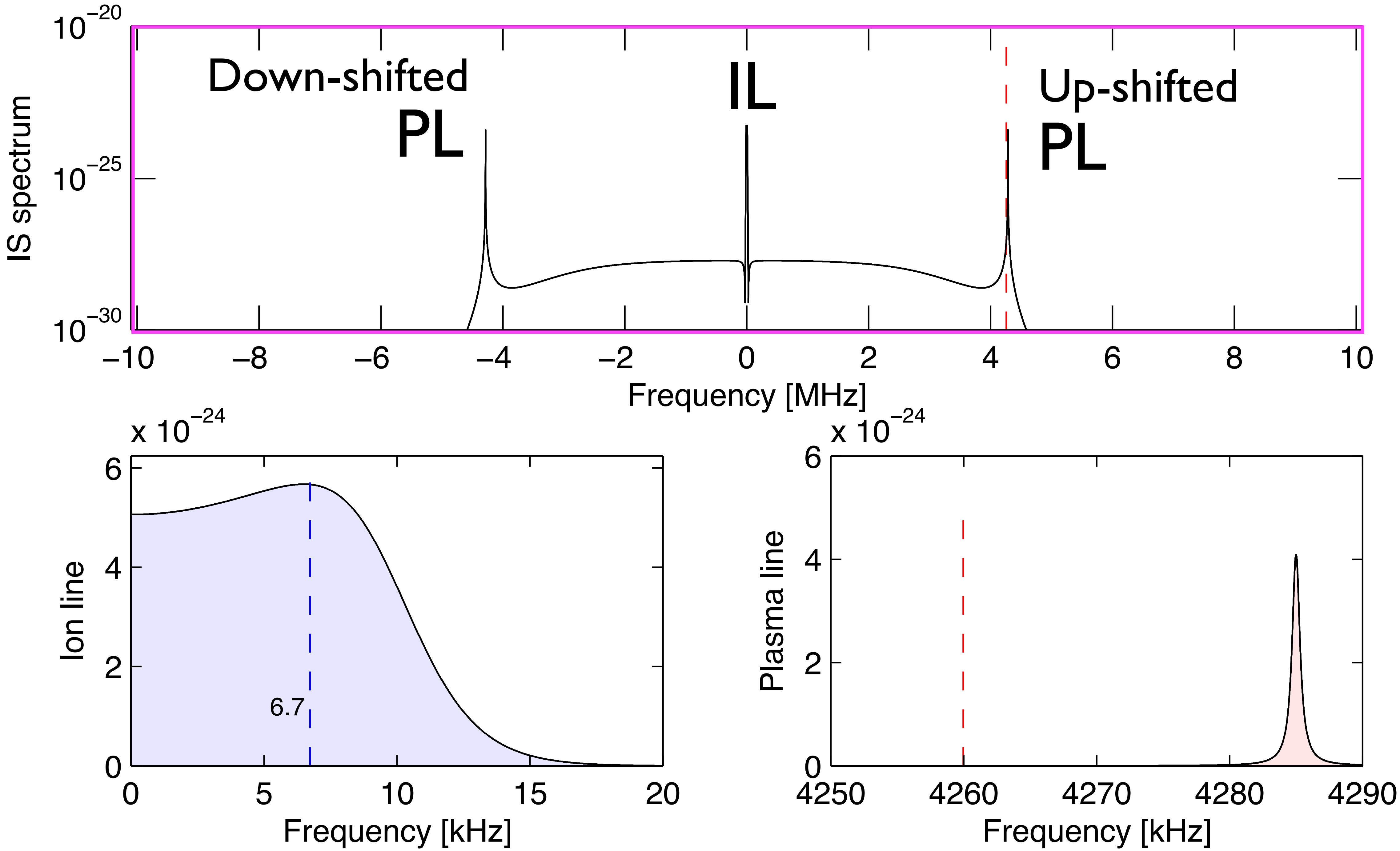
IS spectrum WHz^{-1}

IS spectrum

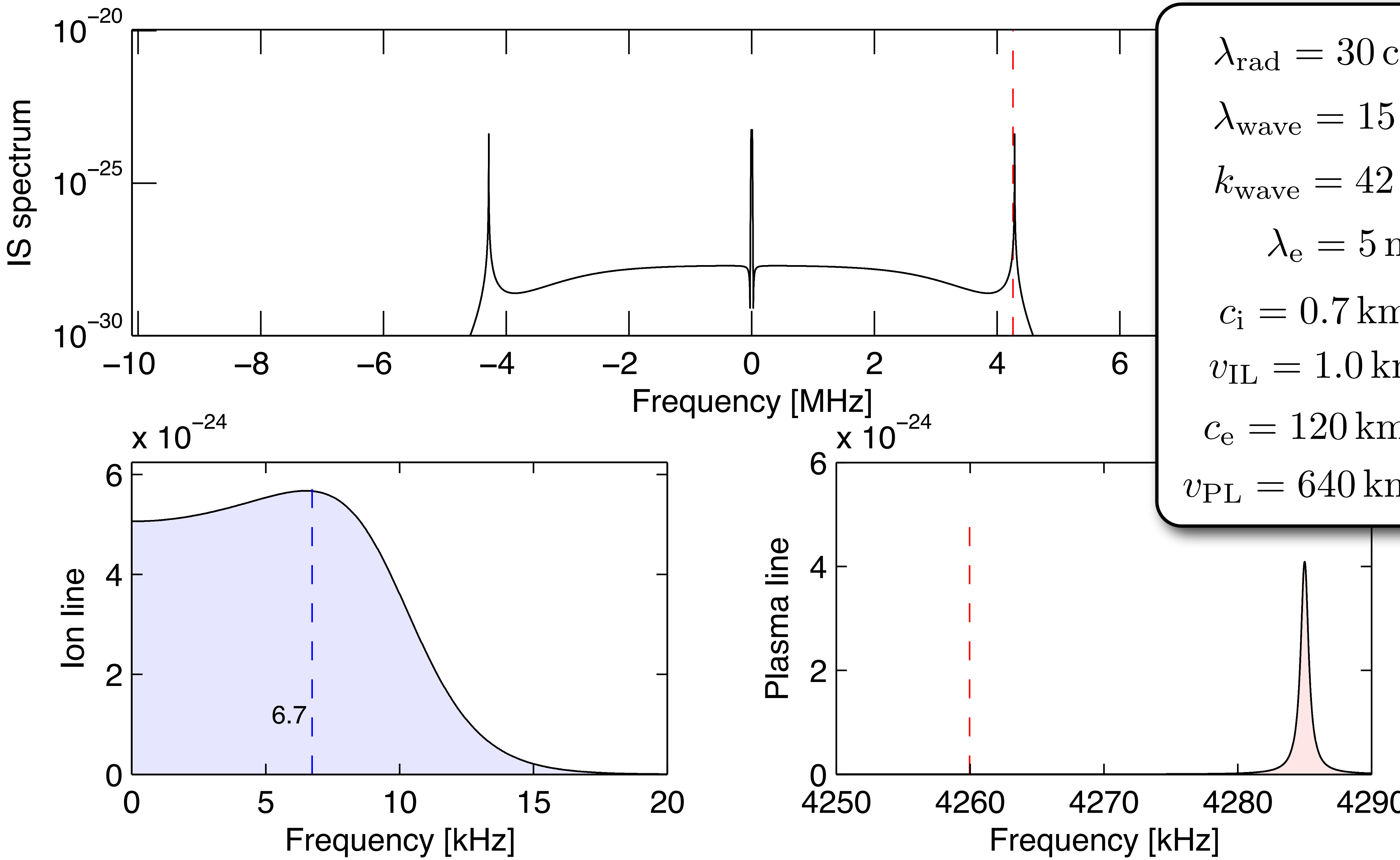
$$f_{\mathrm{rad}} = 1\,\mathrm{GHz}$$

$$N_{\mathrm{e}} = 2.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_{\mathrm{e}} = 1000\,\mathrm{K} \quad T_{\mathrm{i}} = 1000\,\mathrm{K} \quad m_{\mathrm{i}} = 16\,\mathrm{amu}$$



$f_{\mathrm{rad}} = 1\,\mathrm{GHz}$

$N_\mathrm{e} = 2.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_\mathrm{e} = 1000\,\mathrm{K} \quad T_\mathrm{i} = 1000\,\mathrm{K} \quad m_\mathrm{i} = 16\,\mathrm{amu}$



$\lambda_{\mathrm{rad}} = 30\,\mathrm{cm}$

$\lambda_{\mathrm{wave}} = 15\,\mathrm{cm}$

$k_{\mathrm{wave}} = 42\,\mathrm{m}^{-1}$

$\lambda_\mathrm{e} = 5\,\mathrm{mm}$

$c_\mathrm{i} = 0.7\,\mathrm{kms}^{-1}$

$v_{\mathrm{IL}} = 1.0\,\mathrm{kms}^{-1}$

$c_\mathrm{e} = 120\,\mathrm{kms}^{-1}$

$v_{\mathrm{PL}} = 640\,\mathrm{kms}^{-1}$

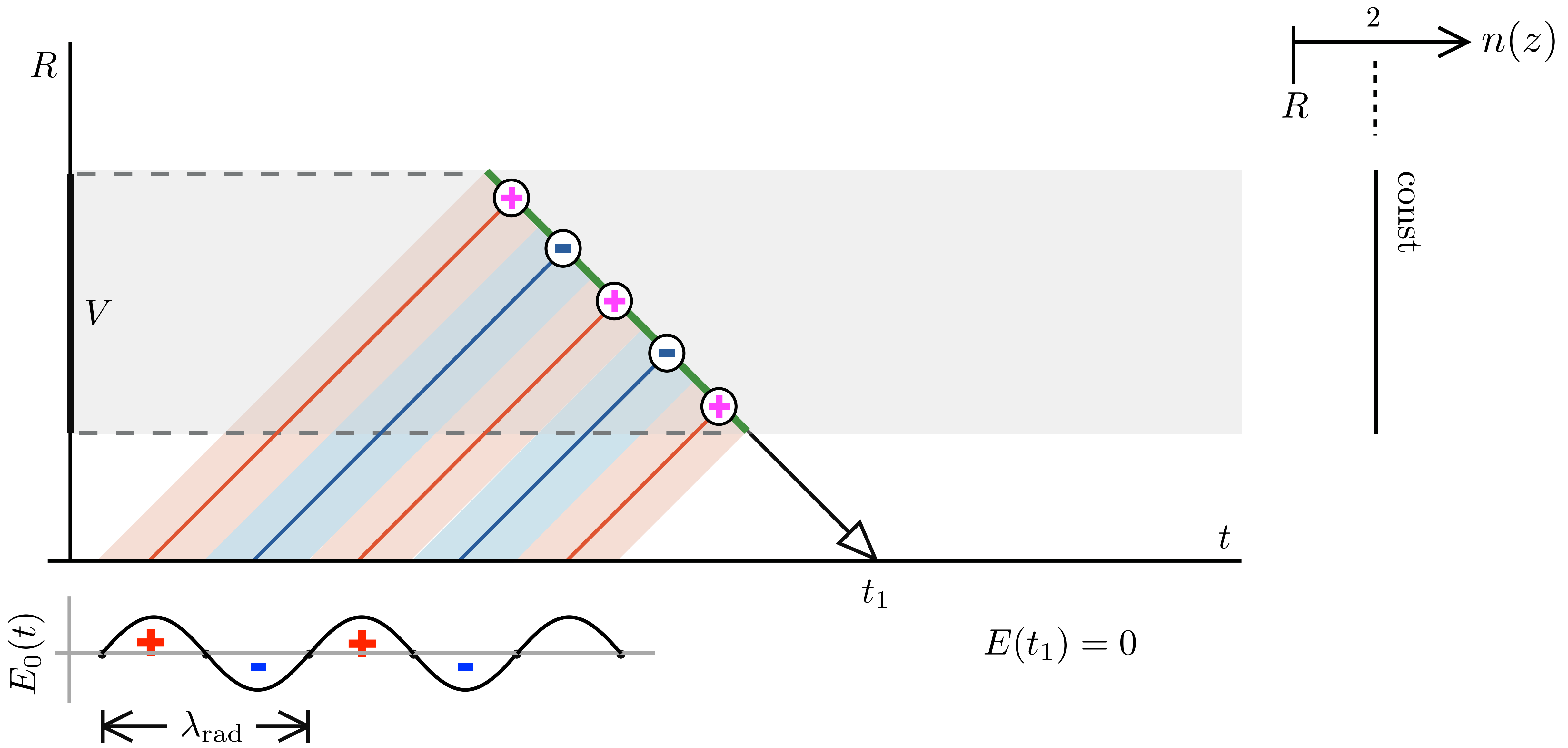
1. Constant plasma density ==> No net scatter

2. TX-matched density fluctuations enhance net scatter

3. Moving fluctuations (=waves) ==> spectral lines

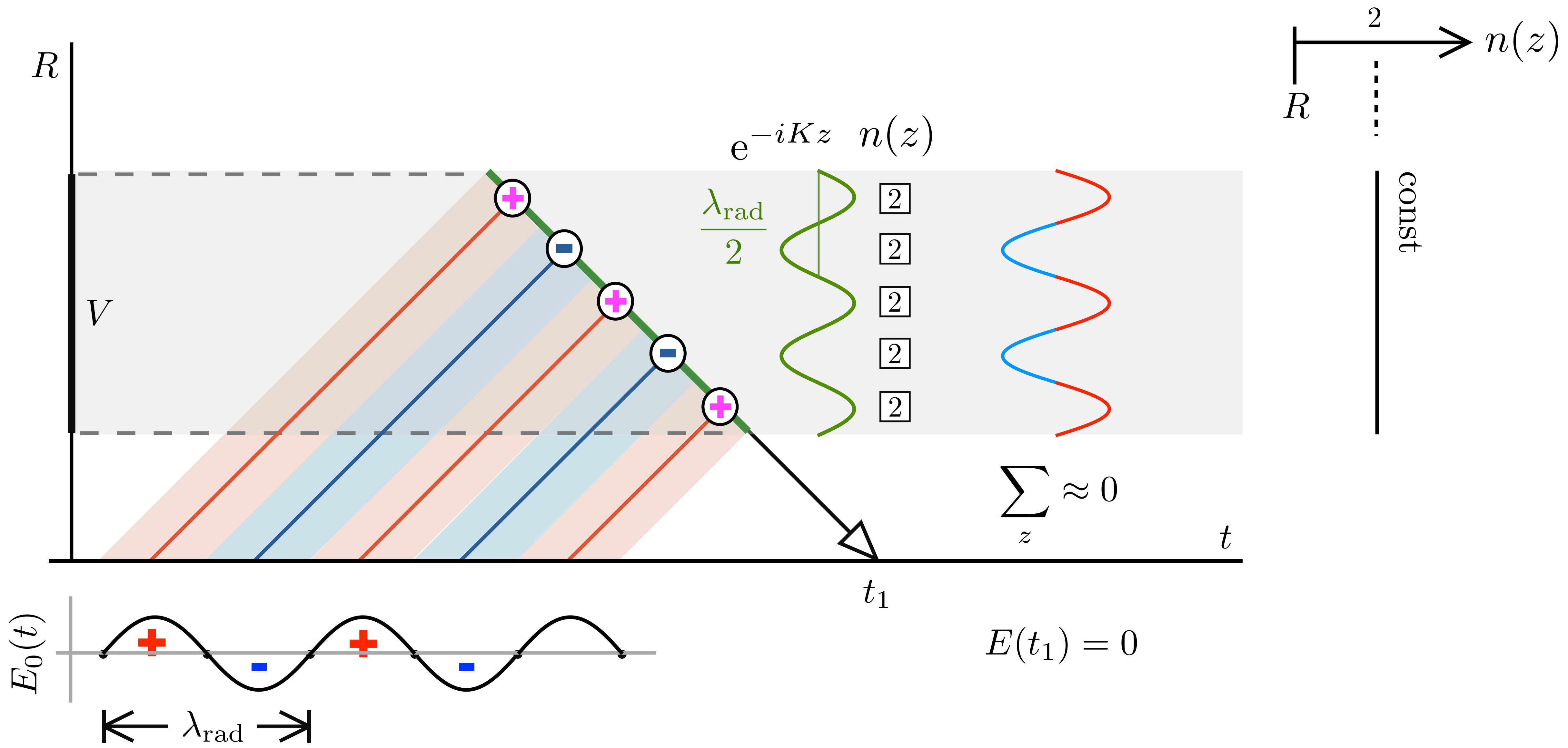
4. Two classes of waves ==> 2+2 line positions (IL & PL)

