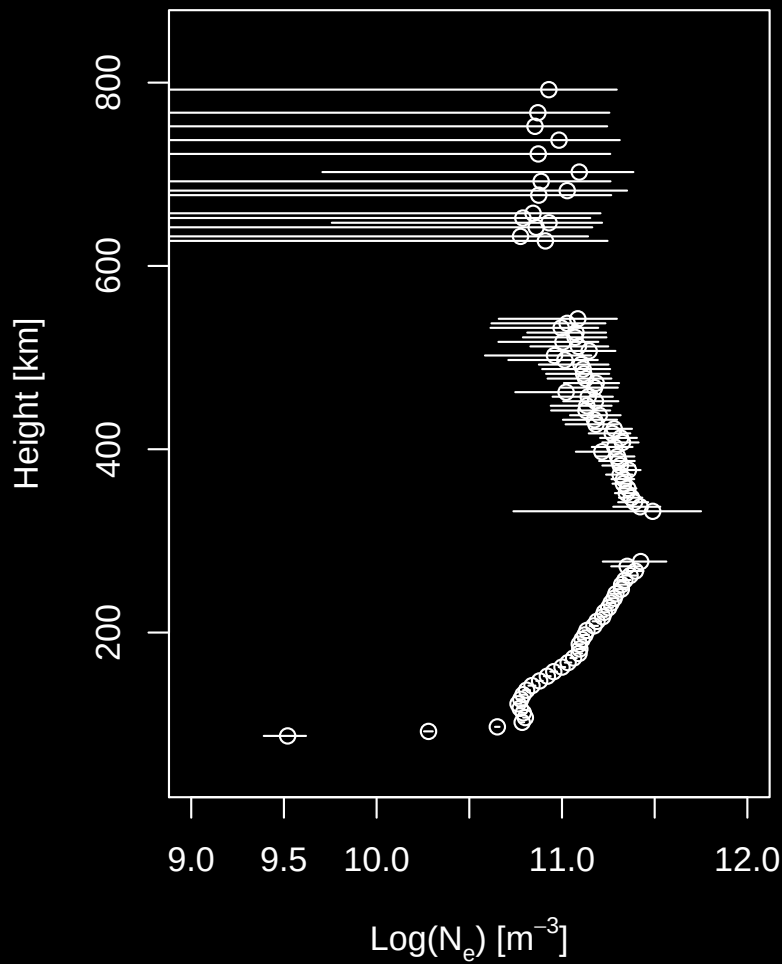
Simulated data of EISCAT manda

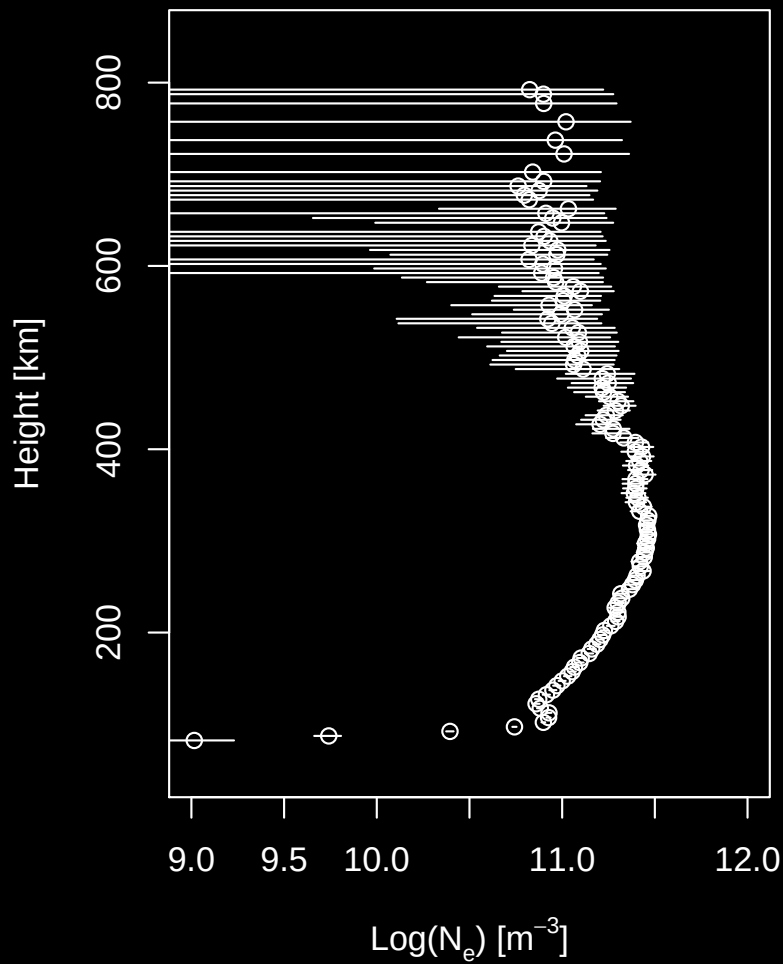


