Simplified experiment design

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Outline

1. Optimization of radar pulse length in IS experiment for any combinations of **SNR** and spatial resolution
   - Formulating an incoherent scatter measurements as an inverse problem
   - Results and conclusions
2. State of the art in IS experiment design
   - Perfect codes
   - Code comparison
   - Multipurpose codes
3. Discussion on the performance specifications for the new radar
   - Original performance specifications
   - Specifications revisited
4. Summary
   - So - what was simple here ?
   - Future

Itó measure = incoherent scatter target

- Let us assume the target is illuminated with an uninterrupted (CW) sine wave of unit transmission power. We assume a time-coherent but spatially incoherent target. This means that the signal received from a volume A can be described by a set function $\mu(A)$.
- Complex function giving the phase and amplitude of the signal.
- Incoherence: $\langle \mu(A)\overline{\mu(B)} \rangle = 0$, if $A \bigcap B = \emptyset$.
- Additivity: $\mu(A \bigcup B) = \mu(A) + \mu(B)$, if $A \bigcap B = \emptyset$.
- Also, it is understood as a random variable.
- In mathematics, this is known as Itó measure. In physics, it is just the basic idea of spatially incoherent scatter!
- It is characterized by $\langle \mu(dr)\overline{\mu(dr')} \rangle = X(r)\delta(r - r')dr\,dr'$
- $X(r)$ is the structure function of the measure. Physically: target density.

Simple use of these relations

What is $\langle \mu(A)\overline{\mu(B)} \rangle$. We can calculate this by using the formal equations above and the fact

$$\mu(A) = \int_{r \in A} \mu(dr) \tag{1}$$

Then

$$\langle \mu(A)\overline{\mu(B)} \rangle = \int_{r \in A} \int_{r' \in B} \langle \mu(dr)\overline{\mu(dr')} \rangle = \int_{r \in A} \int_{r' \in B} X(r)\delta(r - r')dr\,dr' \tag{2}$$

or simply,

$$\langle \mu(A)\overline{\mu(B)} \rangle = \int_{r \in A \bigcap B} X(r)dr \tag{3}$$

This gives a simple physical explanation of the structure function of the measure $X(r)$: It is simply the scattering power density.

Incoherent scatter radar signal model (simplified, time-coherent)

The signal corresponding to a coded transmission is simply given by

$$z^q(t) = \int_r \epsilon^q(t-r)\mu^q(dr) + \sqrt{T}\xi^q(t). \tag{4}$$

We can now use the formal equation

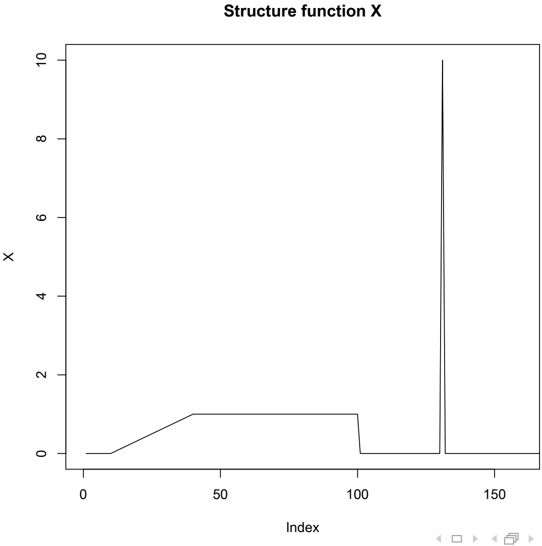
$$\langle \mu^q(dr)\overline{\mu^q(dr')}\rangle = X(r)\delta(r-r')drdr' , \tag{5}$$

where $X(r)$ is the structure function (or target density), we get the covariance equations of the signal

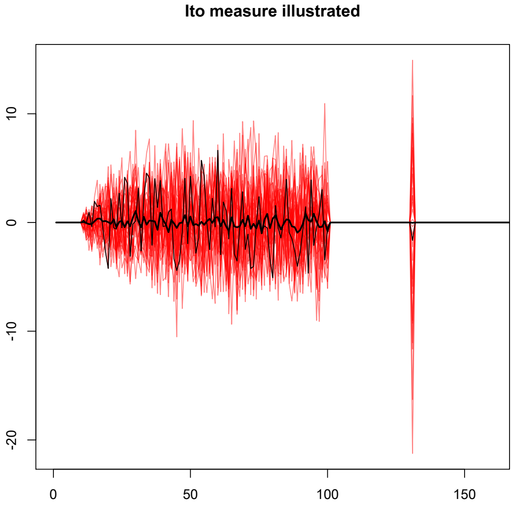
$$\begin{aligned} \langle z^q(t)\overline{z^q(t')}\rangle &= \int\int \langle \mu^q(dr)\overline{\mu^q(dr')}\rangle \epsilon^q(t-r)\overline{\epsilon^q(t'-r')} + T\delta(t-t'), \\ &= \int X(r)\epsilon^q(t-r)\overline{\epsilon^q(t'-r)}dr + T\delta(t-t'). \end{aligned} \tag{6}$$

VanTrees, H., L., Detection, Estimation, and Modulation theory, part III, New York: John Wiley and Sons, 1971