5. The lines are broadened by damping ==> IS spectrum



R
V
t
t_1
$E_0(t)$
λ_{rad}
$E(t_1) = 0$
2
$n(z)$
R
const

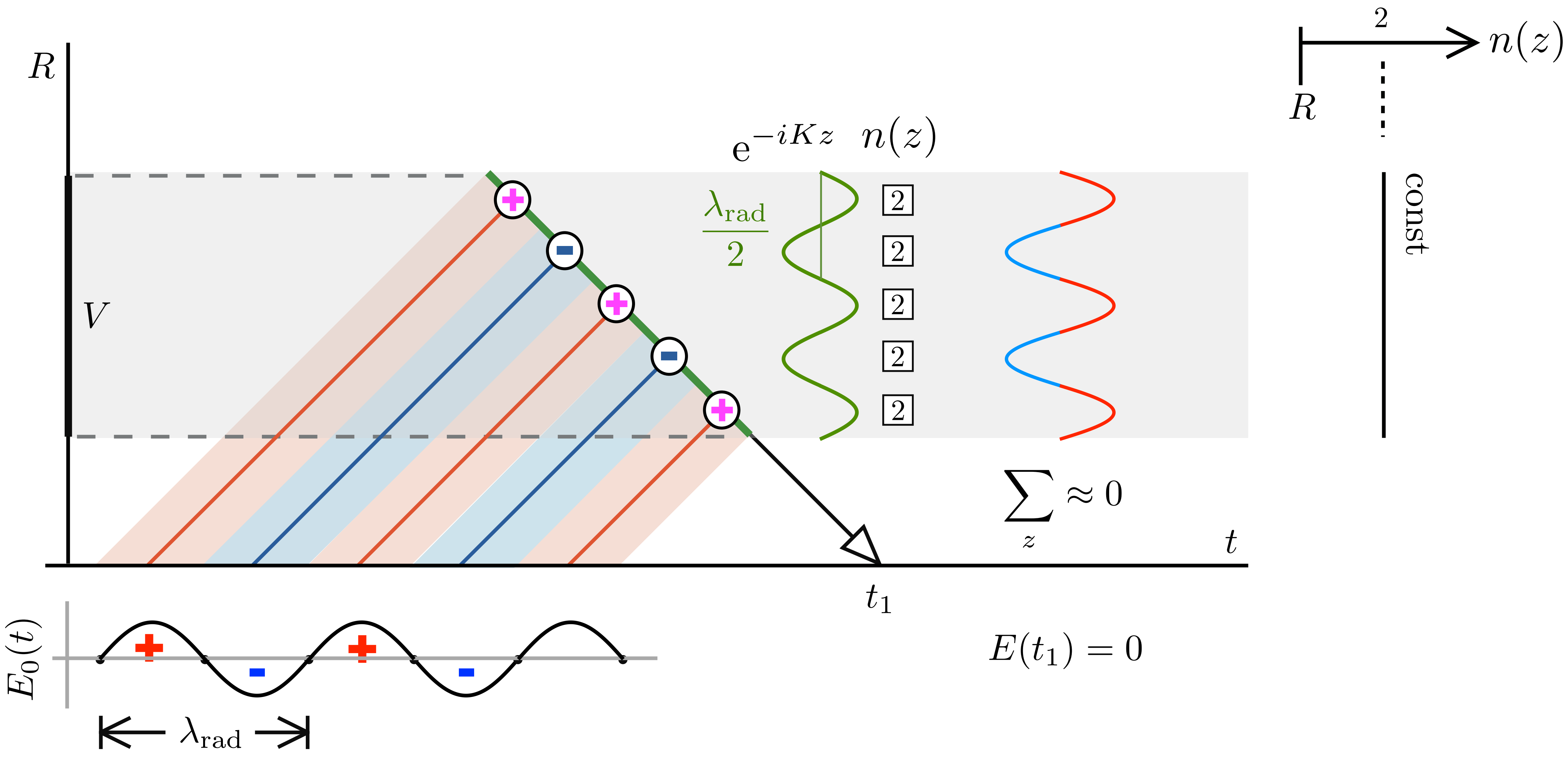
Const density ==> no net scatter



Const density ==> no net scatter

$$E \sim \int_L n(z) \mathrm{e}^{-iKz}\, dz$$

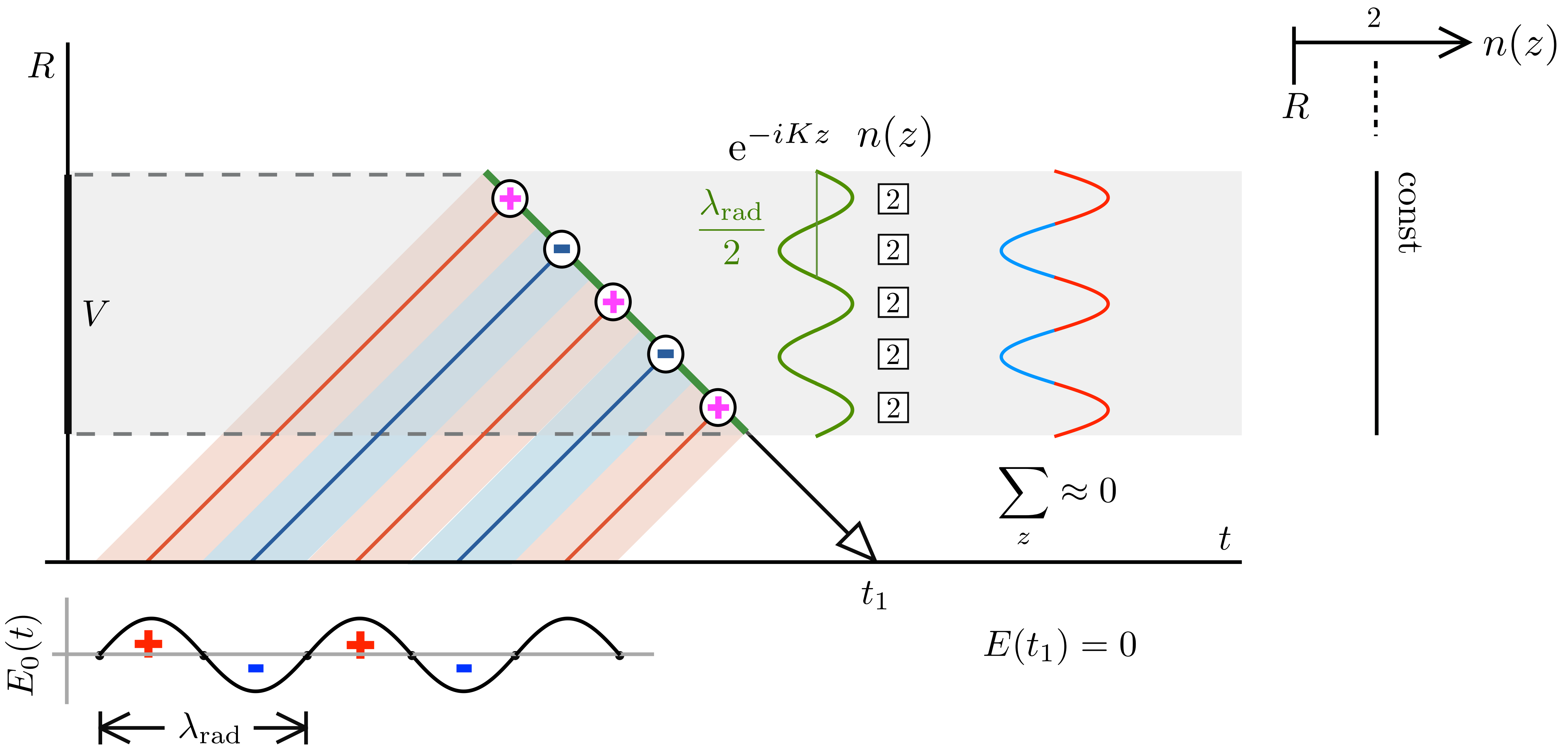
$$K = \frac{2\pi}{\lambda_{\mathrm{rad}}/2}$$



$E(t_1) = 0$

$$E \sim \int_L n(z)\mathrm{e}^{-iKz}\,dz \times \mathrm{e}^{i\omega_0 t}$$

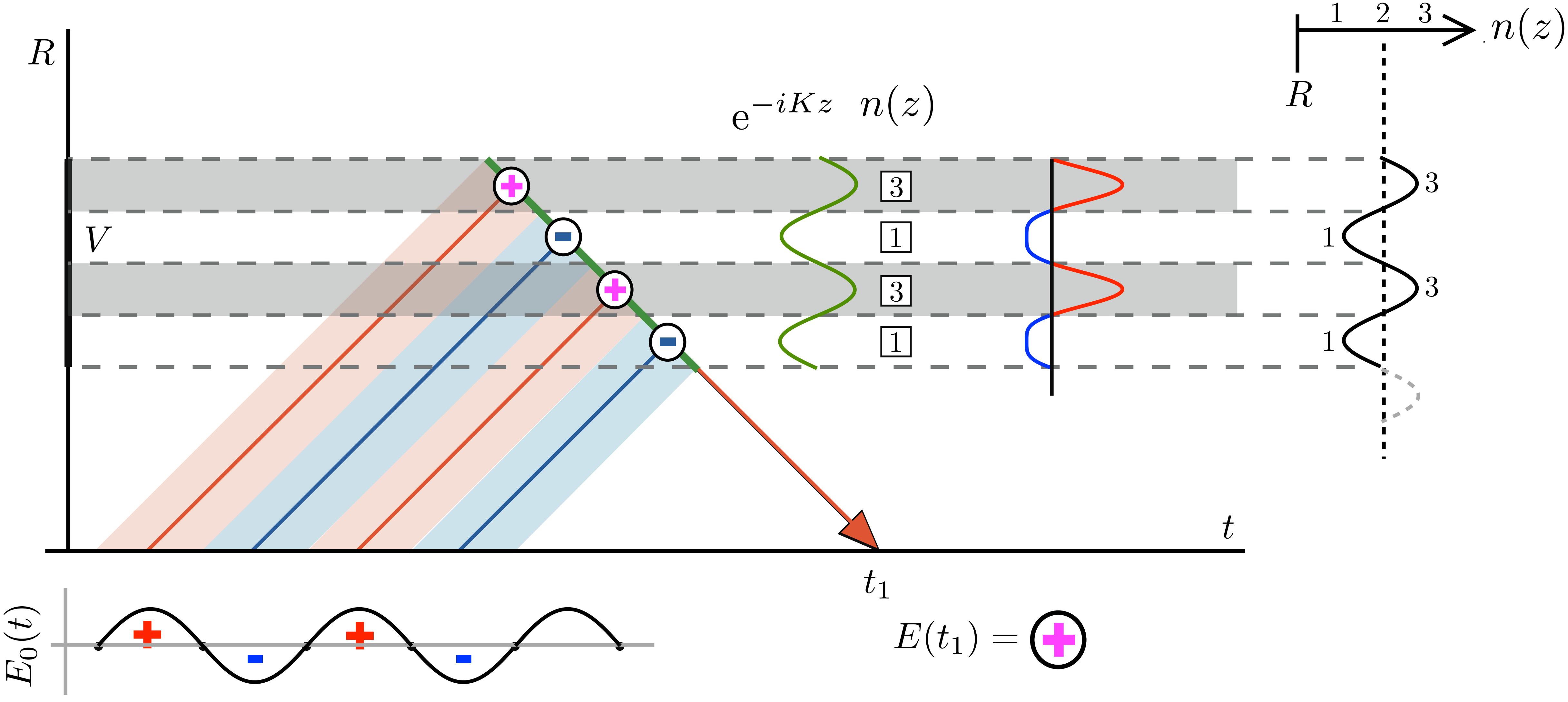
$$K = \frac{2\pi}{\lambda_{\mathrm{rad}}/2}$$



Const density ==> no net scatter

$$E \sim \int_L [n_0 + n_1 \cos(Kz)] \; \mathrm{e}^{-iKz} \, dz \; \times \mathrm{e}^{i\omega_0 t}$$

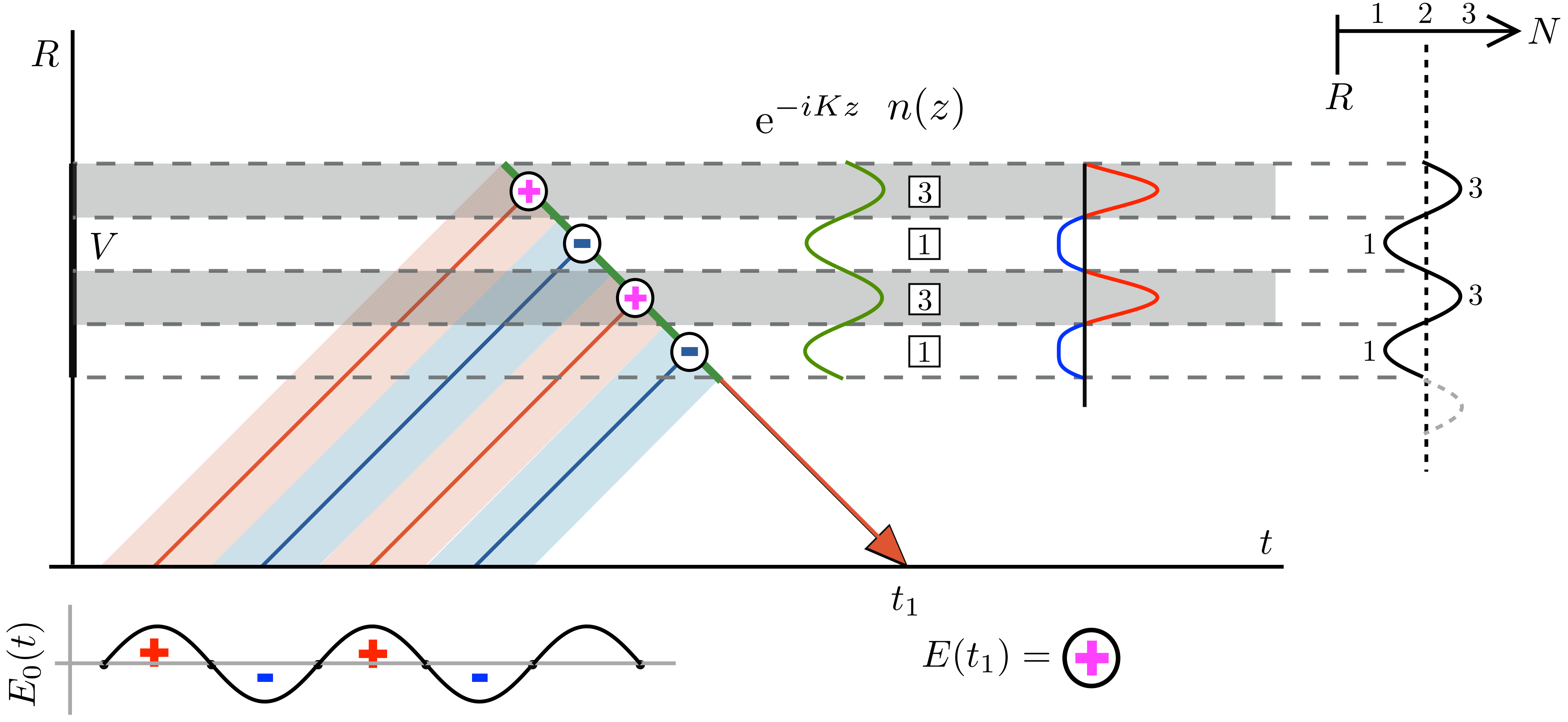
$$K = \frac{2\pi}{\lambda_{\mathrm{rad}}/2}$$