Simulated data of an aperiodic modification

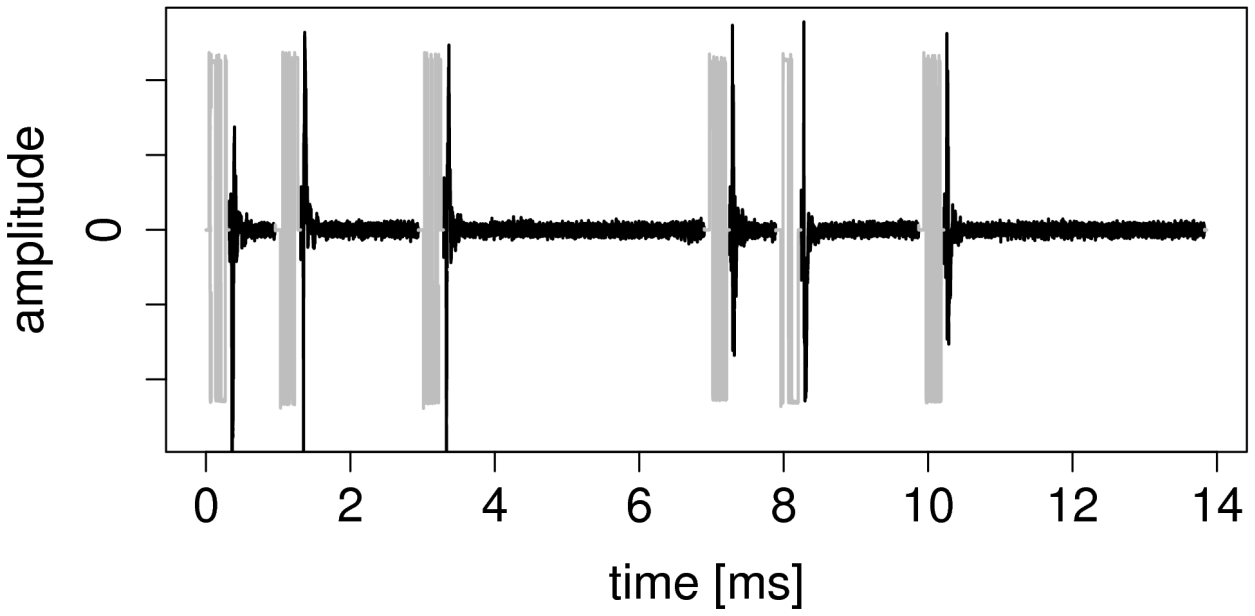