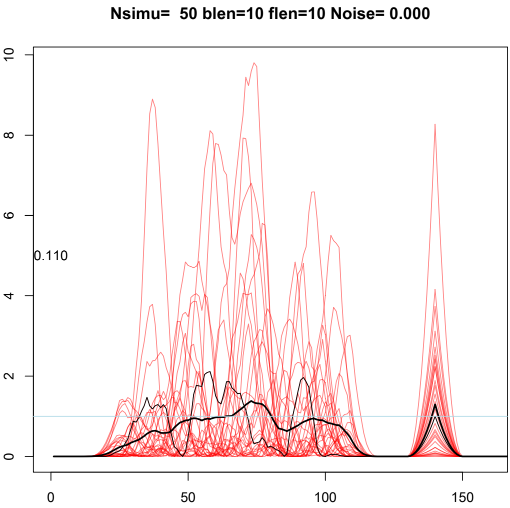
Example: simple example for the structure function



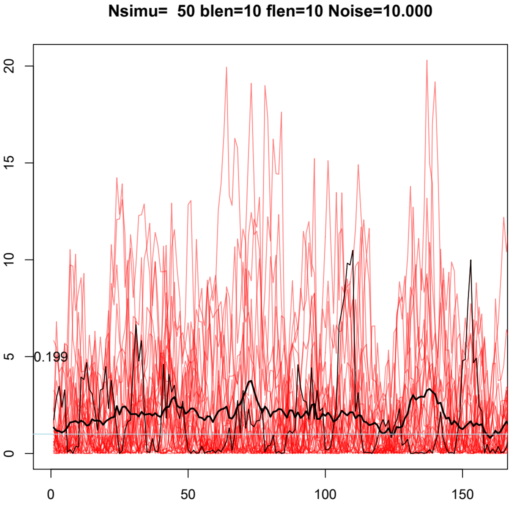
Samples of the measure and their mean



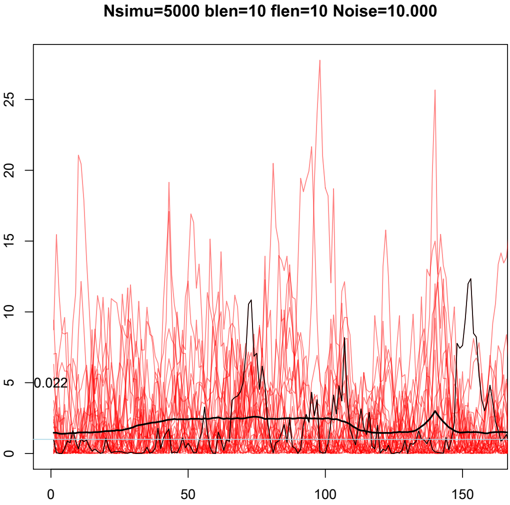
Analysis result of 50 integrations



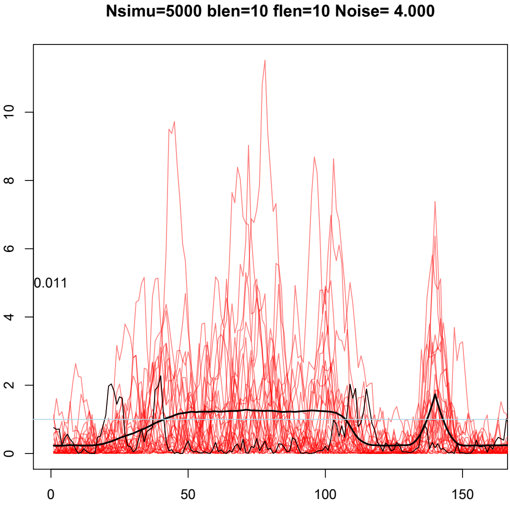
White noise added



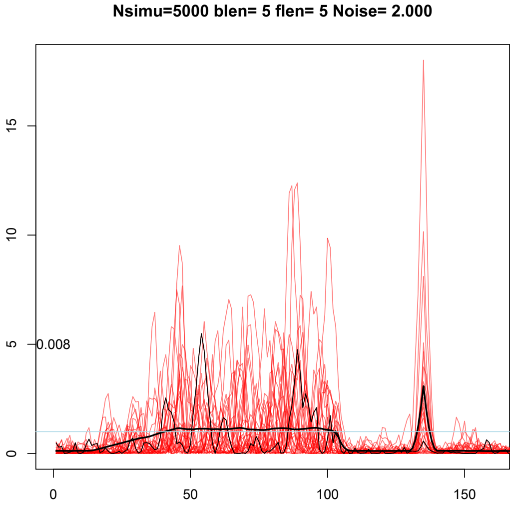
We need more integration



With less noise this looks very good:



We get sharper response by shortening the pulse, but have to reduce noise:



General signal model (time-dependent)

The signal corresponding to a coded transmission is simply given by

$$z^q(t) = \int_r \epsilon^q(t-r)\mu^q(dr;t) + \sqrt{T}\xi^q(t). \tag{7}$$

We can now use the formal equation

$$\langle \mu^q(dr;t)\overline{\mu^q(dr';t')}\rangle = X(r;t-t')\delta(r-r')drdr' \ , \tag{8}$$

where $X(r;t-t')$ is now the plasma autocorrelation function. Then

$$\langle z^q(t)\overline{z^q(t')}\rangle = \int\int \langle \mu^q(dr;t)\overline{\mu^q(dr';t')}\rangle \epsilon^q(t-r)\overline{\epsilon^q(t'-r')} + T\delta(t-t'),$$

Even more complication arises from the fact that the signal is convolved with the receiver impulse response $p(t)$.

$$s(t) = \int_\tau p(\tau)z(t-\tau)d\tau \tag{9}$$

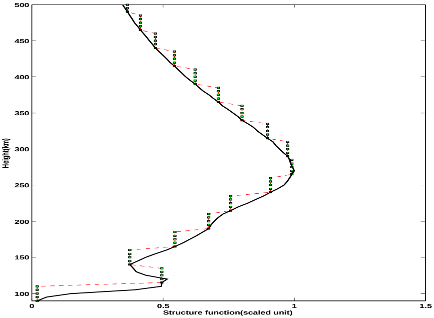
In our new way of signal processing $p(t)$ is typically so narrow that this last complication is avoided.