Matched, “frozen” fluctuation

$$E \sim \int_L \left[n_0 + n_1 \mathrm{e}^{iKz}\right] \mathrm{e}^{-iKz}\, dz \times \mathrm{e}^{i\omega_0 t}$$

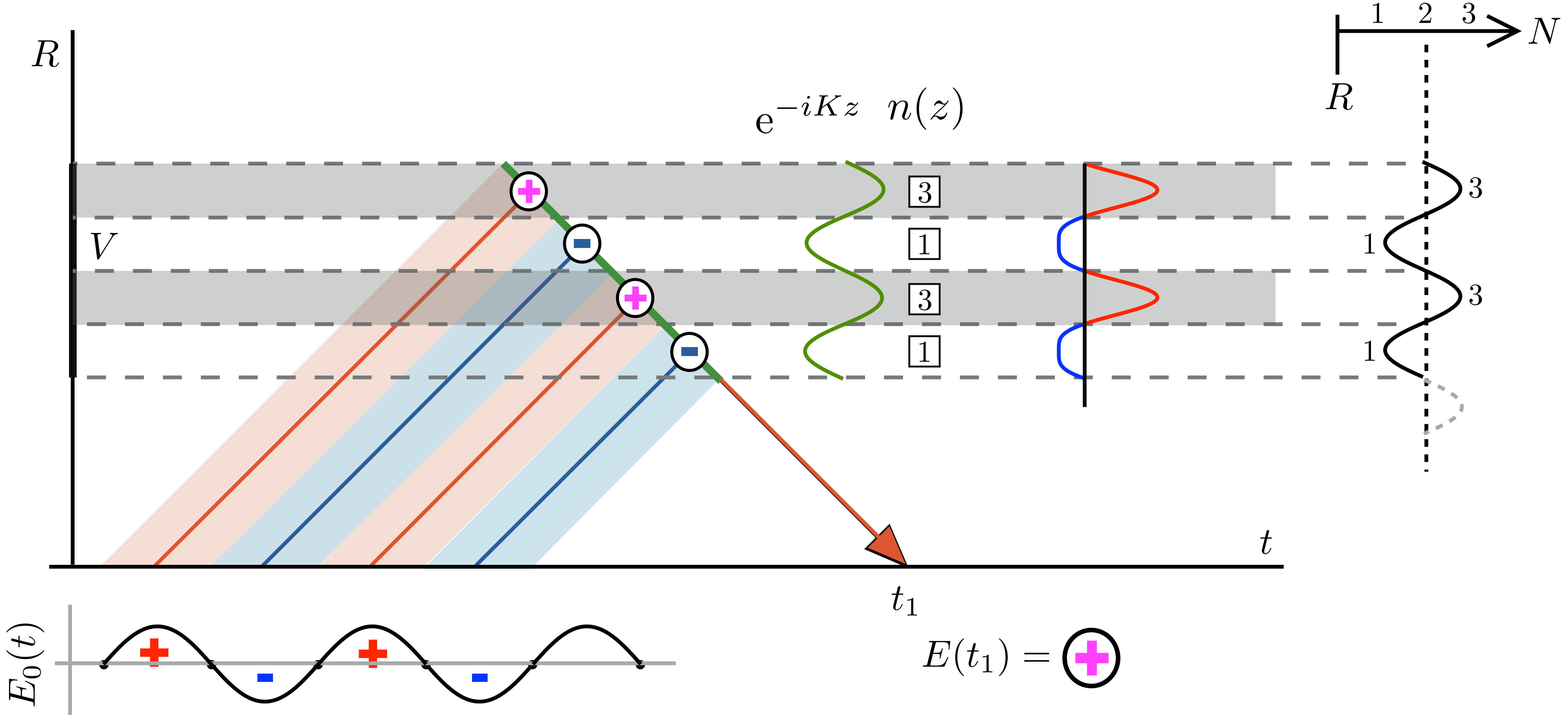
$$K = \frac{2\pi}{\lambda_{\mathrm{rad}}/2}$$



Matched, moving fluctuation

$$E \sim \int_L \left[n_0 + n_1 \mathrm{e}^{i(\pm\omega t + Kz)}\right] \mathrm{e}^{-iKz}\, dz \times \mathrm{e}^{i\omega_0 t}$$

$$K = \frac{2\pi}{\lambda_{\mathrm{rad}}/2}$$

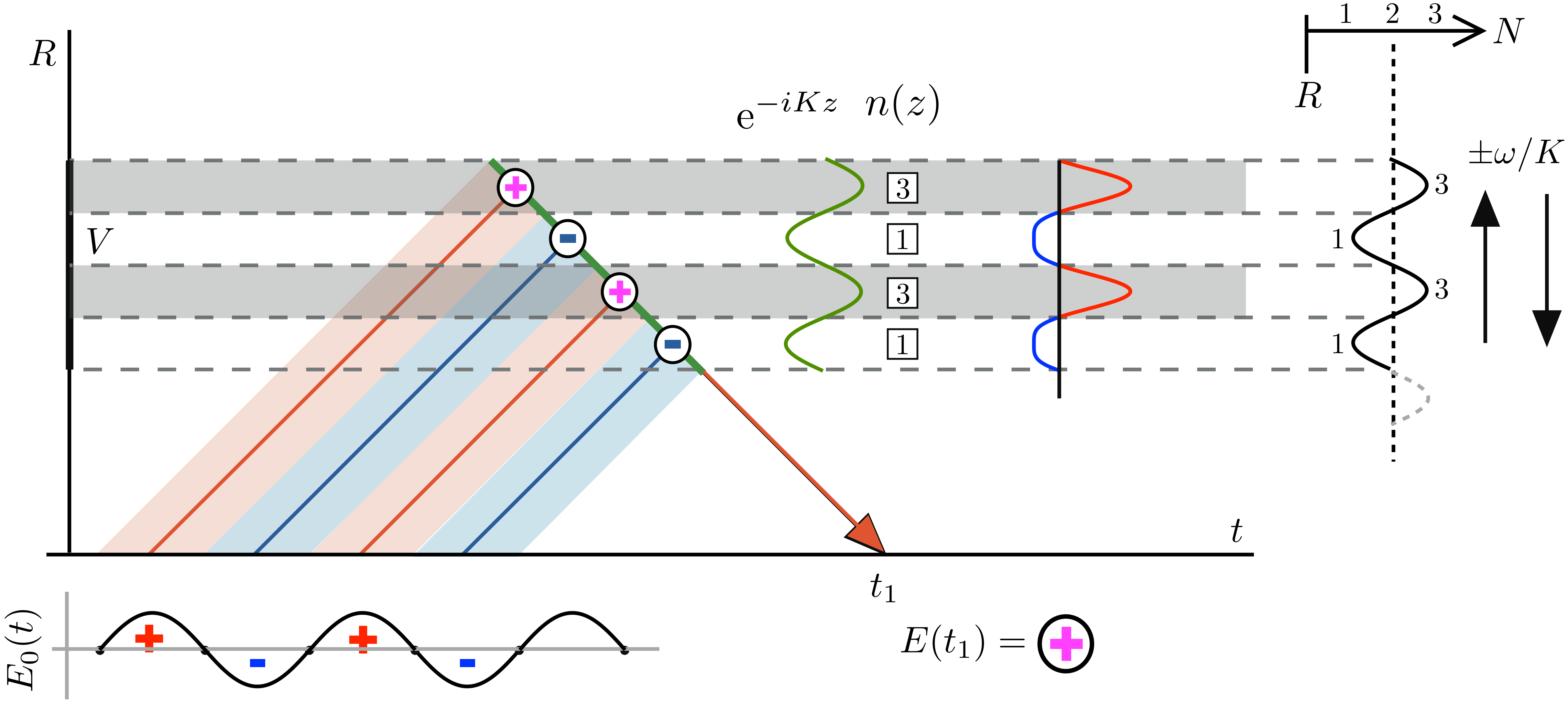


Matched, moving fluctuation

$$E \sim \int_L \left[n_0 + n_1 e^{iKz}\right] e^{-iKz}\, dz \ \times e^{i(\omega_0 \pm \omega)t}$$

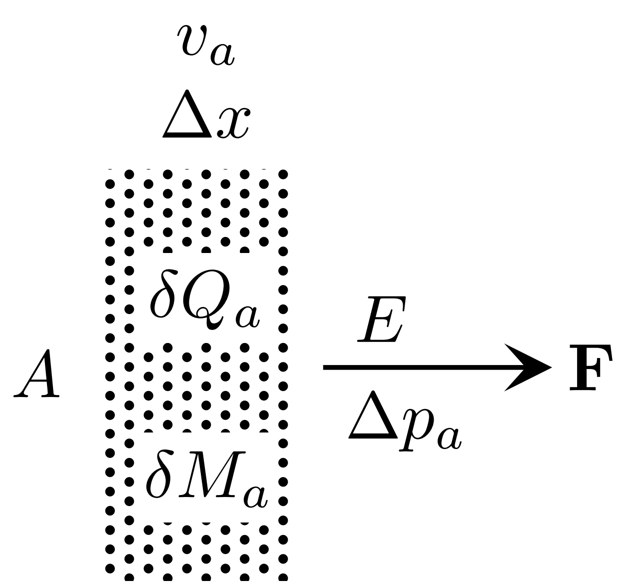
Reception freq. shifted by the wave's angular frequency

$$K = \frac{2\pi}{\lambda_{\rm rad}/2}$$



Plasma waves

$m_a \quad q_a \qquad a \in \{e, i\}$



$$n_a = n_{a0} + n_{a1}$$
$$p_a = p_{a0} + p_{a1}$$
$$T_a = T_{a0} + T_{a1}$$
$$\rho_a = n_a q_a = \rho_{a0} + \rho_{a1}$$

2-component plasma: electrons (e) and ions (i)

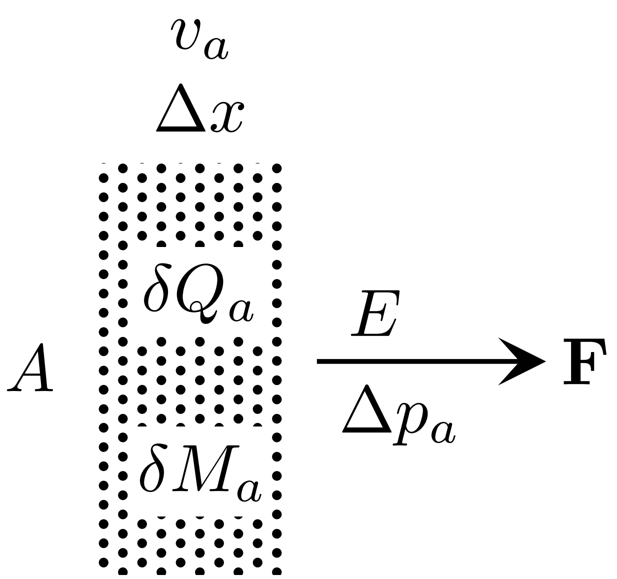
Large mass difference ... what do we expect?

Fast charge disturbance ==>
Only electrons have time to respond.
Electrons oscillate (in the mean) like in pressure wave, ions form an unmoving background. (e.a. wave).

Slowly oscillating driving force ==>
Ions oscillate like in pressure wave, electrons follow to ensure charge neutrality

Plasma waves

$$m_a \quad q_a \qquad a \in \{e, i\}$$



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$$T_a = T_{a0} + T_{a1}$$
$$\rho_a = n_a q_a = \rho_{a0} + \rho_{a1}$$

$$\boxed{\delta M_a \frac{dv_a}{dt} = \Delta p_a A + \delta Q_a\, E}$$

$$n_{a0}\frac{dv_a}{dt} = \frac{-\partial_x p_a}{m_a} + \frac{n_{a0} q_a}{m_a} E$$

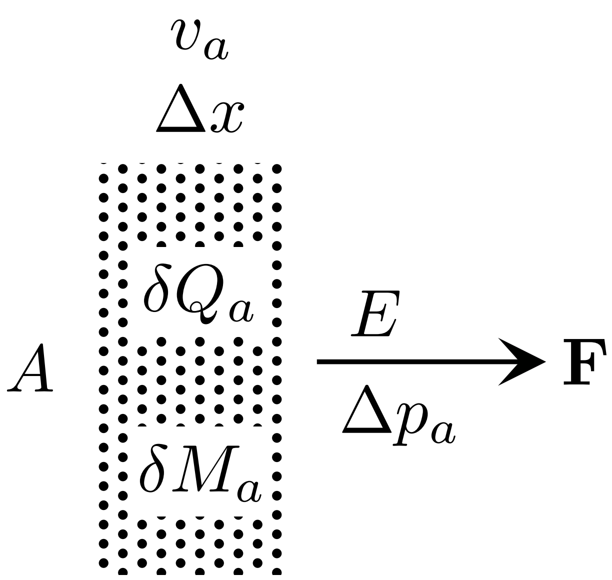
$$\boxed{\partial_x E = \frac{1}{\epsilon_0} \sum_b n_{b1} q_b}$$

$$\frac{dv_a}{dt} = \partial_t v_a + \cancel{v_a \partial_x v_a}$$

$$\partial_t \partial_x (n_a v_a) = \frac{-\partial_{xx} p_{a1}}{m_a} + \frac{n_{a0} q_a}{\epsilon_0 m_a} \sum n_{b1} q_b$$

$$-\partial_{tt} n_{a1} = \frac{-\partial_{xx} p_{a1}}{m_a} + \frac{n_{a0} q_a}{\epsilon_0 m_a} \sum n_{b1} q_b$$