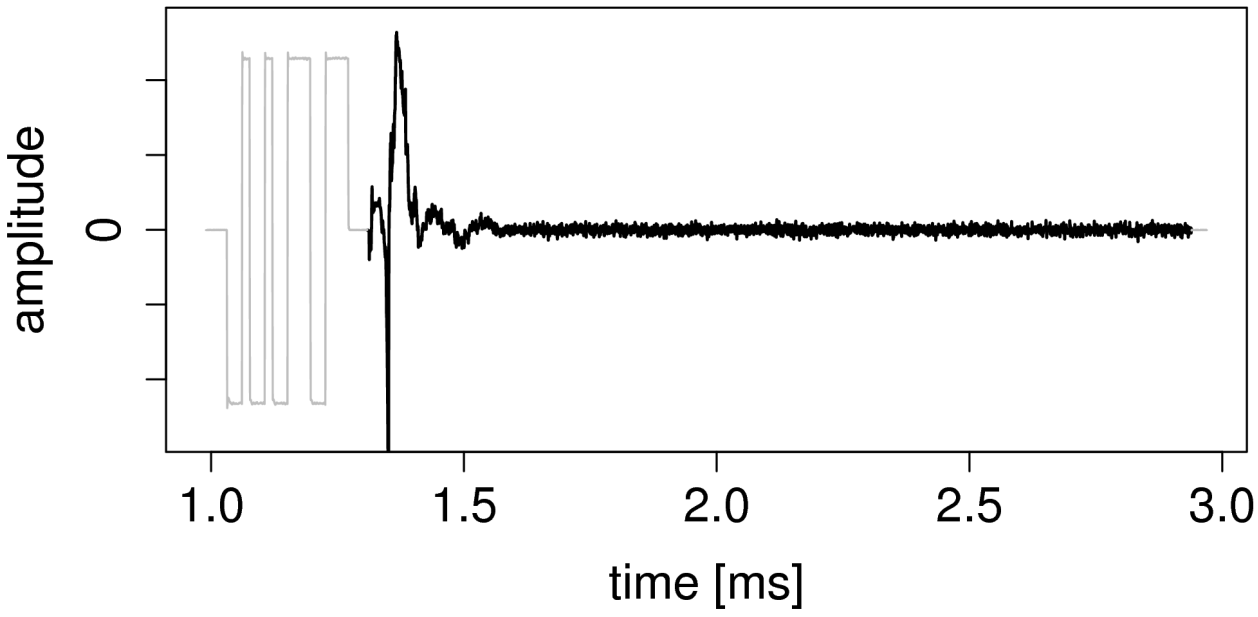
Electron densities with the two experiments