Discretization of the incoherent scatter signal model

Discretization based on staircase spline basis function, which is defined by

$$\psi(r) = 1, \;\text{if}\; 0 \leq r < 1, \;\text{and}\; \psi(r) = 0, \;\text{otherwise.} \tag{10}$$

$$X(r) = \sum_n x_n \psi(r/\Delta r - n), \tag{11}$$



Discretization of the received signal

- The incoherent scatter signal model can be discretized by taking the average of the signal in a time interval Δt
- If we denote the signal sampled at $i\Delta t$ by z_i, one may get

$$z_i^q = \frac{1}{\Delta t} \int_{(i-1)\Delta t}^{i\Delta t} z^q(t) dt. \tag{12}$$

- Similarly, the transmission radar waveform can be described in discrete form by

$$\epsilon(t) = \sum_i \epsilon_i \psi(t/\Delta t - i). \tag{13}$$

- Here we consider a simple boxcar pulse by choosing

$$\begin{aligned} \epsilon_i &= N_b^{-1/2}, \text{ if } 0 \le i < N_b \text{ and} \\ \epsilon_i &= 0, \text{ otherwise.} \end{aligned} \tag{14}$$

The measurement as an inverse problem

- The lag estimates of the received signal can be described in discrete form by

$$\begin{aligned} \langle z_i z_j \rangle &= \sum_k x(k\Delta t)\epsilon(i - k\Delta t)\epsilon(j - k\Delta t) + \frac{T}{\Delta t}\delta_{ij}, \\ &= \sum_k x(k)\epsilon(i-k)\epsilon(j-k) + \frac{T}{\Delta t}\delta_{ij}, \end{aligned} \tag{15}$$

where the index k represents the time instant $k\Delta t$.

- Using the discrete representation of the structure function, one gets

$$\langle z_i z_j \rangle = \sum_k \sum_n x_n \psi(k/\Delta r - n)\epsilon(i-k)\epsilon(j-k) + \frac{T}{\Delta t}\delta_{ij}. \tag{16}$$

The measurement as an inverse problem

- we can carry out the summation in Eq.18 along k within the range resolution cell to obtain

$$\langle z_i z_j \rangle = \sum_n x_n \sum_k \psi(k/\Delta r - n)\epsilon(i-k)\epsilon(j-k) + \frac{T}{\Delta t}\delta_{ij}. \qquad (17)$$

- The inner summation in Eq.17 is carried out step by step for each range resolution cell and this yields a more compact equation of the form

$$m_{ij} = \sum_n W_{ij}^n x_n + \varepsilon, \qquad (18)$$

where

- $m_{ij} = \langle z_i z_j \rangle$
- $W_{ij}^n = \sum_k \psi(k/\Delta r - n)\epsilon(i-k)\epsilon(j-k)$
- $\varepsilon = \frac{T}{\Delta t}\delta_{ij}$

The measurement as an inverse problem

- By merging the measurements from all the possible time pairs into Eq.18, we readily obtain the matrix equation (the equation for the inverse problem)

$$\mathbf{m} = \mathbf{W}\mathbf{x} + \varepsilon, \tag{19}$$

  where
  - $\mathbf{m}$ contains all the possible lagged product estimates
  - $\mathbf{W}$ is the theory matrix that contains the corresponding range ambiguity functions
  - ε is the measurement error
  - $\mathbf{x}$ consists of the unknown coefficients of the structure function.
- We focus on investigations of the characteristics of posteriori variance of the unknown coefficients by considering several combinations of radar pulse length and spatial resolution for any kind of **SNR** scenarios.

The posteriori variance calculation

- The posteriori variance can be extracted from the fluctuation of the covariance of the measurements (lag estimates) by using the associated theory matrix.
- Let us denote the measurement fluctuation covariance matrix by Σ_m
- It is then easy to show that posterior covariance of the unknown vector Σ_p is given by

$$\boldsymbol{\Sigma_p} = (\mathbf{W}^T \boldsymbol{\Sigma_m^{-1}} \mathbf{W})^{-1}. \tag{20}$$

- The values of the diagonal of the matrix Σ_p are the variance of the unknown coefficients.
- We know how to calculate **W** from radar pulse after discretization based on the basis functions.
- If we get Σ_m, we can readily calculate Σ_p.
- How do we determine Σ_m from the measurements?

Determining the fluctuation of the measurements

- Let M_{ij} denotes the different lag estimates, which can be calculated by

$$M_{ij} = \frac{1}{N_q} \sum_{q=1}^{N_q} z_i^q z_j^q, \tag{21}$$

where N_q denotes the number of times the experiment is repeated.