$m_a \quad q_a \quad a \in \{e, i\}$



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$$T_a = T_{a0} + T_{a1}$$
$$\rho_a = n_a q_a = \rho_{a0} + \rho_{a1}$$

$$-\partial_{tt} n_{a1} = \frac{-\partial_{xx} p_{a1}}{m_a} + \frac{n_{a0} q_a}{\epsilon_0 m_a} \sum n_{b1} q_b$$

$$\boxed{p_a n_a^{-\gamma_a} = \text{const}}$$

$$\gamma_a = 1 \leftrightarrow \text{ isothermal}$$
$$\gamma_a = 3 \leftrightarrow \text{ adiabatic}$$

$$\boxed{p_a = n_a k_\mathrm{B} T_a}$$

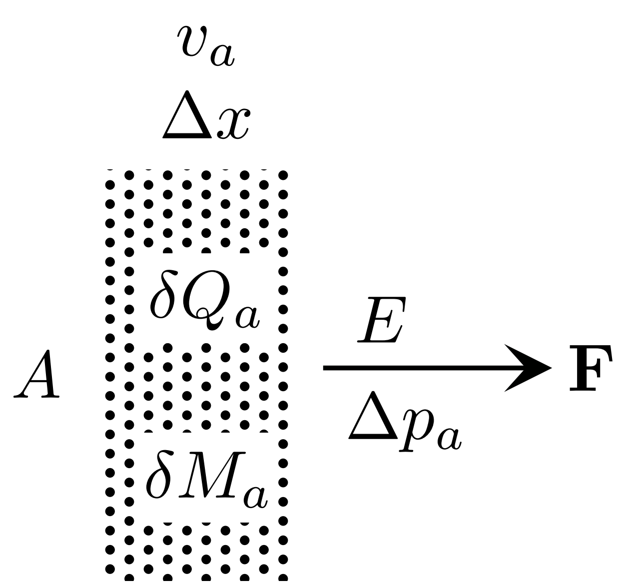
$$c_a^2 \equiv \frac{k_\mathrm{B} T_{a0}}{m_a}$$

$$\frac{-\partial_{xx} p_{a1}}{m_a} = -\gamma_a c_a^2 \, \partial_{xx} n_{a1}$$

$$\omega_a^2 \equiv \frac{n_{a0} q_a^2}{\epsilon_0 m_a}$$

$$-\partial_{tt} \rho_{a1} = -\gamma_a c_a^2 \, \partial_{xx} \rho_{a1} + \omega_a^2 \sum \rho_{b1}$$

$$m_a \quad q_a \quad a \in \{e, i\}$$



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$$p_a = p_{a0} + p_{a1}$$
$$T_a = T_{a0} + T_{a1}$$
$$\rho_a = n_a q_a = \rho_{a0} + \rho_{a1}$$

$$c_a^2 \equiv \frac{k_{\mathrm{B}} T_{a0}}{m_a}$$

$$\omega_a^2 \equiv \frac{q_a^2\, n_{a0}}{\epsilon_0\, m_a}$$

$$\lambda_a^2 \equiv \frac{c_a^2}{\omega_a^2} = \frac{\epsilon_0 k_{\mathrm{B}}\, T_a}{q_a^2\, n_a}$$

$$-\partial_{tt}\rho_{a1} = -\gamma_a c_a^2\, \partial_{xx}\rho_{a1} + \omega_a^2 \sum \rho_{b1}$$

$$\rho_{a1} = \rho_{a1}^0 \mathrm{e}^{i(\omega t - kx)}$$

$$\omega^2 \rho_{a1}^0 = k^2 \gamma_a c_a^2 \rho_{a1}^0 + \omega_a^2 \sum \rho_{b1}^0$$

$$\rho_{a1}^0 = \frac{\omega_a^2}{\omega^2 - \gamma_a k^2 c_a^2} \sum \rho_{b1}^0$$

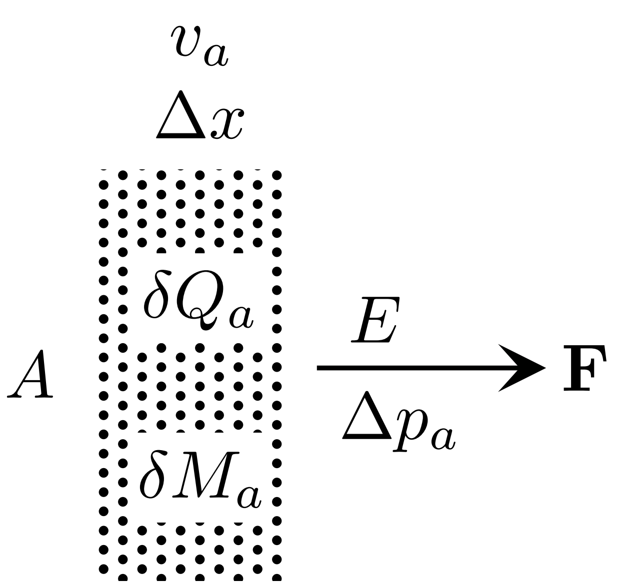
$$1 - \frac{\omega_e^2}{\omega^2 - \gamma_e k^2 c_e^2} - \frac{\omega_i^2}{\omega^2 - \gamma_i k^2 c_i^2} = 0$$

$$\omega = vk$$

$$k^2 - \frac{\omega_e^2}{v^2 - \gamma_e c_e^2} - \frac{\omega_i^2}{v^2 - \gamma_i c_i^2} = 0$$

Plasma dispersion relation

$m_a \quad q_a \quad a \in \{e, i\}$



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$$p_a = p_{a0} + p_{a1}$$
$$T_a = T_{a0} + T_{a1}$$
$$\rho_a = n_a q_a = \rho_{a0} + \rho_{a1}$$

$$c_a^2 \equiv \frac{k_\mathrm{B} T_{a0}}{m_a}$$

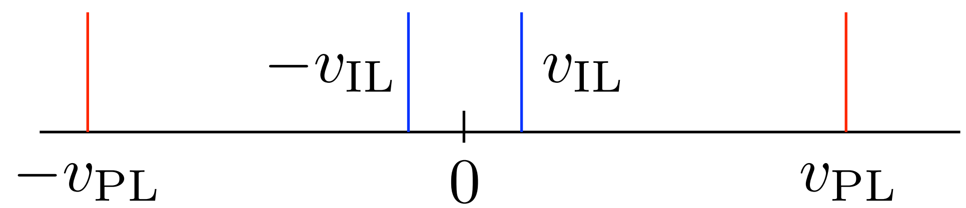
$$\omega_a^2 \equiv \frac{q_a^2\, n_{a0}}{\epsilon_0\, m_a}$$

$$\lambda_a^2 \equiv \frac{c_a^2}{\omega_a^2} = \frac{\epsilon_0 k_\mathrm{B}\, T_a}{q_a^2\, n_a}$$

$$k^2 - \frac{\omega_e^2}{v^2 - \gamma_e c_e^2} - \frac{\omega_i^2}{v^2 - \gamma_i c_i^2} = 0$$

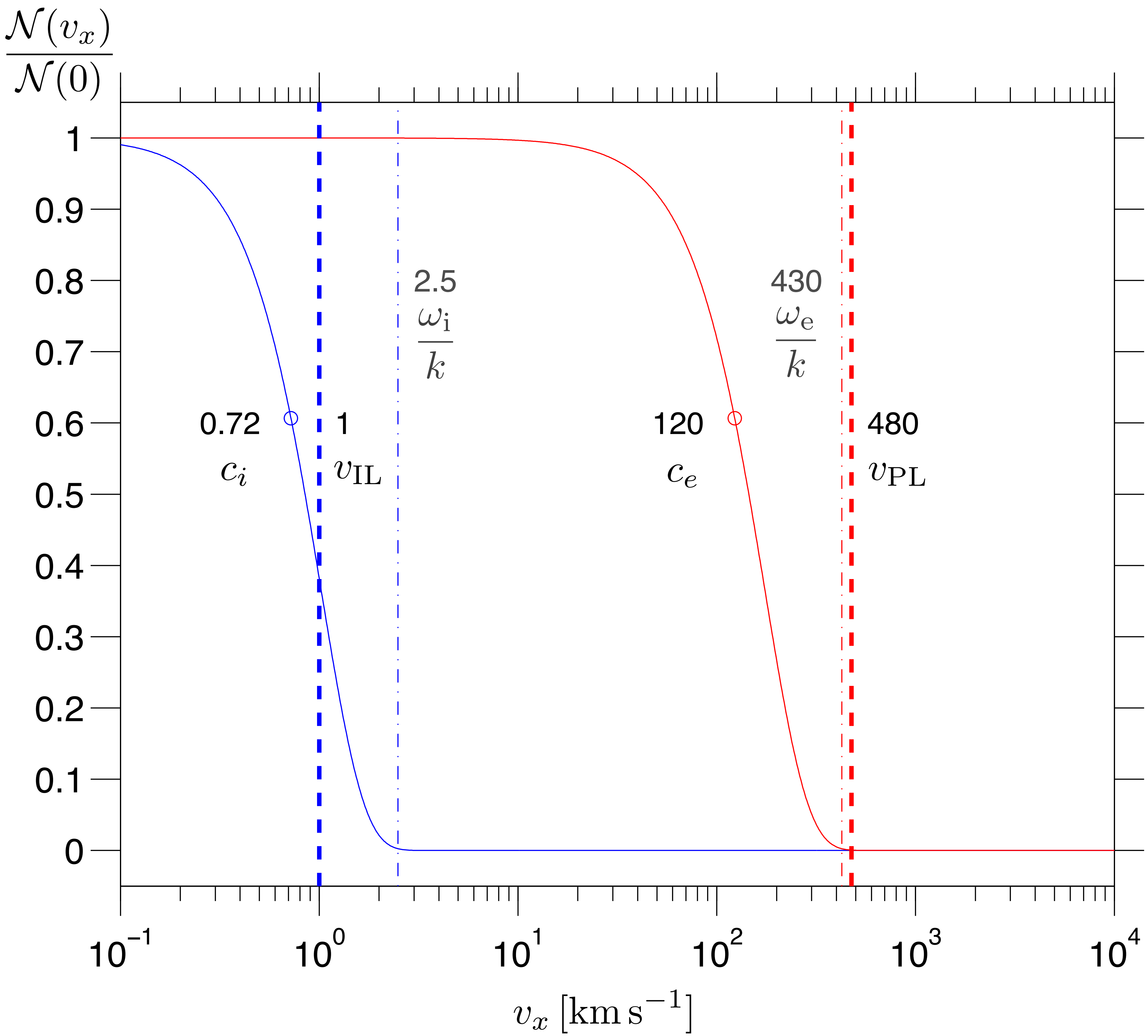
$$v^2 = (\dots) \pm \sqrt{(\dots)}$$

$$f = \frac{2k_\mathrm{rad}}{2\pi} \times v$$



2+2 solutions => Two wave modes:
Ion-acoustic => ion line
Electron-acoustic (Langmuir) => plasma line

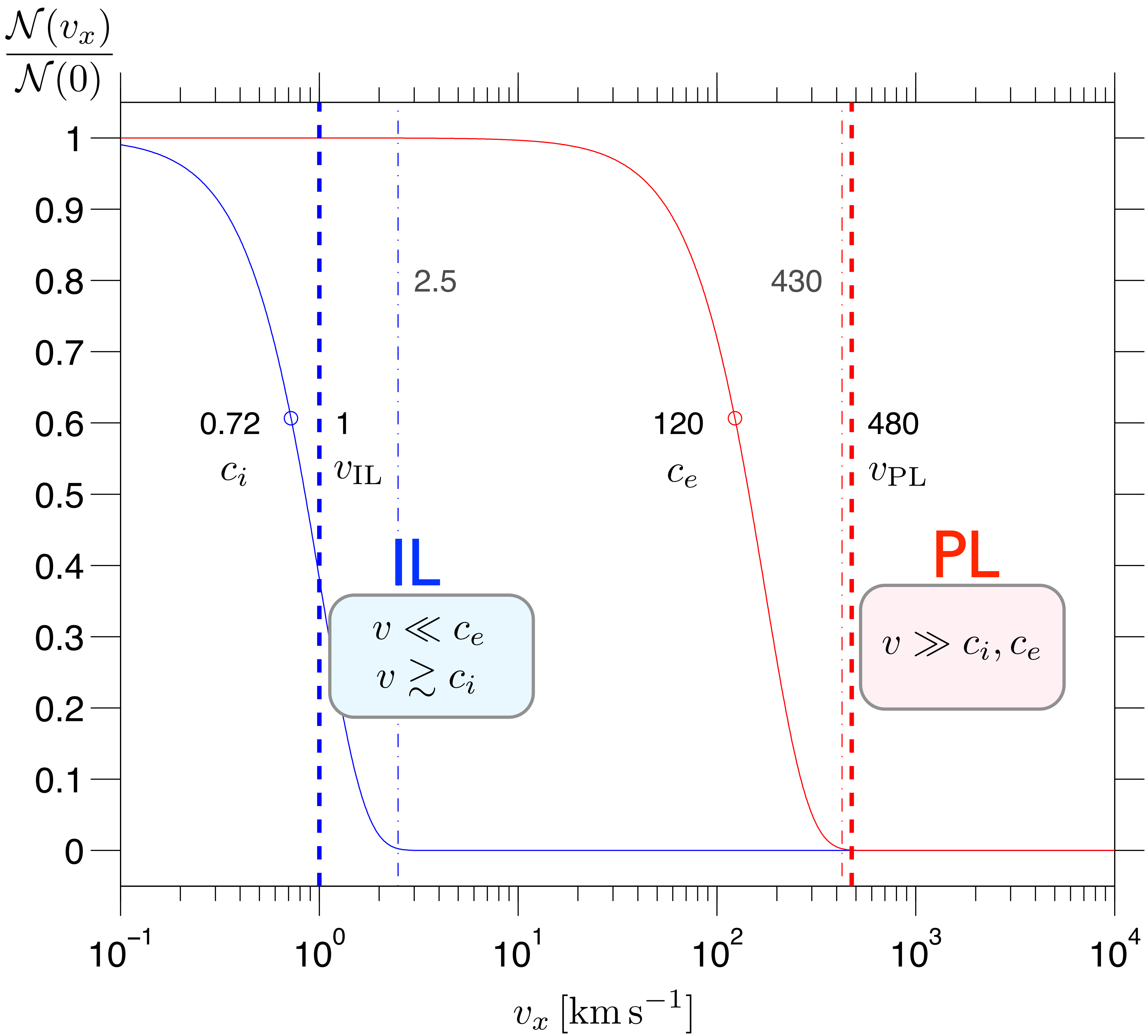
$N_e = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_e = 1000\,\mathrm{K} \quad T_i = 1000\,\mathrm{K} \quad m_i = 16\,\mathrm{amu}$



$$v^2 = (\dots) \pm \sqrt{(\dots)}$$

Based on a "typical" exact solution ....

$N_e = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_e = 1000\,\mathrm{K} \quad T_i = 1000\,\mathrm{K} \quad m_i = 16\,\mathrm{amu}$



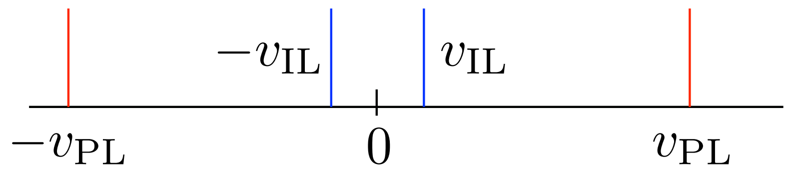
$$v^2 = (\ldots) \pm \sqrt{(\ldots)}$$

Based on a "typical" exact solution ....

approximations seem possible ...

