Appleton-Hartree Equation

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2\sin^2\theta}{1-X-iZ} \pm \frac{1}{1-X-iZ}\left(\frac{1}{4}Y^4\sin^4\theta + Y^2\cos^2\theta\,(1-X-iZ)^2\right)^{1/2}}$$

or, alternatively[4]:

$$n^2 = 1 - \frac{X\,(1-X)}{1 - X - \frac{1}{2}Y^2\sin^2\theta \pm \left(\left(\frac{1}{2}Y^2\sin^2\theta\right)^2 + (1-X)^2\,Y^2\cos^2\theta\right)^{1/2}}$$

n = complex refractive index

$i = \sqrt{-1}$

$X = \frac{\omega_0^2}{\omega^2}$

$Y = \frac{\omega_H}{\omega}$

$Z = \frac{\nu}{\omega}$

ε_0 = permittivity of free space

μ_0 = permeability of free space

B_0 = ambient magnetic field strength

e = electron charge

m = electron mass

θ = angle between the ambient magnetic field vector and the wave vector

ν = electron collision frequency

$\omega = 2\pi f$ (radial frequency)

f = wave frequency (cycles per second, or Hertz)

$\omega_0 = 2\pi f_0 = \sqrt{\frac{Ne^2}{\epsilon_0 m}}$ = electron plasma frequency

$\omega_H = 2\pi f_H = \frac{B_0|e|}{m}$ = electron gyro frequency

Ionospheric Range Correction

$$n \approx \left(1 - \frac{\omega_N^2}{\omega^2}\right)^{\frac{1}{2}} \approx 1 - \frac{\omega_N^2}{2\omega^2} \approx 1 - \frac{AN_e}{f^2}$$

$$\Delta R_{ion}(meters) = \frac{40.3}{f^2} \int_0^R N_e \, dr$$

Range Delay

TEC	S-Band	L-Band	UHF	VHF	Elev	Mapping Function
50	2.4 m	12 m	104 m	787 m	90 °	x 1
110	5.1 m	26 m	223 m	1.7 km	20 °	x 2.12