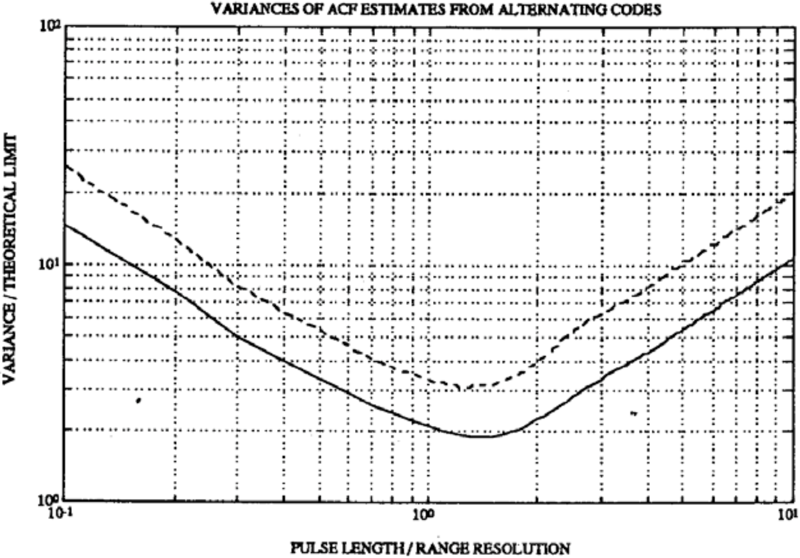
- The covariances of these estimates around their expected values, which is by definition Σ_m, is given by

$$\boldsymbol{\Sigma_m = \langle \{ (M_{ij} - \langle M_{ij} \rangle)(M_{i'j'} - \langle M_{i'j'} \rangle) \} \rangle}. \tag{22}$$

- Using the fourth moments theorem for Gaussian random variables, the right side of Eq.22 can be described in terms of lag estimates to obtain

$$\boldsymbol{\Sigma_m = \{ \langle z_i z_{i'} \rangle \langle z_j z_{j'} \rangle + \langle z_i z_{j'} \rangle \langle z_{i'} z_j \rangle \} / (2N_q)}. \tag{23}$$

Optimal radar pulse for the case of low **SNR** (Lehtinen, 1989)



Optimal radar pulse for different range resolution and **SNR** scenarios

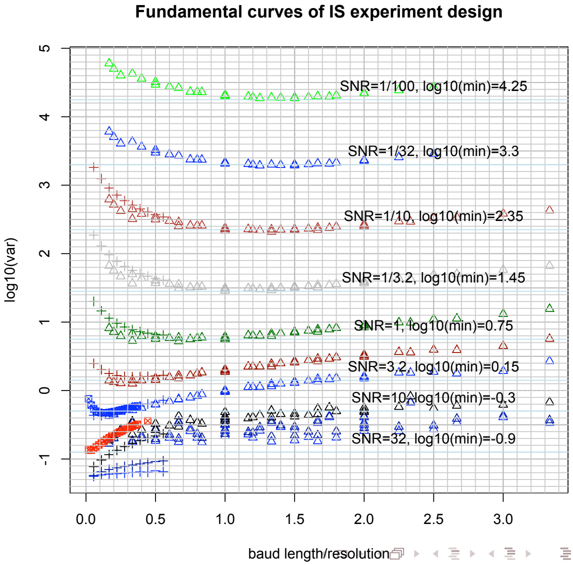
At high SNR we need very dense signal samples to get useful variance estimates

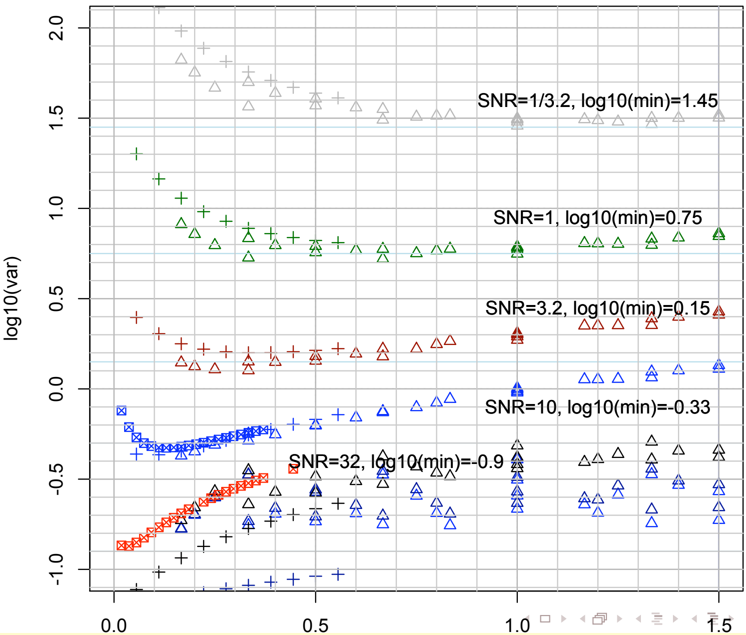
- Optimal baud =
- For SNR> 1:

$$\Delta t_{opt} = \frac{1.2 \cdot \Delta r}{\mathrm{SNR}}$$

- For SNR< 1:

$$\Delta t_{opt} = 1.4 \cdot \Delta r$$




SNR=1/3.2, log10(min)=1.45
SNR=1, log10(min)=0.75
SNR=3.2, log10(min)=0.15
SNR=10, log10(min)=-0.33
SNR=32, log10(min)=-0.9
log10(var)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.5
1.0
1.5