Approximative solutions

$$k^2 - \frac{\omega_e^2}{v^2 - \gamma_e c_e^2} - \frac{\omega_i^2}{v^2 - \gamma_i c_i^2} = 0$$



IL

$$v \ll c_e$$
$$v \gtrsim c_i$$

$$\gamma_i = \gamma_e = 1 \ \text{(isothermal)}$$

$$\cancel{k^2} - \frac{\omega_e^2}{\cancel{v^2} - \gamma_e c_e^2} - \frac{\omega_i^2}{v^2 - \gamma_i c_i^2} = 0$$

$$v_{\mathrm{IL}}^2 \approx c_i^2 (1 + \frac{c_e^2}{c_i^2} \frac{\omega_i^2}{\omega_e^2})$$

$$= c_i^2 (1 + \frac{T_e}{T_i})$$

$$= \frac{k_{\mathrm{B}}(T_i + T_e)}{m_i}$$

PL

$$v \gg c_i, c_e$$

$$\gamma_i = \gamma_e = 3 \ \text{(adiabatic)}$$

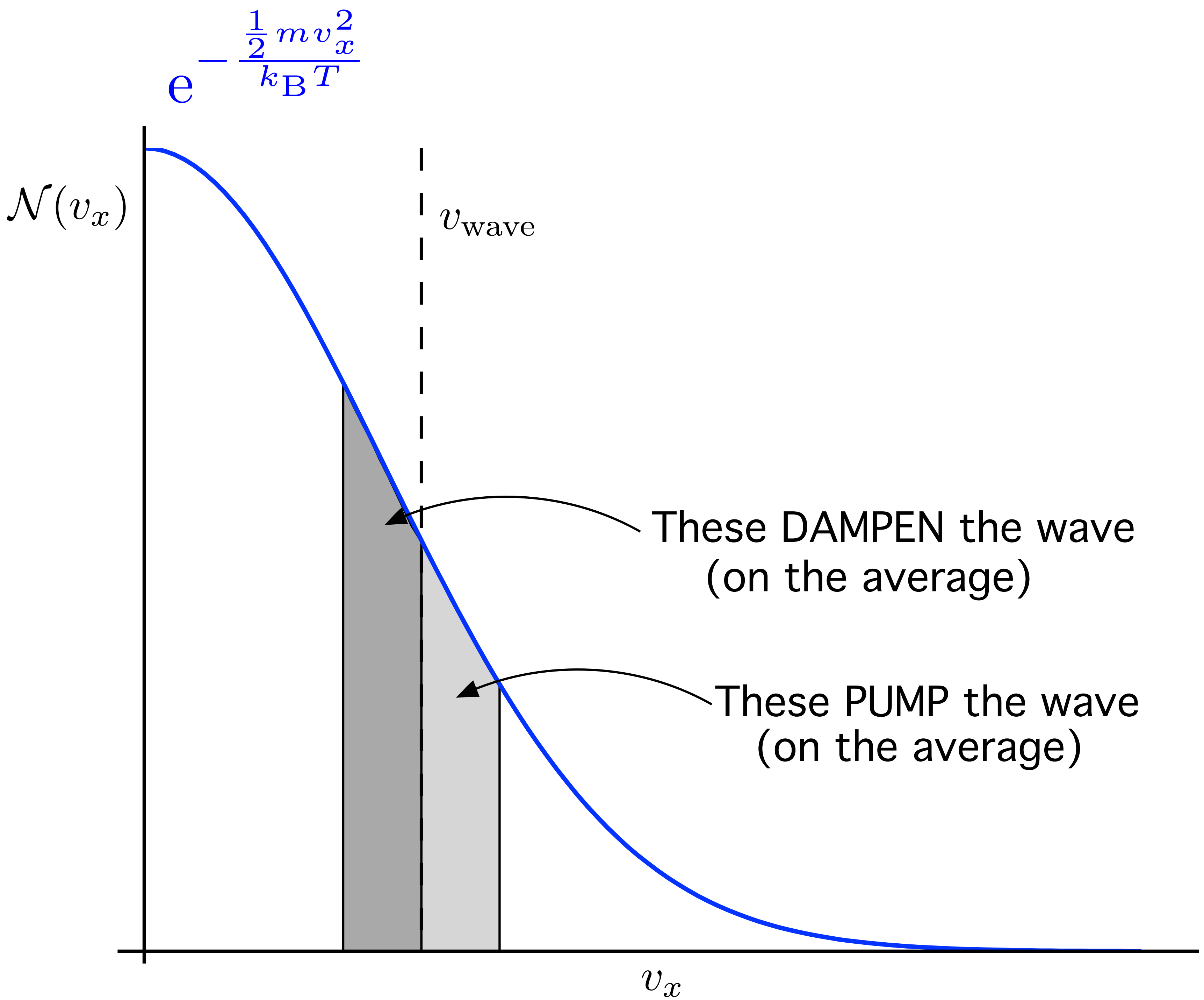
$$k^2 - \frac{\omega_e^2}{v^2 - \gamma_e c_e^2} - \cancel{\frac{\omega_i^2}{v^2 - \gamma_i c_i^2}} = 0$$

$$v_{\mathrm{PL}}^2 = \frac{\omega_e^2 + 3 c_e^2 k^2}{k^2}$$

$$= \frac{\omega_e^2}{k^2} (1 + 3 \lambda_e^2 k^2)$$

$$= \frac{1}{k^2} \frac{1}{m_e} (\frac{e^2 n_e}{\epsilon_0} + k^2 \, 3 k_{\mathrm{B}} T_{\mathrm{e}})$$

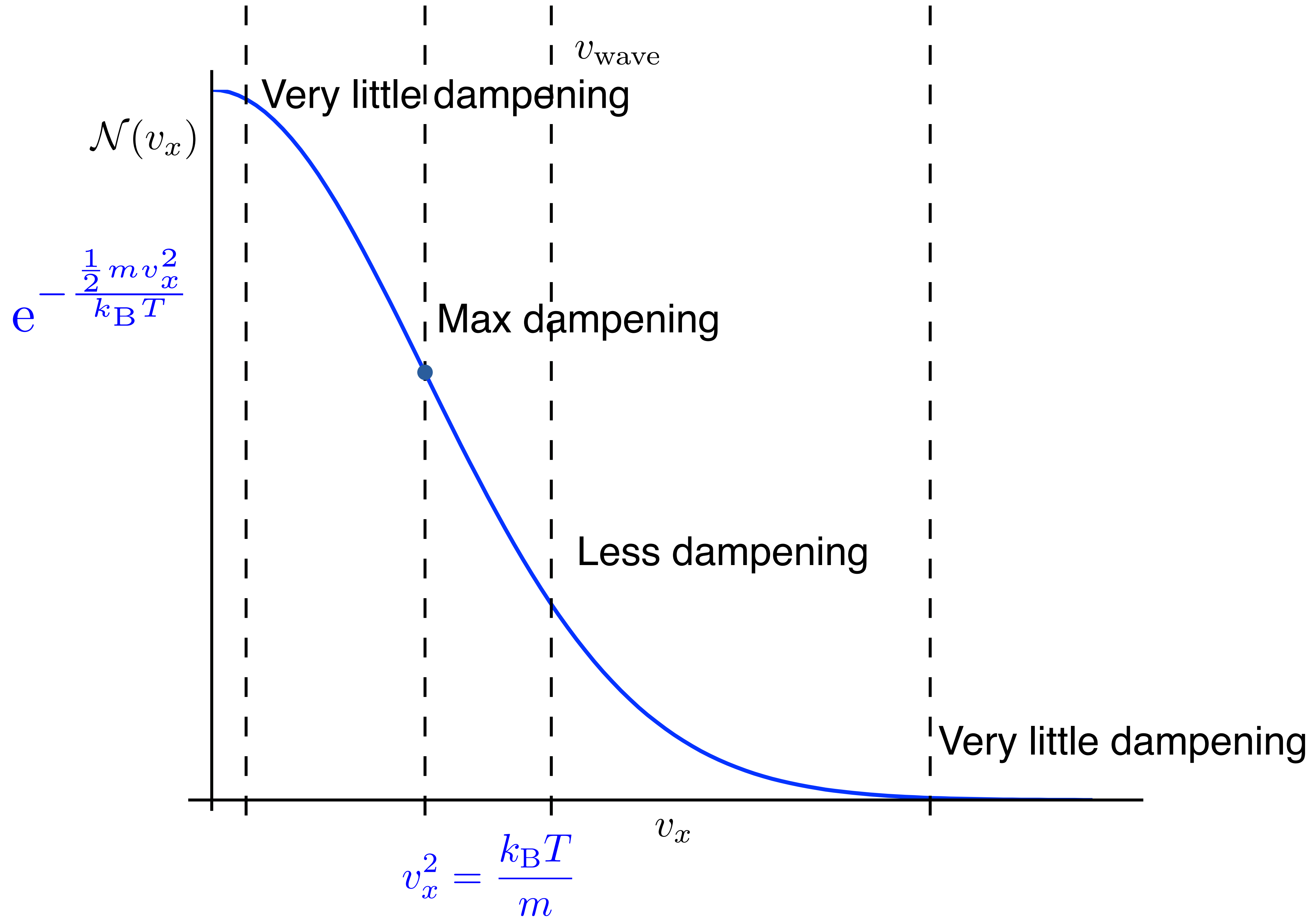
Broadening of the spectral lines due to collisionless (Landau-) damping of the plasma waves.



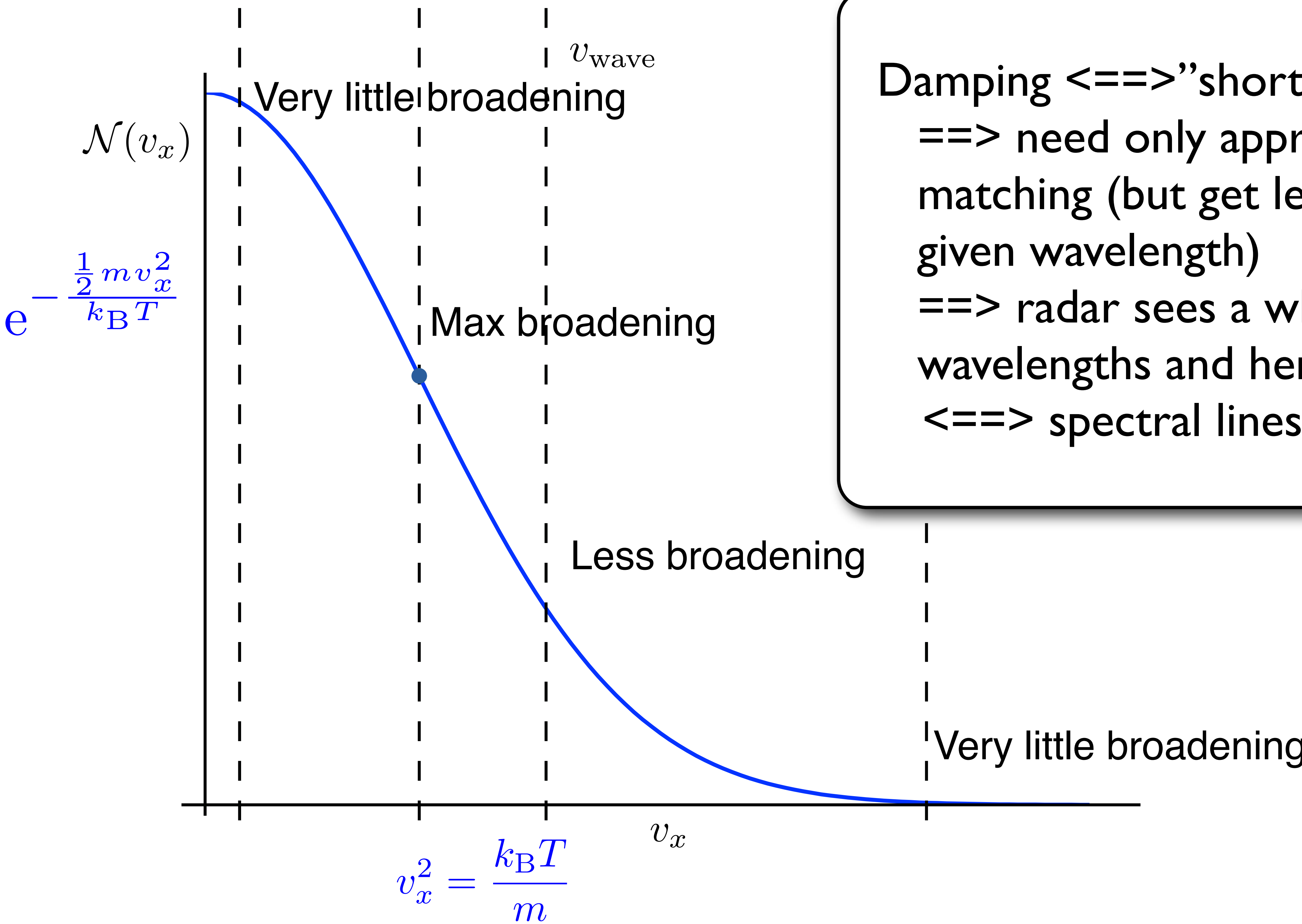
Net effect: DAMPEN

Broadening of the spectral lines due to collisionless (Landau-) damping of the plasma waves

$$\lambda_{\rm wave} = \frac{\lambda_{\rm rad}}{2}$$



Broadening of the spectral lines due to collisionless (Landau-) damping of the plasma waves

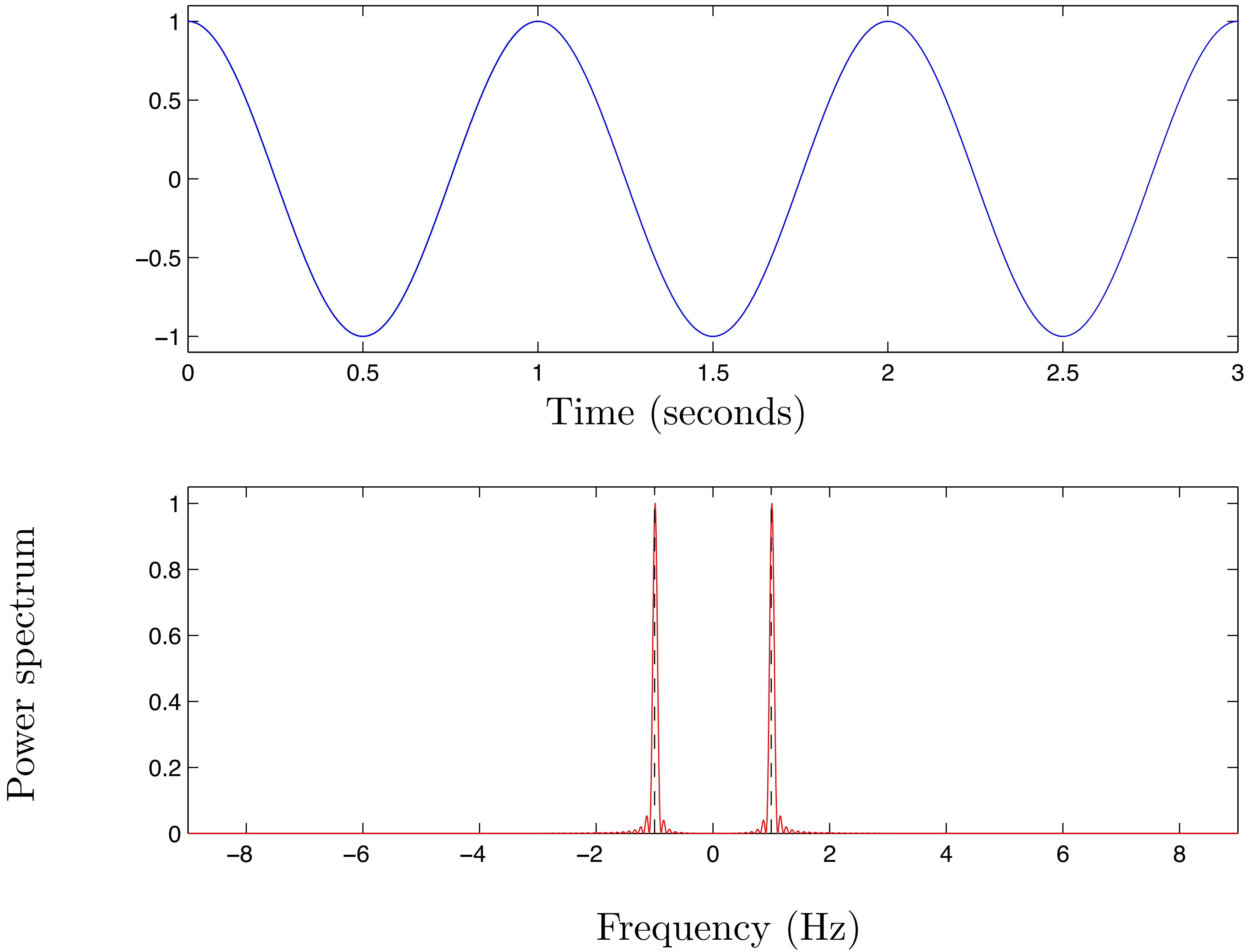


Damping <==>"short" fluctuations
==> need only approx. wavelength matching (but get less scatter at any given wavelength)
==> radar sees a whole bunch of wavelengths and hence frequencies
<==> spectral lines broaden