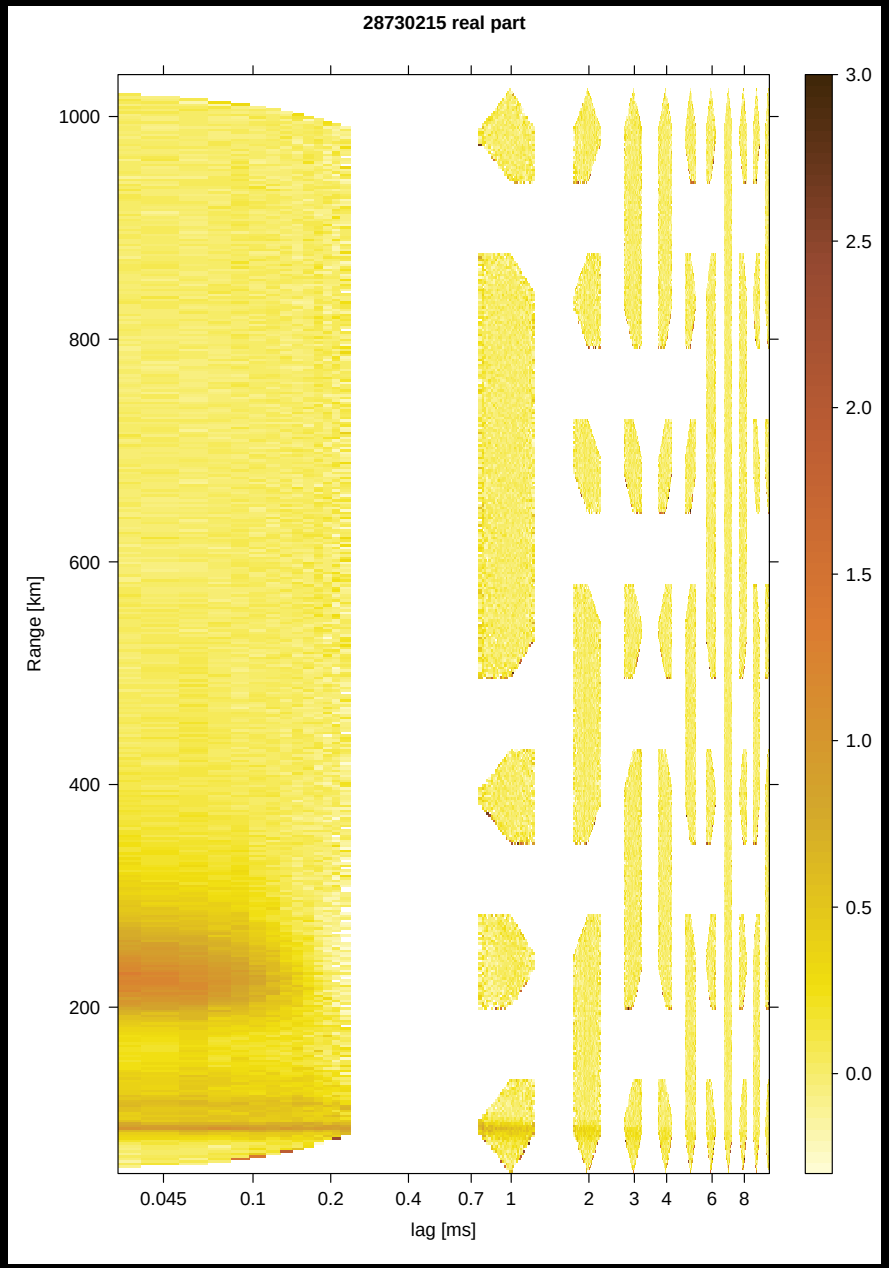


Raw data from a real multi-purpose experiment

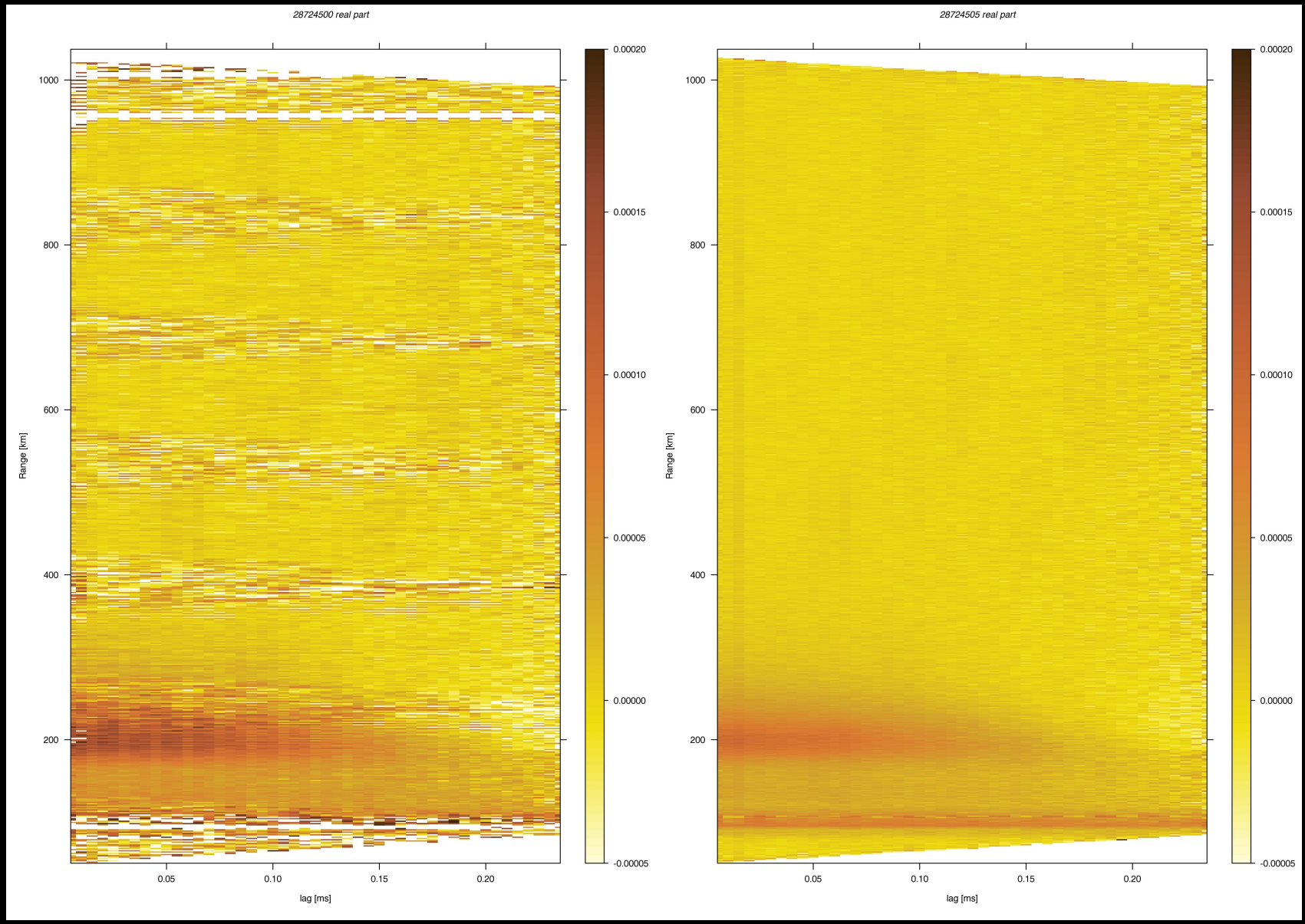


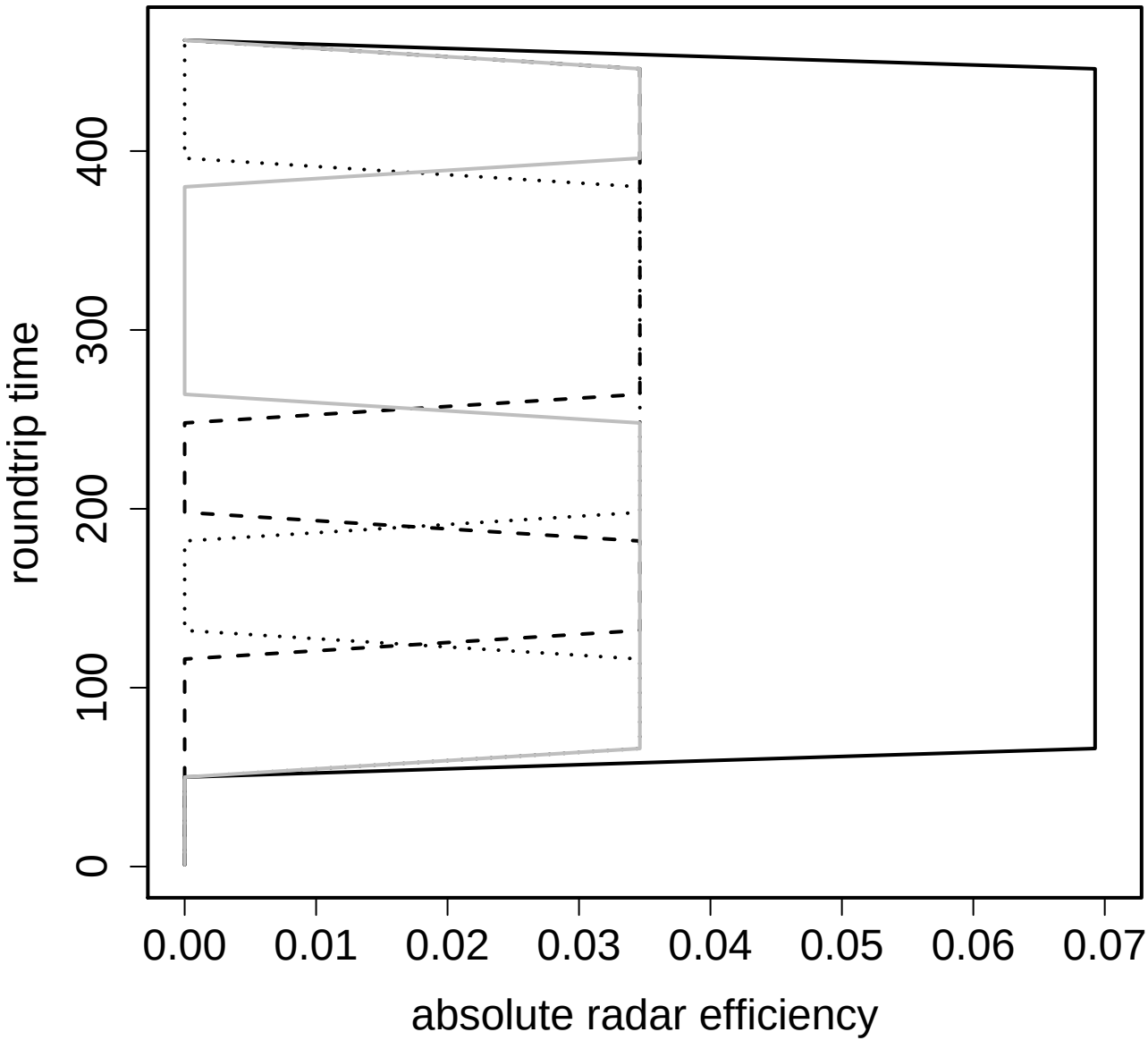