Conclusions

- Improving SNR always improves time resolution
  - With poor SNR integration time goes as SNR^{-2}
  - Then with increasing SNR as SNR^{-1}
- There is a penalty for using bauds not matching resolution and SNR
  - ... and we plan to derive analytic formulas for it in limiting cases
  - Like: for $\Delta t \ll \Delta t_{opt}$, we lose proportionally to $(\Delta t/\Delta t_{opt})^{-1}$
  - And: for $\Delta t \gg \Delta t_{opt}$, we lose proportionally to $(\Delta t/\Delta t_{opt})^{?}$ probably $(\Delta t/\Delta t_{opt})^{1}$
- Proper variance calculations are very expensive
  - We plan SNR$>$ 1 -proof analysis in amplitude space
  - This is because then noise is always white
  - ... but the problem is always underdetermined ...
  - We need real processing power and FLIPS
  - But using correlated data we would need impossible processing power
- HPC power will be necessary with EISCAT3D data analysis

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3D conclusions

- For SNR$\geq$ 1 there are two ways to proceed:
  - Shorten Δt
  - Widen the transmitter beam and use many receiver beams
- Asymptotically both ways work the same way
- But for just moderately good SNR the optimum is probably in between:
  - Widening the transmit beam and using many receive beams produces probably same kind of improvement as fractional lags (factor 1.5)
  - My guess however, because of 2 dimensions, even more, like 1.5^2
- We are just at a start of a learning process in antenna coding !
- The worst mistake we can do now:
- Make the radar non-flexible – fix things in hardware or FPGA firmware
- Next talk: HPC architecture instead of fixed FPGA solution

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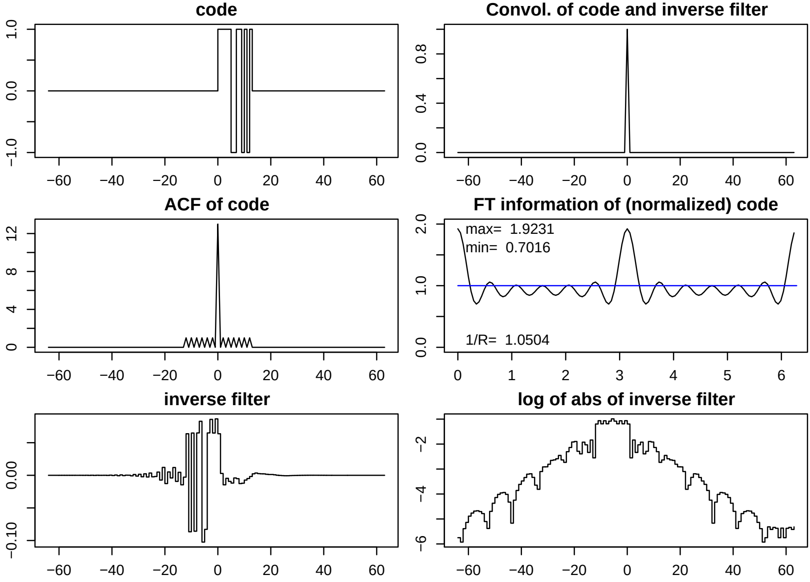
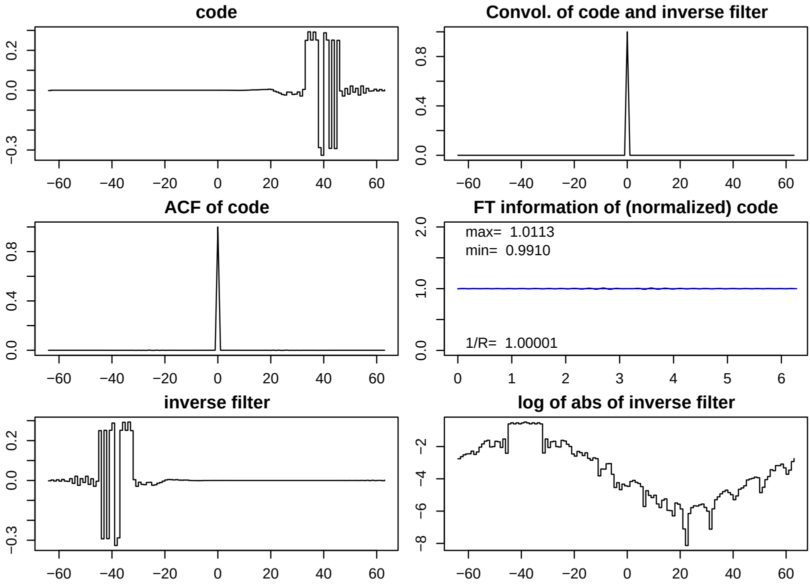
Perfect codes

- First explained in last WS
- Lehtinen, Damtie, Piiroinen and Orispää: PERFECT AND ALMOST PERFECT PULSE COMPRESSION CODES FOR RANGE SPREAD RADAR TARGETS, Inverse Problems and Imaging, Volume 3, No. 3, 2009, 465–486

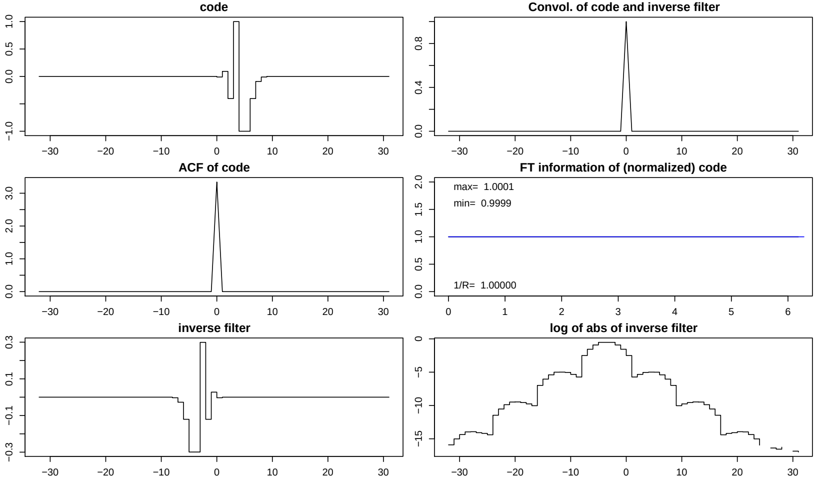
13-bit Barker



Perfect code made from 13-bit Barker



A very nice almost perfect 3-main-bit code:



Code comparison

- First fully rigorous results for SNR> 1
- However, just for stationary targets
- Anyway valid over short enough codes for any targets

Comparison theorem

Theorem

Let us denote the minimum and maximum values of the spectrum of the code by $P_{min} = \min_{\omega \in [0,2\pi)} |\hat{e}_n(\omega)|^2$ *and* $P_{max} = \max_{\omega \in [0,2\pi)} |\hat{e}_n(\omega)|^2$*, respectively. It is possible to add independent simulated noise to the measurement values so that the modified measurements are equivalent to measurements using a simple pulse of any power* $P \leq P_{min}$*. Also, it is possible to add independent simulated noise to a single-pulse measurement of power* $P \geq P_{max}$ *so that the modified measurement is equivalent to the radar measurement coded with* e_n*.*

This provides us to a rigorous way to compare (almost) perfect codes e to single pulses with same total power:

$$P_{min}^{1/2}\delta_{\cdot,0} \underset{x}{\prec} e \underset{x}{\prec} P_{max}^{1/2}\delta_{\cdot,0} \, , \tag{24}$$

where a single pulse code with unit power is denoted by with $\delta_{\cdot,0}$.

Proof. In the first case, consider a Gaussian stationary noise process ξ_1 independent of all the other processes considered. Adding this to our measurement values gives our measurement values gives

$$z = e * \mu + \xi + \xi_1, \tag{25}$$

which is equivalent to

$$\sqrt{P}\widehat{\hat{e}^{-1}} * z = \sqrt{P}(\mu + \widehat{\hat{e}^{-1}} * \xi + \widehat{\hat{e}^{-1}} * \xi_1). \tag{26}$$

The spectrum of the noise $P(|\hat{\xi}_1(\omega)|^2 + 1)|\hat{e}(\omega)|^{-2}$ is equal to a constant 1, if

$$|\hat{\xi}_1(\omega)|^2 = P^{-1}|\hat{e}(\omega)|^2 - 1. \tag{27}$$

This is non-negative and thus specifies a Gaussian stationary process ξ_1 for any

$$P \leq \min_{\omega\in[0,2\pi)} |\hat{e}(\omega)|^2. \tag{28}$$

With this choise the modified measurement equation corresponds to that of a single pulse of power P and normalized additive white noise, which proves the first part of the theorem.

Multipurpose codes

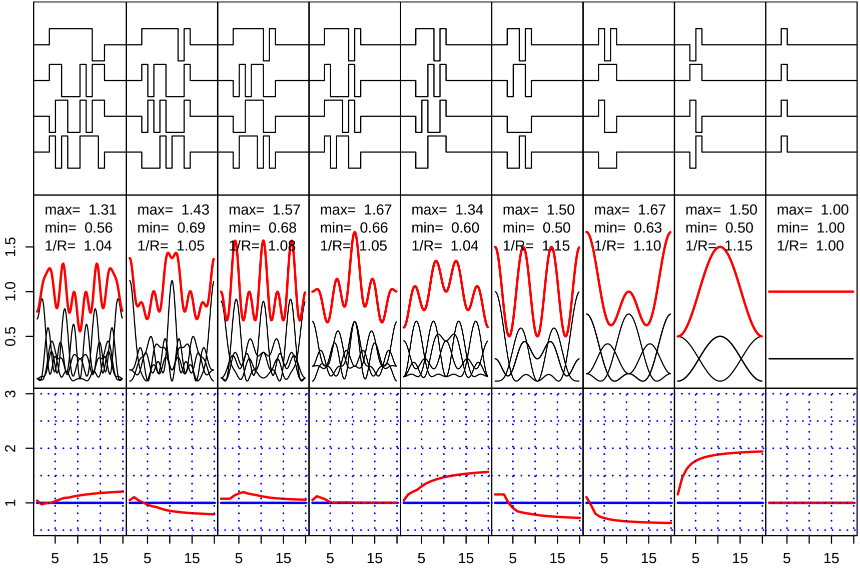
The idea:

- Extend the perfect properties of alternating codes to pulse-to-pulse
- Can be done trivially with very long (or hierarchic) codes
- Becomes too long to be practical

The solution:

- Lag profiles are convolutions of the profile and range ambiguity
- Invert this convolution away
- Analogy: target = lag profile, radar pulse = ambiguity function
- Code comparison applied to lag profile inversion in this case
- Thus results rigorous just for SNR$\ll 1$
- Computer searches to find almost perfect sets

A group of 4 10-baud codes to replace AC



More conclusions

- With multipurpose codes the choice of code is reduced to choosing of best baud length, compatible with range resolution and SNR.
- The codes then work near-optimally in every layer.
- However, in places where the baud length is not the optimum, one pays a price in integration time or variance – very roughly estimated: proportional in the mismatch of the actual baud length and the optimal baud length.