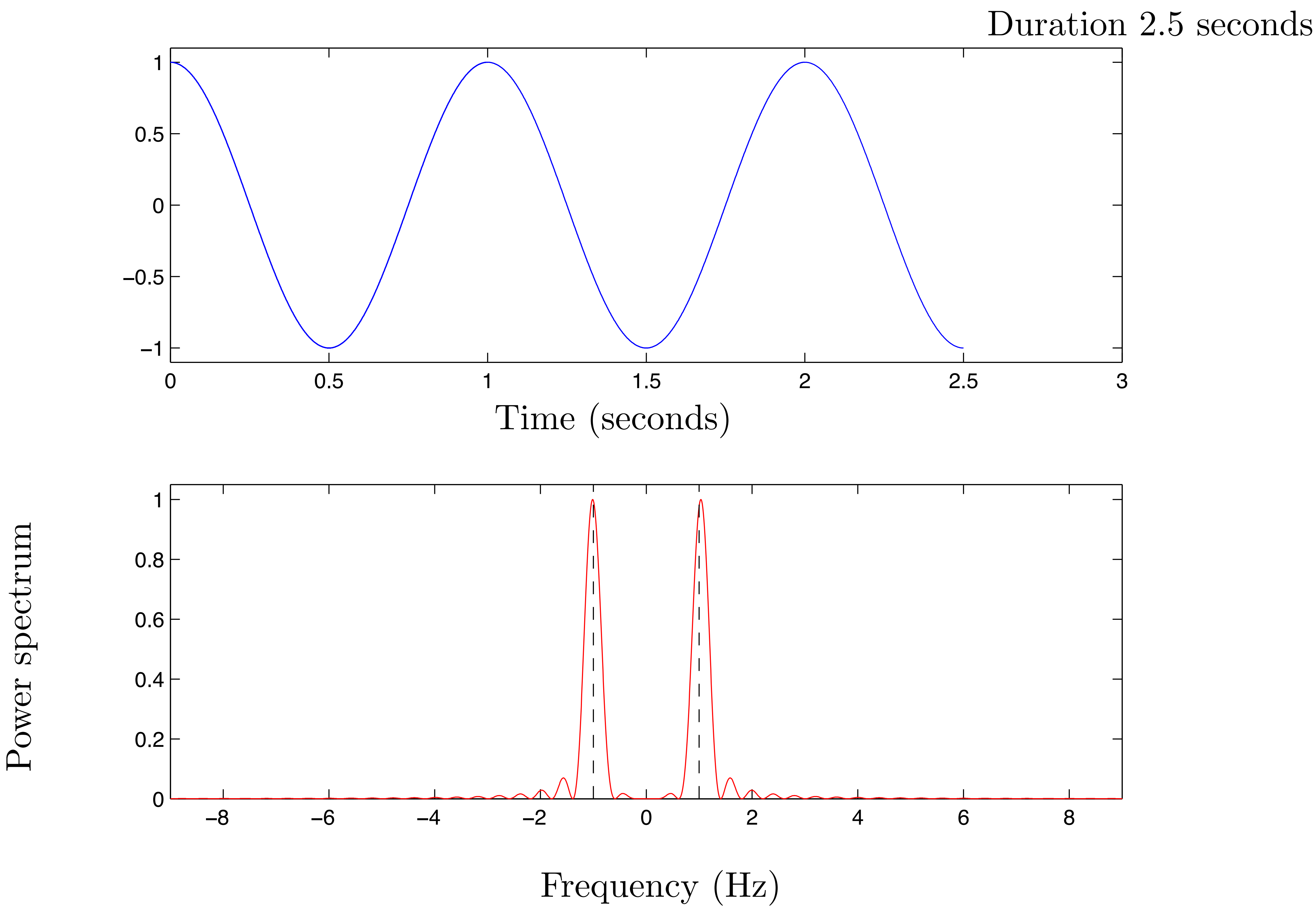
$$\lambda_{\text{wave}} \approx \frac{\lambda_{\text{rad}}}{2}$$

Simulating damping of a sinusoidal wave-train by truncation

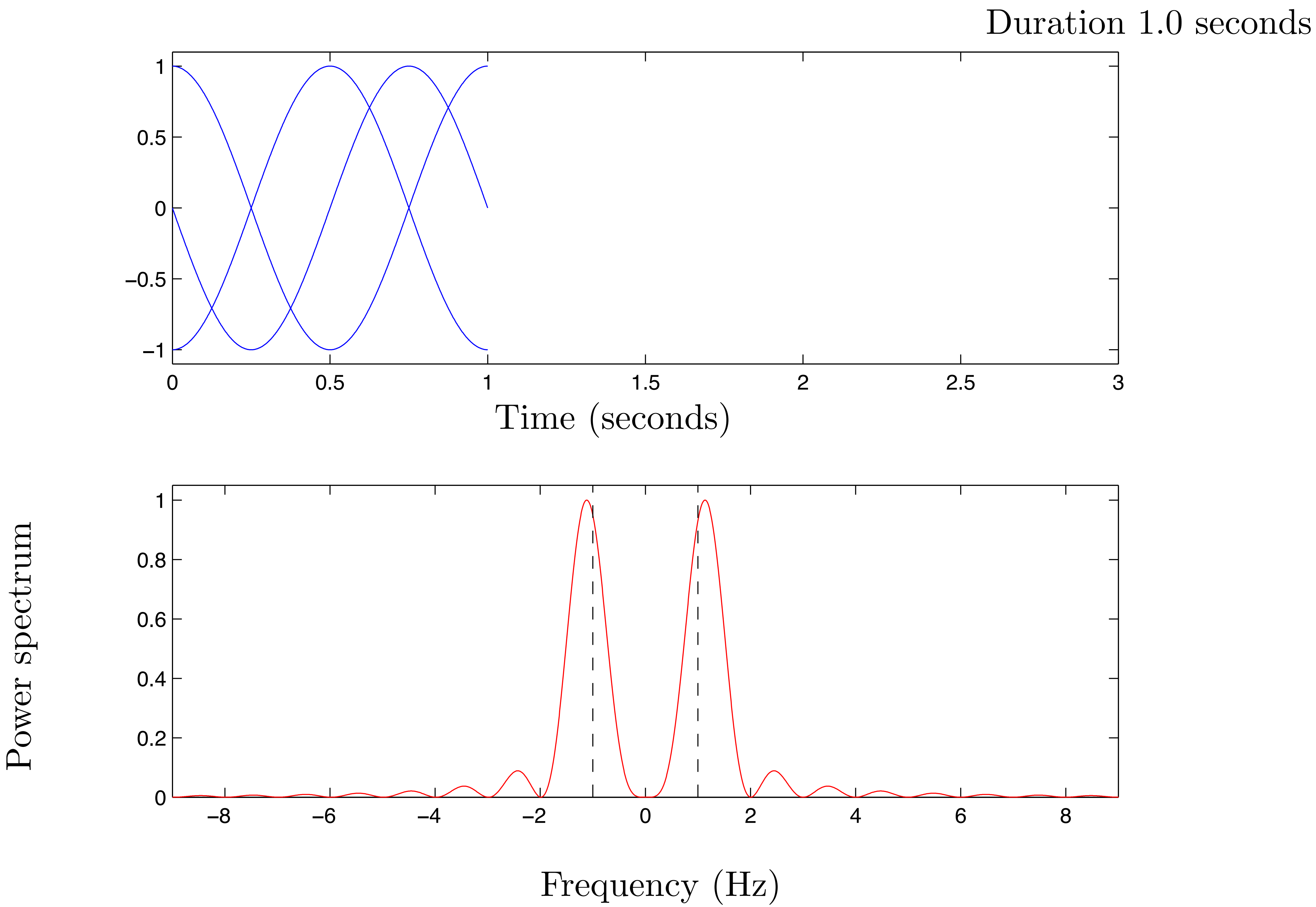
Duration 10 seconds



Simulating damping of a 1 Hz sinusoidal wave-train by truncation



Simulating damping of a 1 Hz sinusoidal wave-train by truncation



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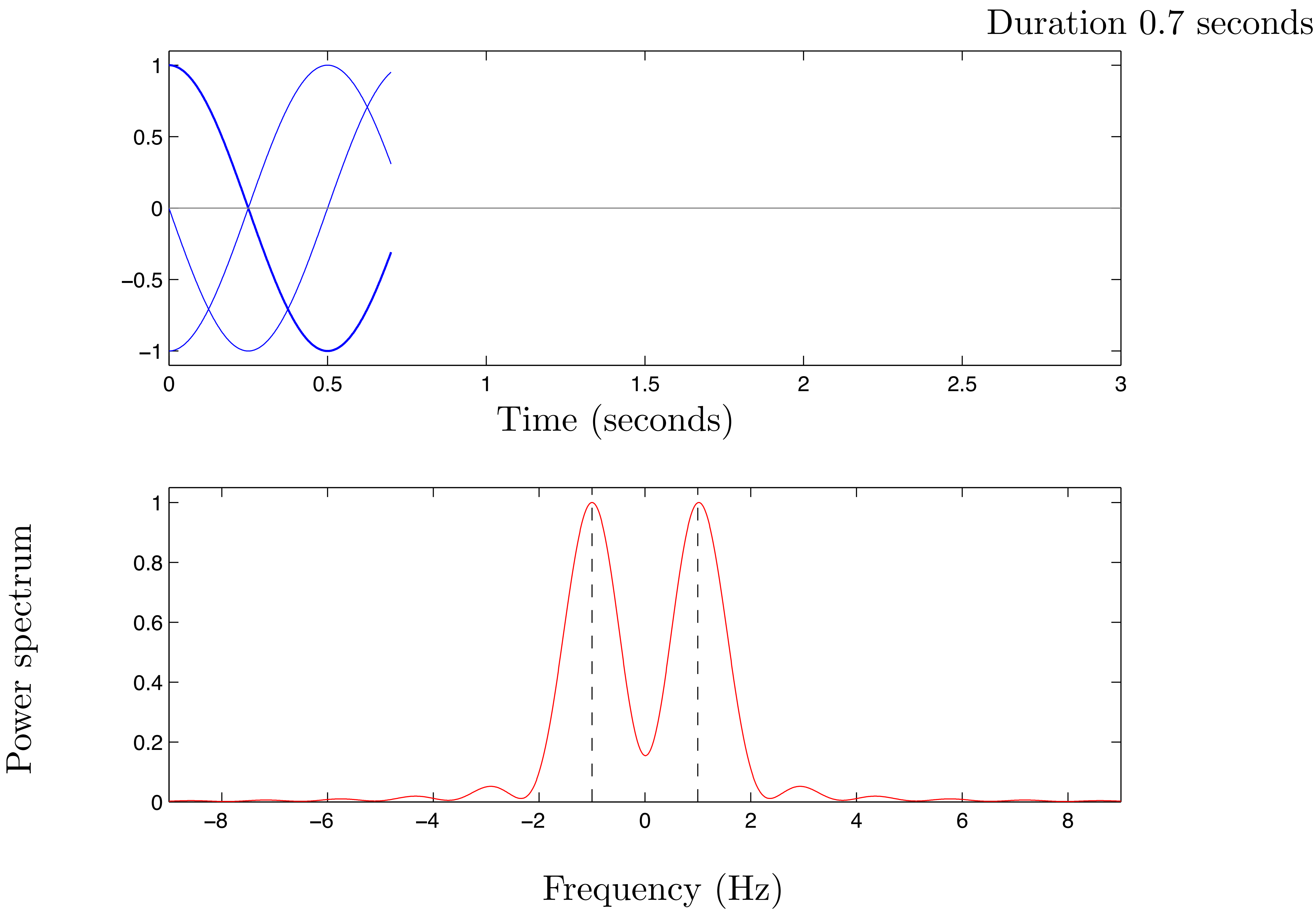

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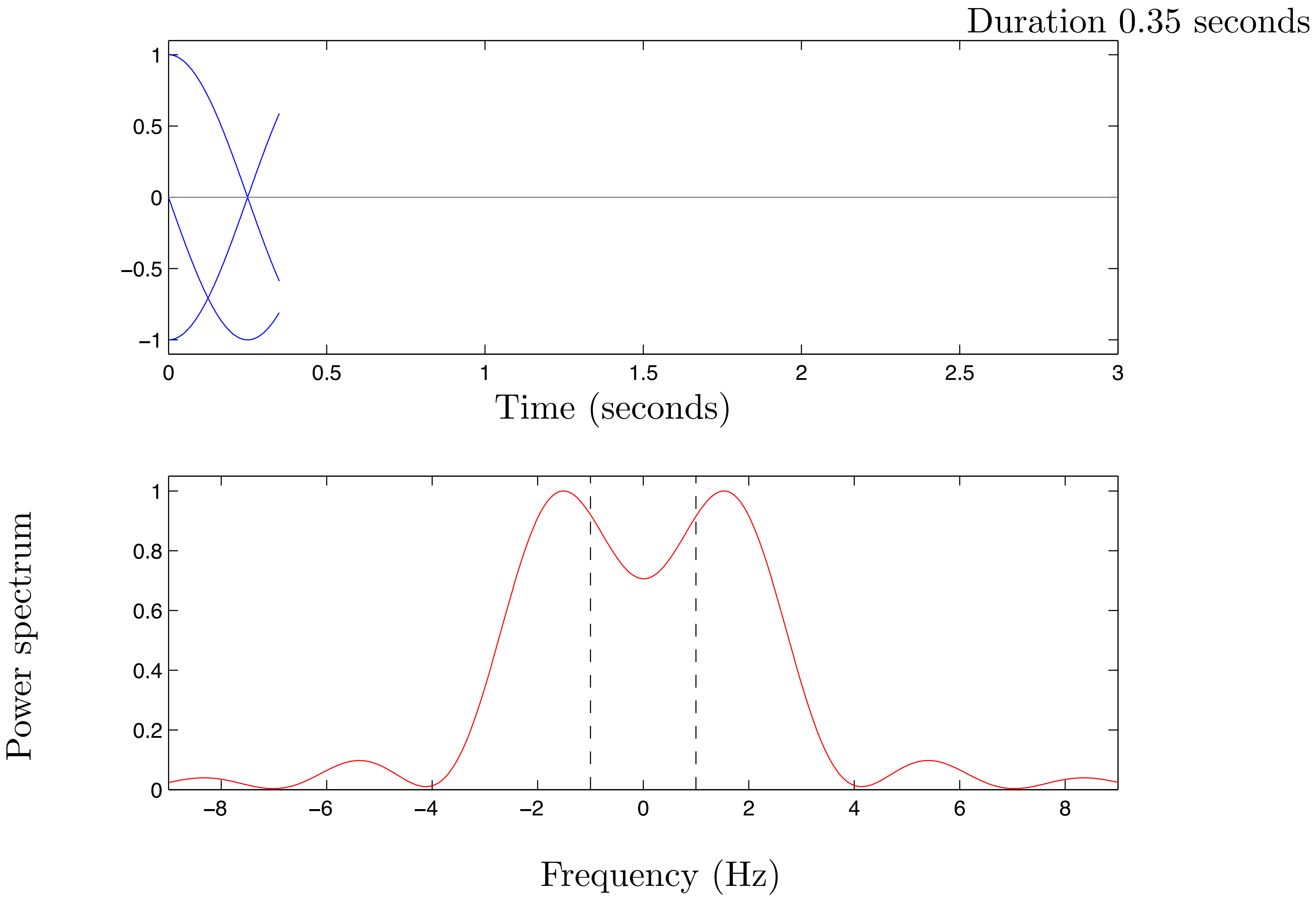