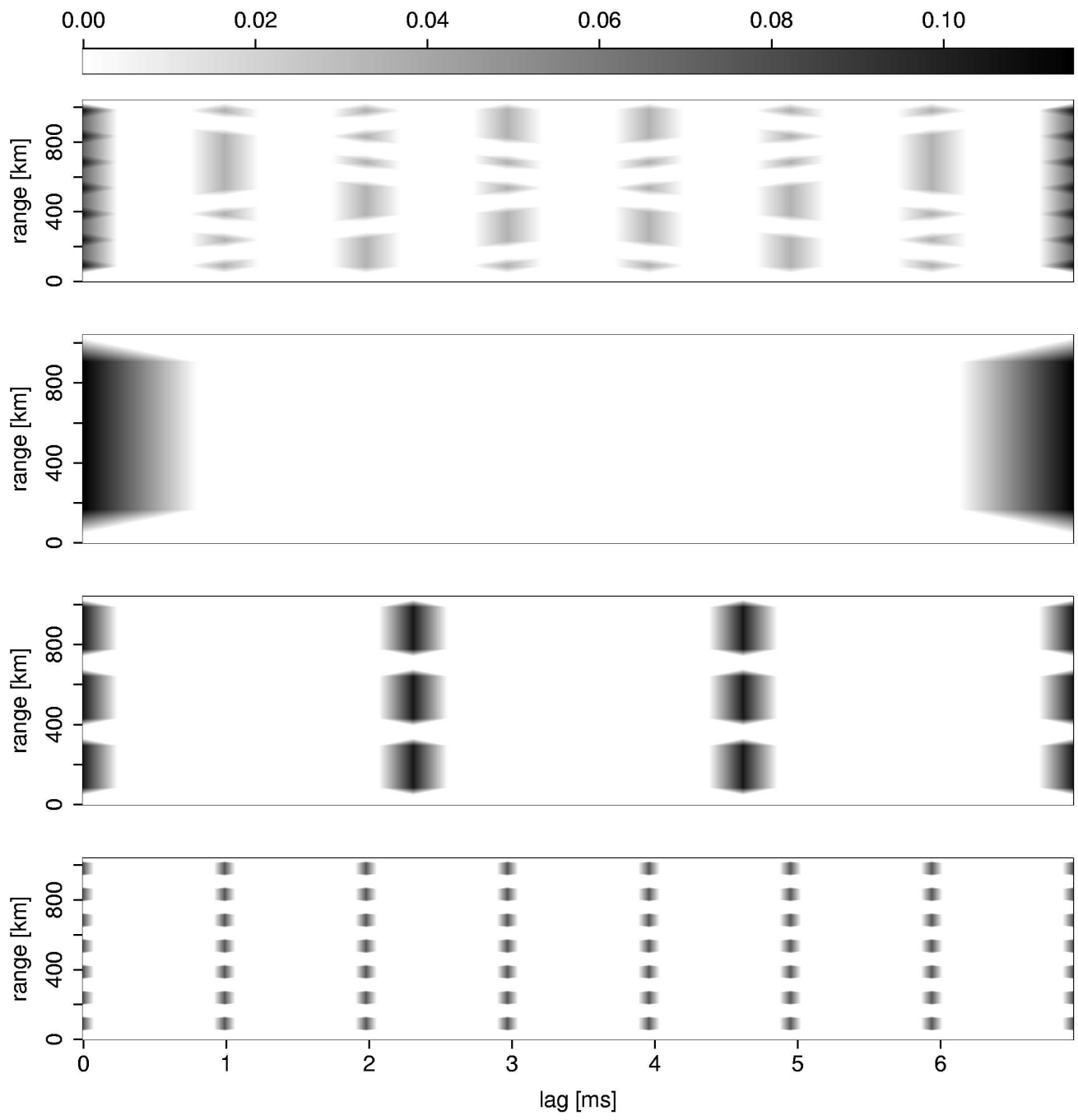
ACF with the multi-purpose experiment

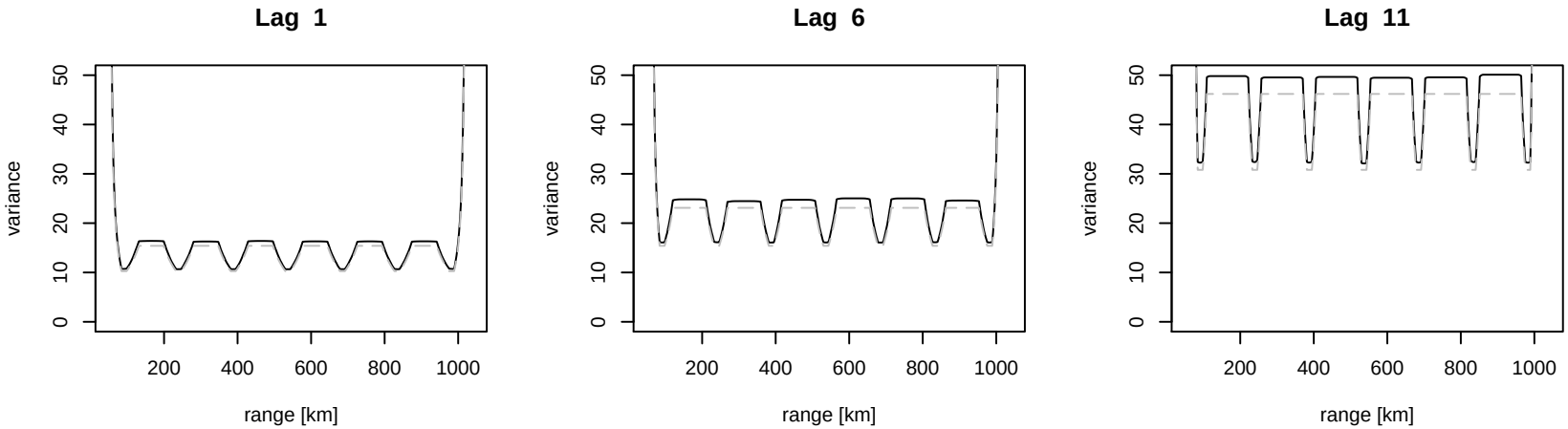


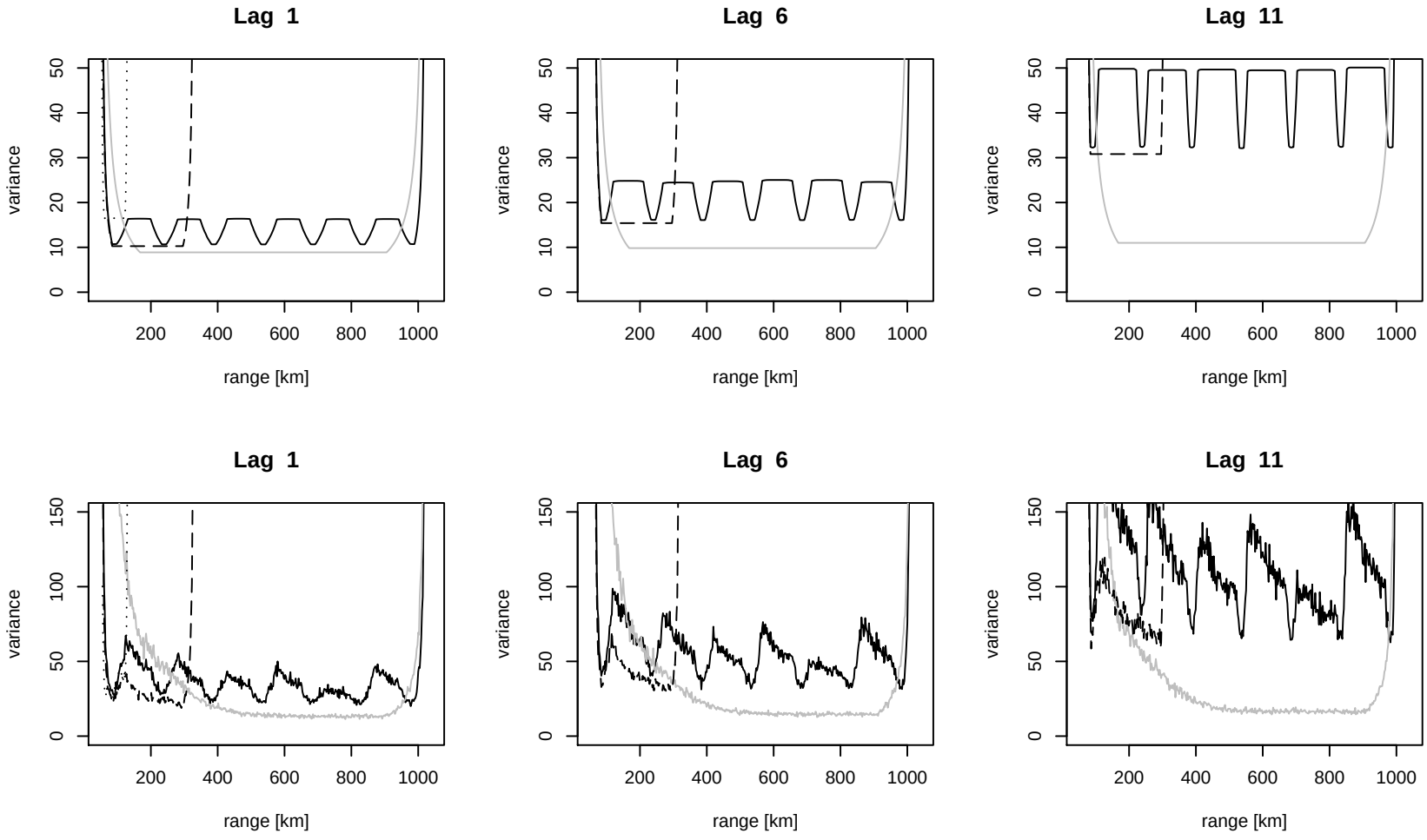
Meteor head echoes in aperiodic modulations




roundtrip time
0
100
200
300
400
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
absolute radar efficiency


0.00
0.02
0.04
0.06
0.08
0.10
range [km]
0
400
800
0
1
2
3
4
5
6
lag [ms]


Lag 1
Lag 6
Lag 11
variance
range [km]
0
10
20
30
40
50
200
400
600
800
1000


Lag 1
Lag 6
Lag 11
Lag 1
Lag 6
Lag 11
variance
range [km]
0
10
20
30
40
50
50
100
150
200
400
600
800
1000

Links to original publications

https://wiki.oulu.fi/x/MIWj