Implications on radar design

- Our algorithms assume band-limited amplitude data.
- Inversion is used a a decoding method.
- Optimal analysis for moderate or high SNR needs to be developed using amplitude analysis and MCMC methods.
- All this implies need of HPC capability

Original performance specifications of the EISCAT3D

Table: Performance requirements set for the EISCAT 3D

R [km]	h [m]	N_e	T_e/T_i	Int time
150	100	$1 \cdot 10^{10}$	1	1
300	300	$3 \cdot 10^{10}$	2	1
800	1000	$3 \cdot 10^{10}$	3	10

Single-pulse performances from the Deliverable 3.2

Table: System #8 single pulse performance

R [km]	h [m]	SNR	N	IPP_1 [ms]	t_p [μs]	$t_{int}[s]$
150	100	$2.2 \cdot 10^{-3}$	$2.1 \cdot 10^{7}$	1	0.67	9200
300	300	$1.7 \cdot 10^{-2}$	$3.5 \cdot 10^{5}$	2	2.0	700
800	1000	$1.3 \cdot 10^{-2}$	$6.4 \cdot 10^{4}$	5.3	6.67	3400

Table: System #9 single pulse performance

R [km]	h [m]	SNR	N	IPP_1 [ms]	t_p [μs]	$t_{int}[s]$
150	100	$1.1 \cdot 10^{-2}$	$8.3 \cdot 10^{5}$	1	0.67	830
300	300	$8.4 \cdot 10^{-2}$	$1.4 \cdot 10^{4}$	2	2.0	28
800	1000	$6.3 \cdot 10^{-2}$	$2.5 \cdot 10^{3}$ [a]	5.3	6.67	133

[a] For 10 s integration time, the total number of samples needed is $2.5 \cdot 10^4$

This raises an alarm:

We see to our dismay that a System 8 configuration using short single pulses under Section 2.12 conditions falls hopelessly short of the target. Even a System 9 configuration fails to meet the desired time resolution at all altitudes; at 150-km by a factor of over 800! This highlights the need for running the system with advanced modulation schemes at all times, as this will allow the extraction of a substantial number of essentially uncorrelated target estimates from each radar cycle, corresponding to a reduction in the required integration time by factor of ≈ 10.

sorry, no problem here!

- *Assuming 1 ms IPP, 200 µs is well available for coding, facilitating a code of 300 pulses 0.66 µs each.*
- *Pulse compression increases the power by 300, and as integration time goes as SNR^{-2}, we have 90 000 instead of 10*

With HPC architecture and advanced coding schemes we avoid the following:

The radar performance analysis in Section 5 demonstrates that the extreme joint time/height resolution requirements laid down in PSD Section 2.12 are quite unrealistic; as shown in Table 5, even a 36000-element array configuration utilising advanced modulation schemes would fail to meet the targets at all altitudes by a factor of (70. . .920)!

However, if the altitude resolution is relaxed by a factor of (2. . .10), even a 16000-element system will be in a position to meet the 1-s/10-s time resolution requirement at 150. . .800 km altitude. As shown in Table 6, the SNR will then exceed 20 % at all altitudes, which already brings the system into the region of diminishing returns. Any substantial improvement in altitude resolution would require significantly longer integration times.

The user community is strongly advised to review its requirements and consider whether incoherent-scatter altitude resolutions better than 1 km are really meaningful and required at altitudes above 150 km, keeping in mind that demands for extreme simultaneous time and height resolution must be bought at very high capital investment and operating cost.

Specifications revisited

- Calculations similar to what will be discussed in Ilkka Virtanen's talk
- high SNR properly accounted.
- Lags up to first zero crossing of the ACF used for raw N_e estimates
- No. lags denoted by k_{max}

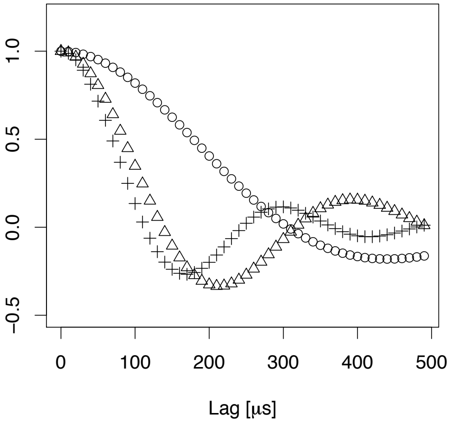


Table: System #8

R [km]	N_b	k_{max}	N_s	PRF [Hz]	$N_{\#8}$	$t_{int,\#8}$ [s]
150	560	450	150,525	666	$2.1 \cdot 10^7$	0.21
300	375	65	22,230	333	$3.5 \cdot 10^5$	0.05
800	300	16	4,664	125	$6.4 \cdot 10^5$	1.10

Table: System #8, corrected for SNR $>\approx 1$

R [km]	N_b	k_{max}	SNR	SNR_{compr}	corr	$t_{int,\#8}$ [s]
150	560	450	$2.2 \cdot 10^{-3}$	0.99	$10^{0.5}$	0.66
300	375	65	$1.7 \cdot 10^{-2}$	1.1	$10^{0.5}$	0.15
800	300	16	$1.3 \cdot 10^{-2}$	0.21	$10^{0.2}$	1.73

Table: System #9

R [km]	N_b	k_{max}	N$_s$	PRF [Hz]	N$_{\#9}$	$t_{int,\#9}$ [s]
150	560	450	150,525	666	$8.3 \cdot 10^5$	0.009
300	375	65	22,230	333	$1.4 \cdot 10^4$	0.003[a]
800	300	16	4,664	125	$2.5 \cdot 10^4$	0.043