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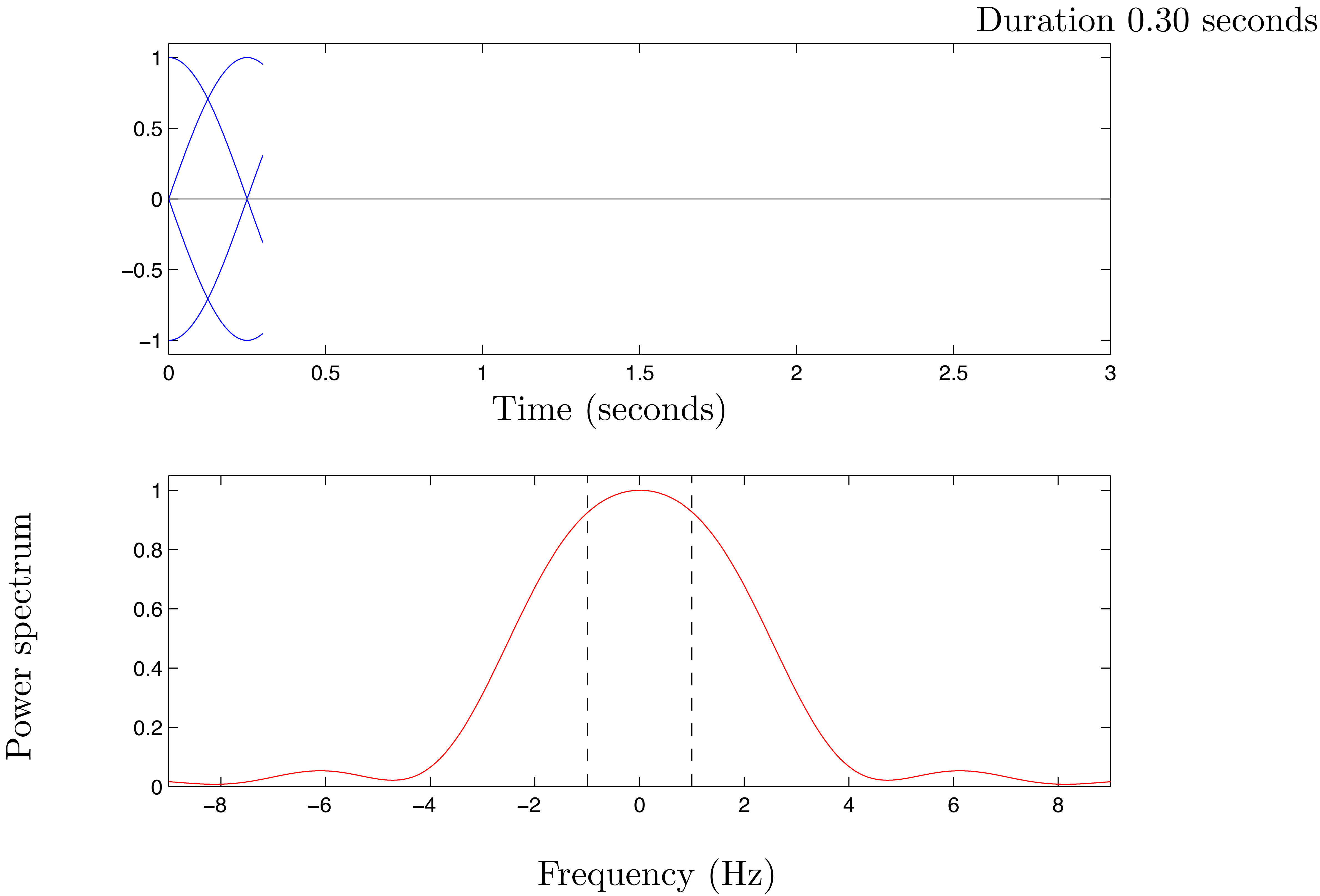
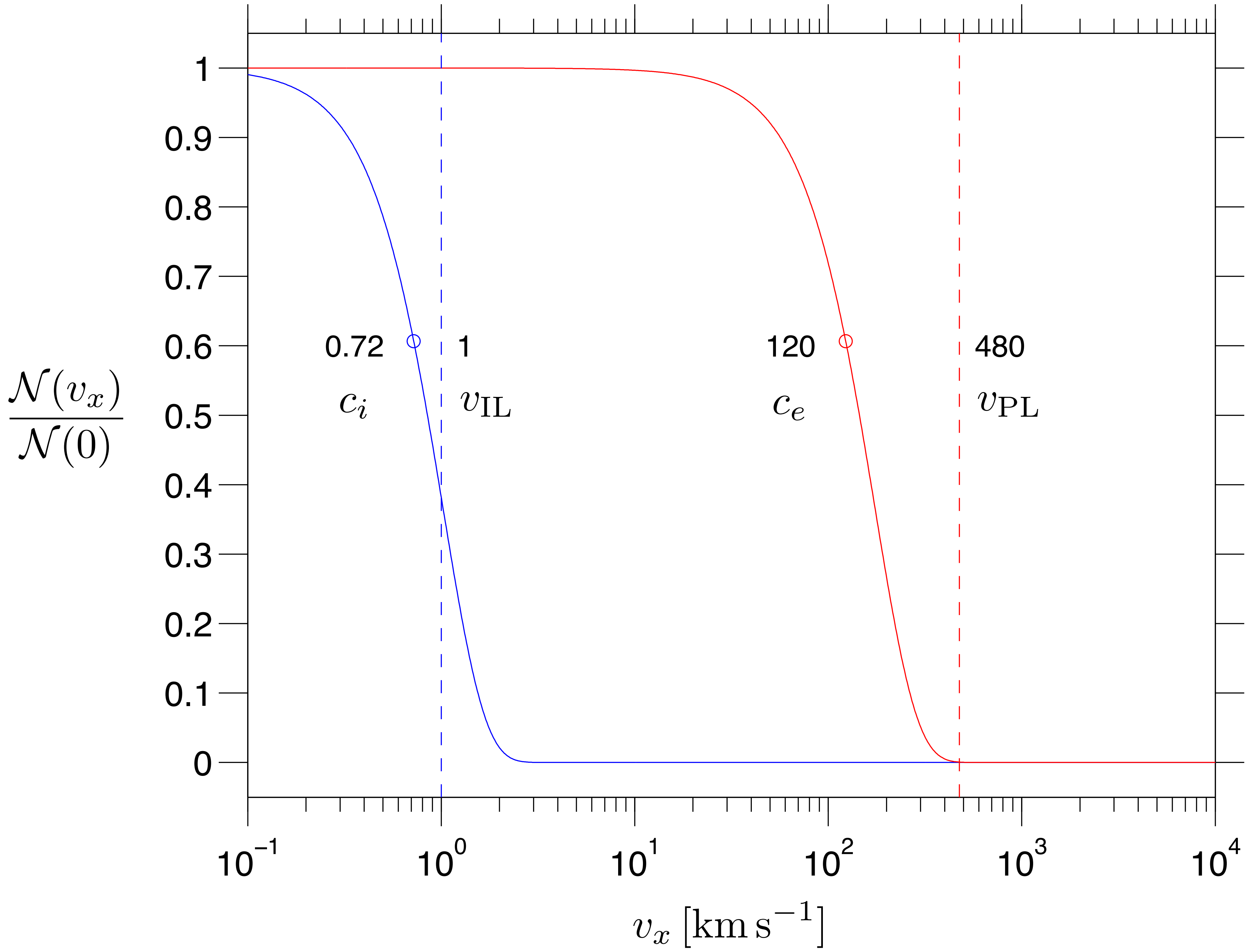


Remember: Wave velocity with respect to e- and ion- velocity distributions controls line broadening.

$$v_{\mathrm{IL}} \approx \left(\frac{k_{\mathrm{B}}(T_{\mathrm{i}} + T_{\mathrm{e}})}{m_{\mathrm{i}}} \right)^{\frac{1}{2}}$$

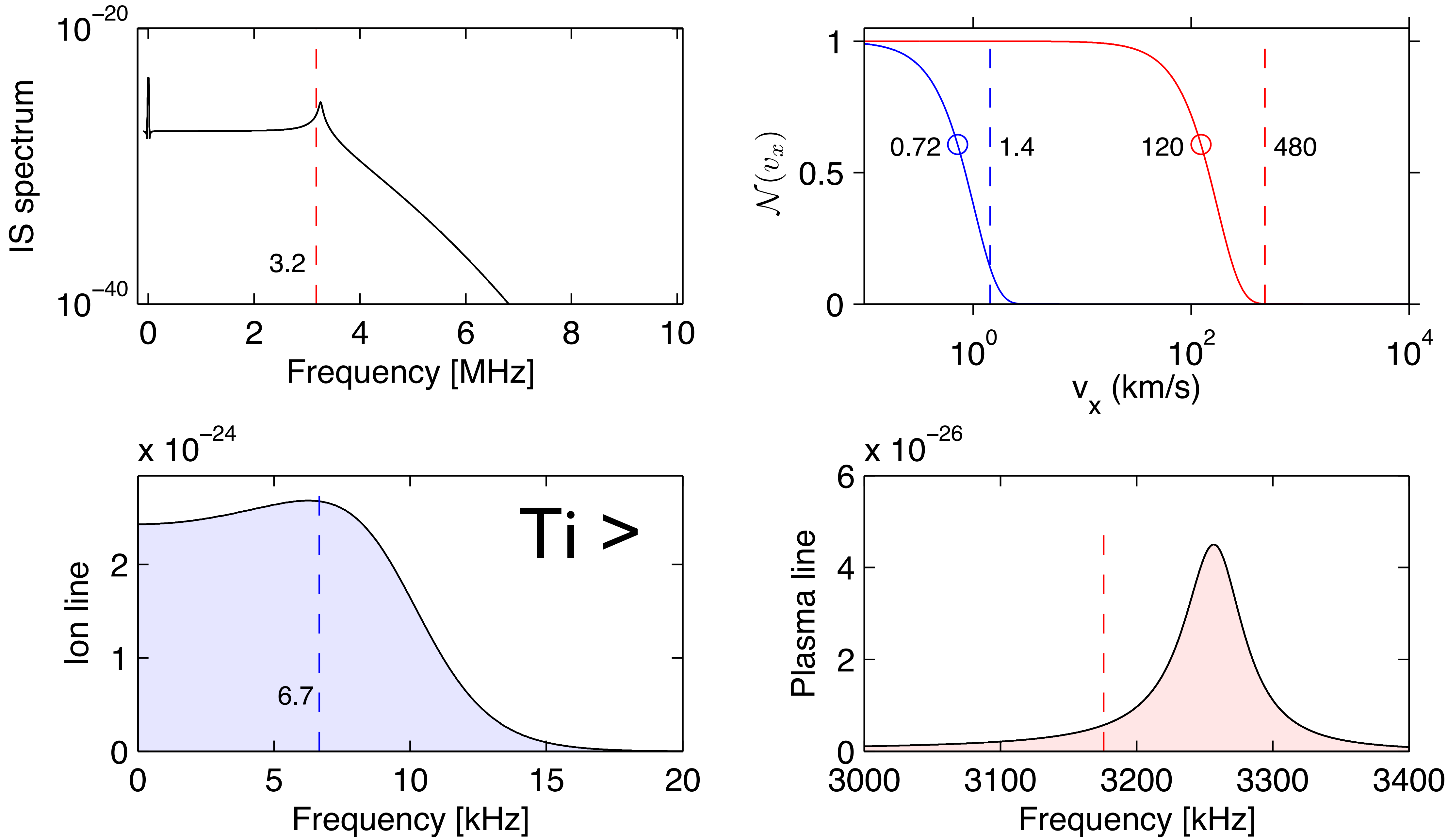
$$v_{\mathrm{PL}} = \frac{1}{k} \left(\frac{e^2 n_{\mathrm{e}}}{m_{\mathrm{e}}\epsilon_0} + k^2 \frac{3k_{\mathrm{B}}T_{\mathrm{e}}}{m_{\mathrm{e}}} \right)^{\frac{1}{2}}$$

$$c_a^2 \equiv \frac{k_{\mathrm{B}}T_a}{m_a}$$



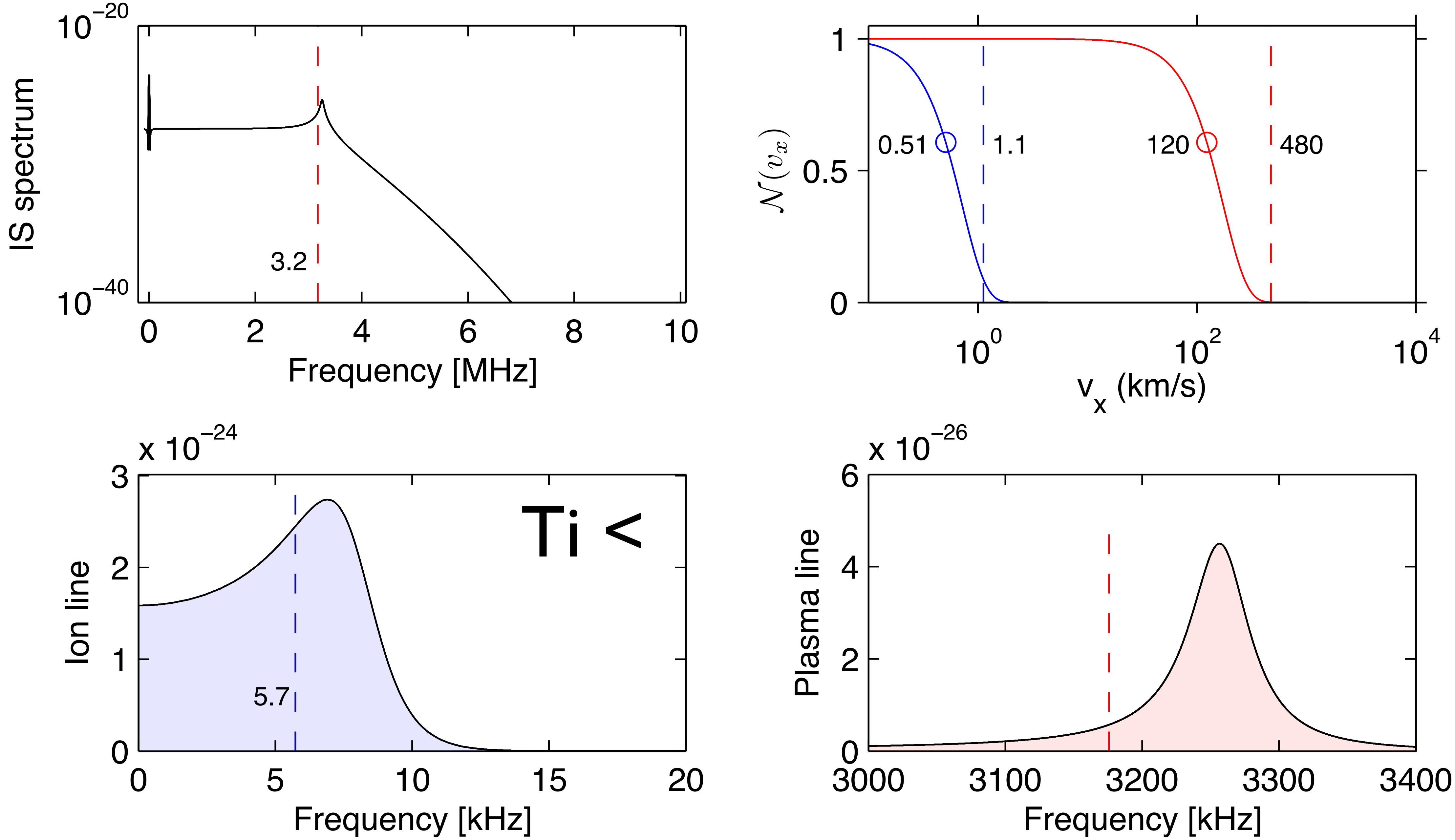
Example of an IS spectrum

$$N_{\mathrm{e}} = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_{\mathrm{e}} = 1000\,\mathrm{K} \quad T_{\mathrm{i}} = 1000\,\mathrm{K}$$