[a] 1/PRF, only one pulse needed

Table: System #9, corrected for SNR $>\approx 1$

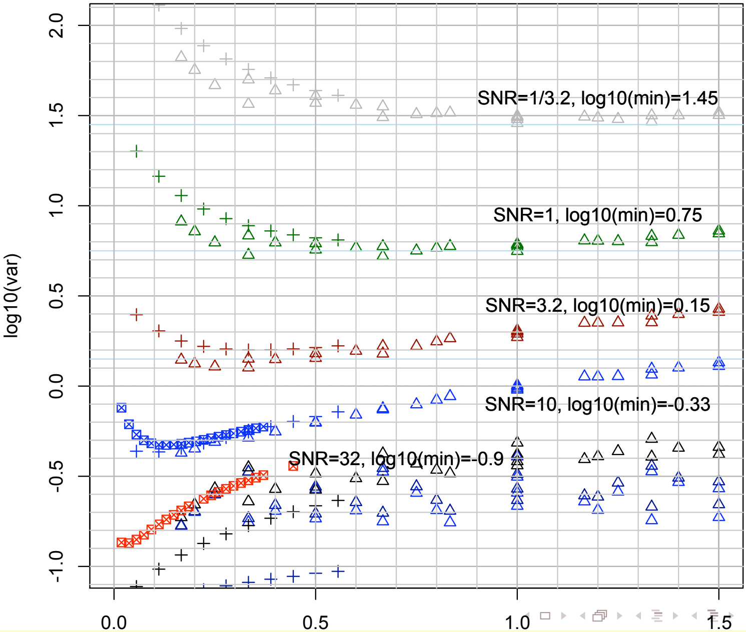
R [km]	N_b	k_{max}	SNR	SNR$_{compr}$	corr	$t_{int,\#9}$ [s]
150	560	450	$1.1 \cdot 10^{-2}$	4.95	$10^{0.9}$	0.071
300	375	65	$8.4 \cdot 10^{-2}$	5.46	$10^{0.9}$	0.024
800	300	16	$6.3 \cdot 10^{-2}$	1.01	$10^{0.5}$	0.14

What does this mean in terms of 3D

- 0.66 s integration time and 100 m range resolution seems of course more than sufficient for most purposes.
- Not for 3D, however
- If we have a 3D grid of $30 \cdot 30$ directions, this is as long as $0.66 \cdot 30^2 = 594$ s.
- This is not too good – maybe we reduce resolution to 1000 m with single-pulse $t_{int} = 0.92$ s. This is reduced to $0.92/45^2$ by compression, but as compressed signal-to-noise goes to 9.9, we get $10^{1.3}$ reduction in this, leading to 0.0091 s with single direction, but 8.15 s for the $30 \cdot 30$ scan. Design study promises $(0.92/10) * 30^2 = 83$ s or little more because of their idea of SNR correction.
- With proper coding, we can scan a $30 \cdot 30$ grid of 1000 m range resolution in 8.15 s. While with the design study we need 83 s.

Remember polarization agility!

- Polarization switching can be used as a coding method with high SNR.
- Tor Hagfors often tried to promote this
- A fresh paper by Gustavsson and Grydeland in Radio Sci explains this in simpler terms. With very high SNR a factor 4 will be gained.
- Look again at the curves: SNR 9.9 reduced to SNR 2.5 – We are not quite yet in the asymptotic regime and have to relax the improvement estimate by $10^{0.1}$. Thus we get $8.15/4 \cdot 10^{0.1}$ =2.6 s.
- 1000 m resolution is 6.7 μs. Optimal baud lengthens from 1 μs to 3 μs.
- We might still get $1.5 \ldots 1.5^2$ by *fractional beams* and antenna coding.
- So – it will be something like $1.5 \ldots 2$ s in a $30 \cdot 30$ grid and 1000 m range resolution.


log10(var)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.5
1.0
1.5
SNR=1/3.2, log10(min)=1.45
SNR=1, log10(min)=0.75
SNR=3.2, log10(min)=0.15
SNR=10, log10(min)=-0.33
SNR=32, log10(min)=-0.9

So - where is the simplicity ?

- Decide what is the time correlation length of your target
- Decide what is the desired spatial resolution
- Use a pulse with total energy equal to radar power times the target correlation time
- ... but compress it in a pulse 1.5 times the desired spatial resolution
- ... if then $SNR > 1$, compress it to a still shorter pulse so that with it $SNR \approx 1$

This has a proven mathematical background. We have found ways to prove theorems that almost perfect codes arbitrarily close to perfect codes of any length exist.

What about time-changing targets ?

- Multipurpose codes of Ilkka Virtanen are the key
- Conjecture: if baud length selection is made as in the previous slide, we are close to optimal

This is still speculation. Actually, we know this method results in good codes. The only thing we do not know is that they truly are the best possible.

How to do this in practise ?

- Forget GUISDAP
- Forget legacy signal processing - take raw data samples instead
- Analysis developed by I. Virtanen
- This will further be developed in the EISCAT3D preparatory phase

Is this the final word ?

- No - LPI is designed to work effectively for poor SNR
- However, with the conjecture that codes are compressed to produce $SNR \approx 1$, this is not too bad
- However, full error analysis calls for amplitude-domain analysis (LPI does inversion on lag profiles internally)
- Very high computational cost – but still lower than through lag profiles

What else ?

- 3D mapping:
- Phased array antenna geometries
- Based on Itó model of radiation fields on directional sphere
- Transmitter and receiver in unsymmetric position