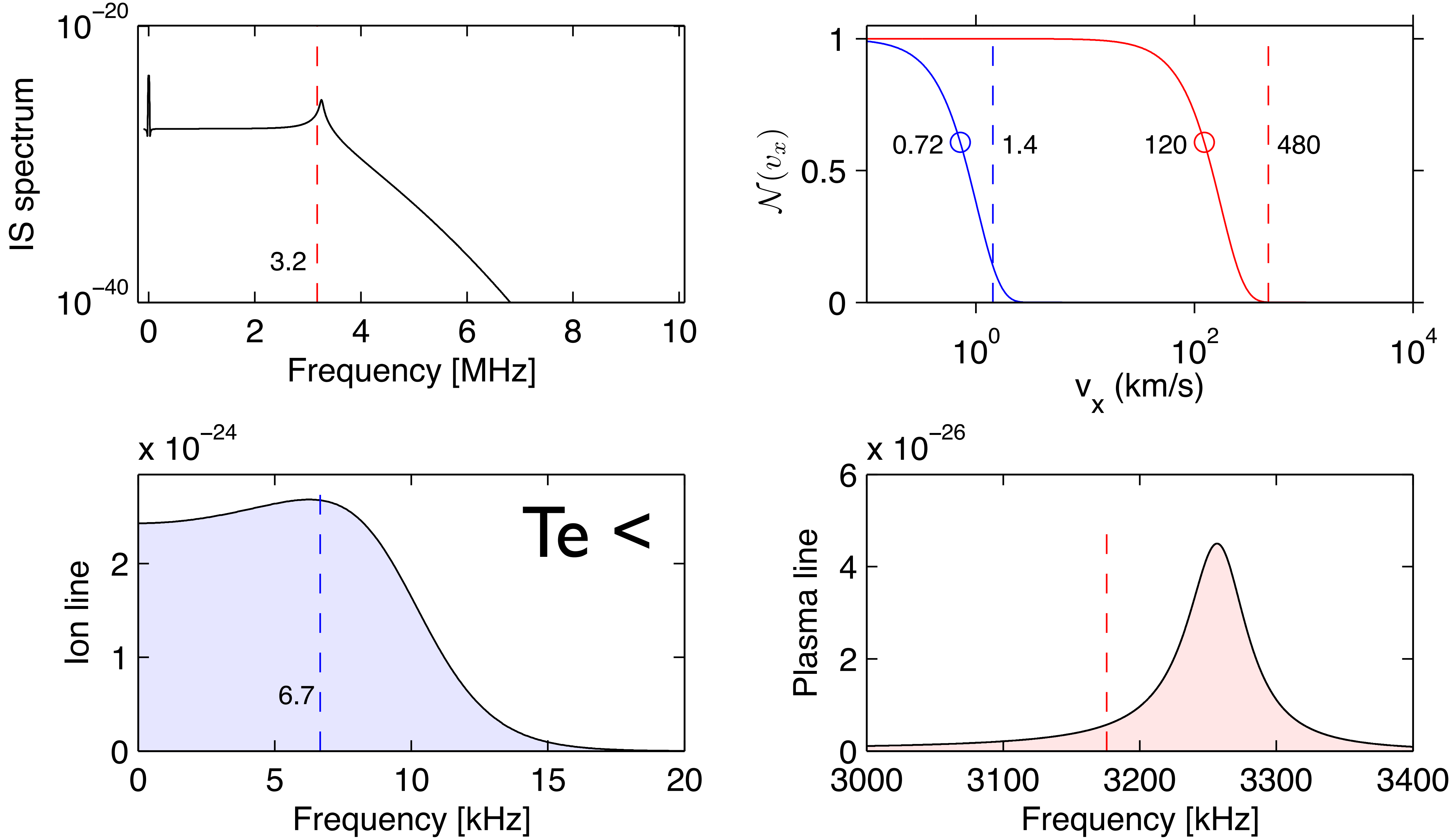
Example of an IS spectrum

$$N_e = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_e = 1000\,\mathrm{K} \quad T_i = 500\,\mathrm{K}$$



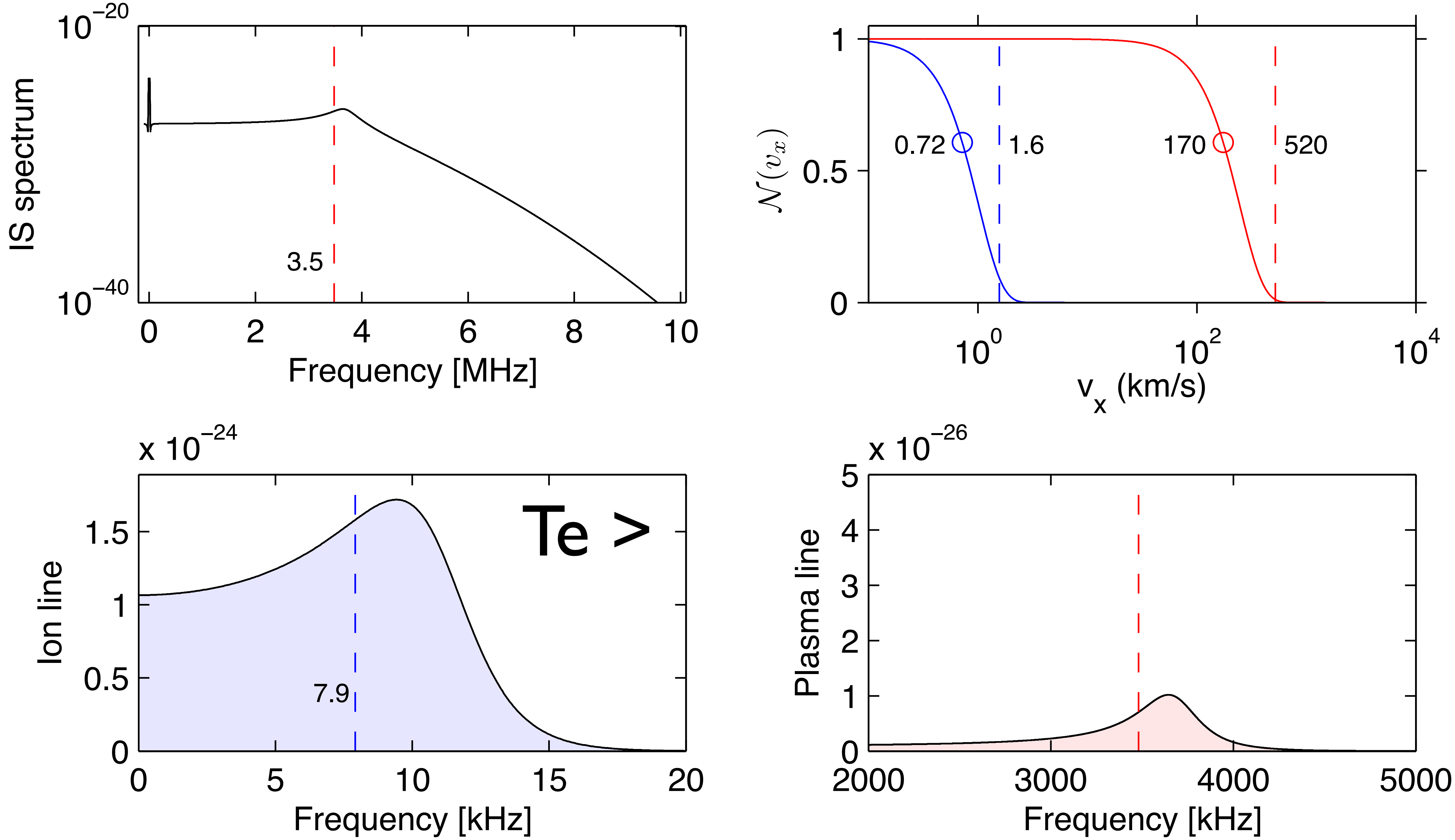
Example of an IS spectrum

$$N_\mathrm{e} = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_\mathrm{e} = 1000\,\mathrm{K} \quad T_\mathrm{i} = 1000\,\mathrm{K}$$



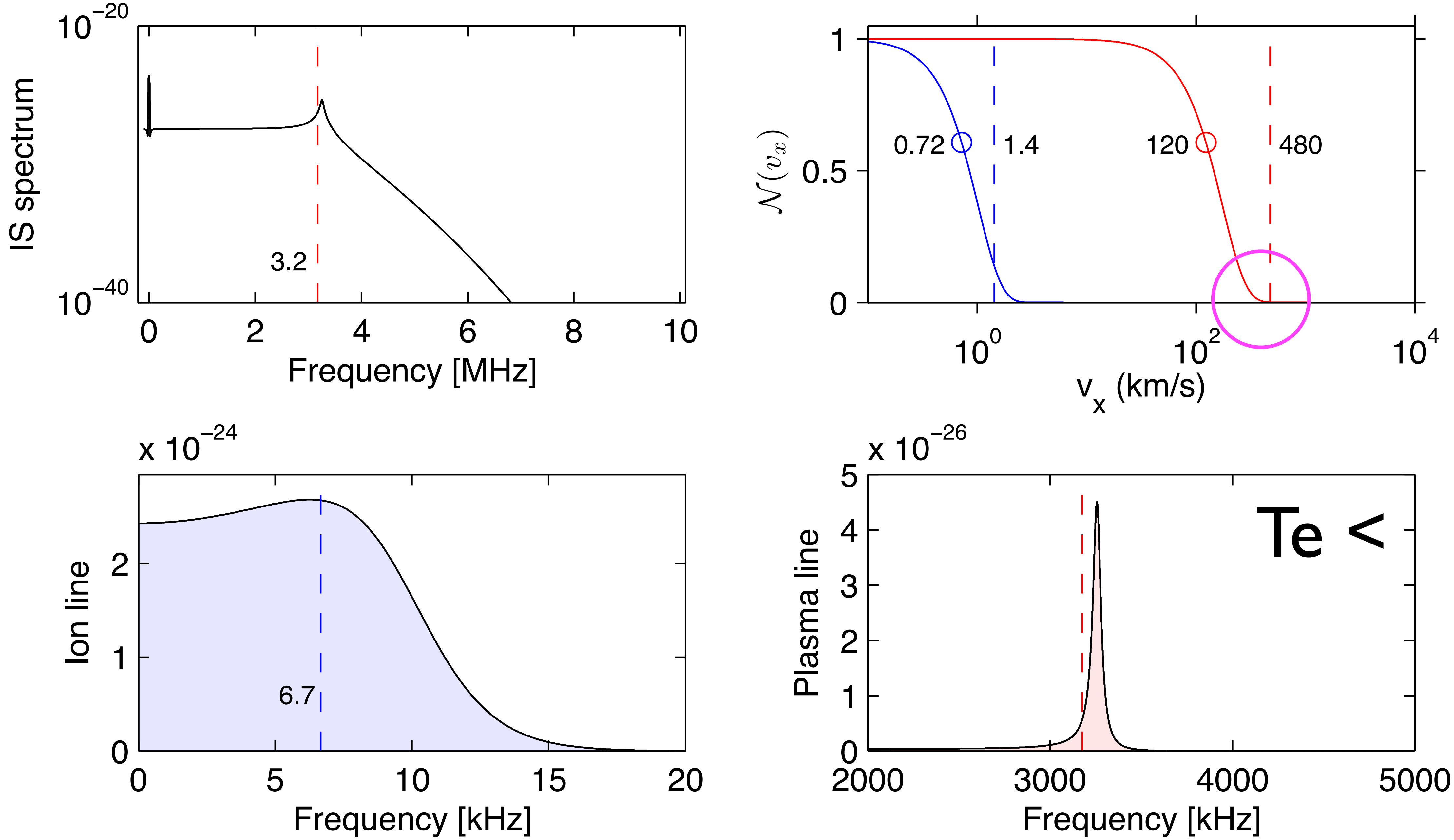
Example of an IS spectrum

$$N_e = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_e = 2000\,\mathrm{K} \quad T_i = 1000\,\mathrm{K}$$



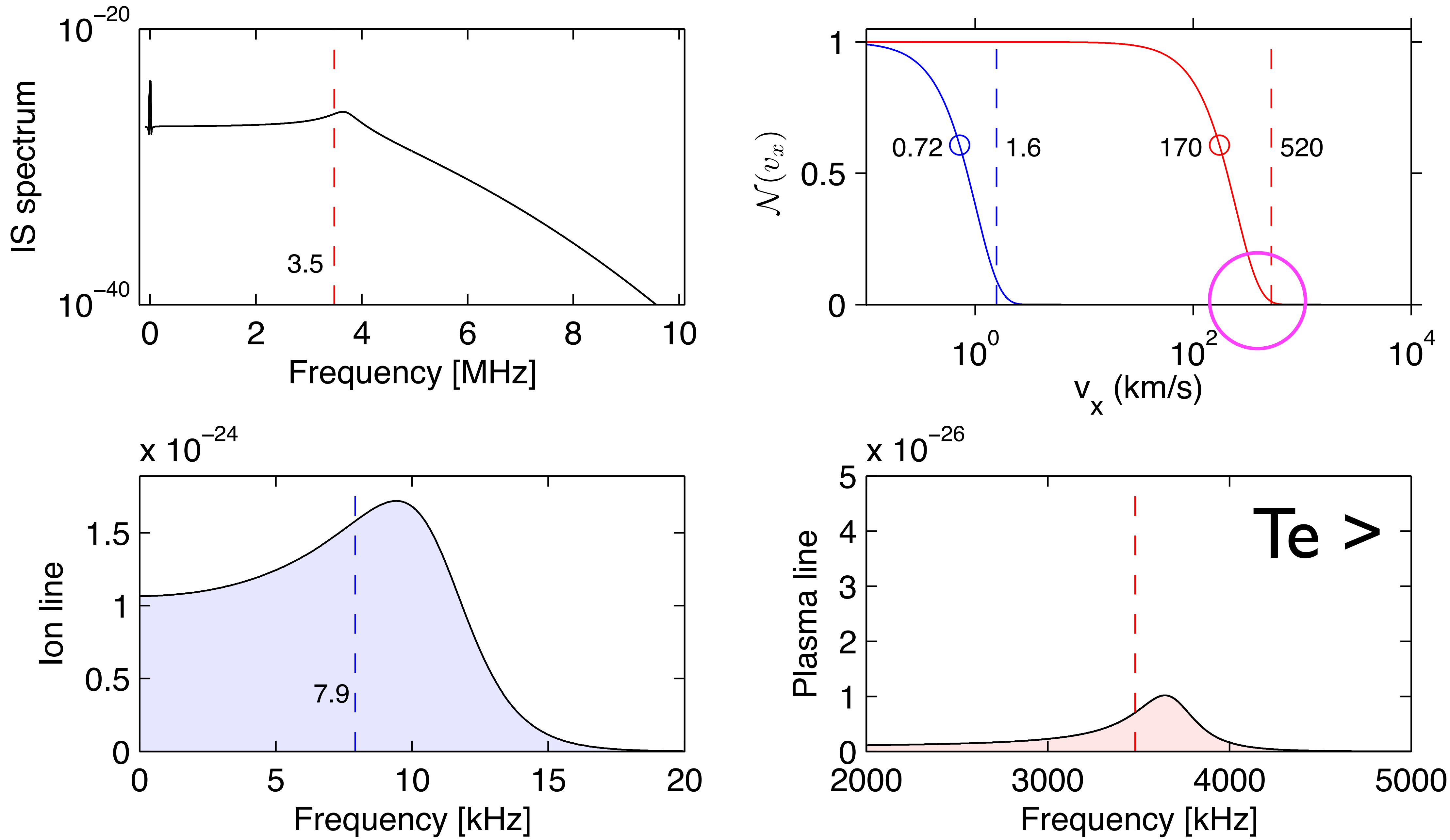
Example of an IS spectrum

$$N_\mathrm{e} = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_\mathrm{e} = 1000\,\mathrm{K} \quad T_\mathrm{i} = 1000\,\mathrm{K}$$



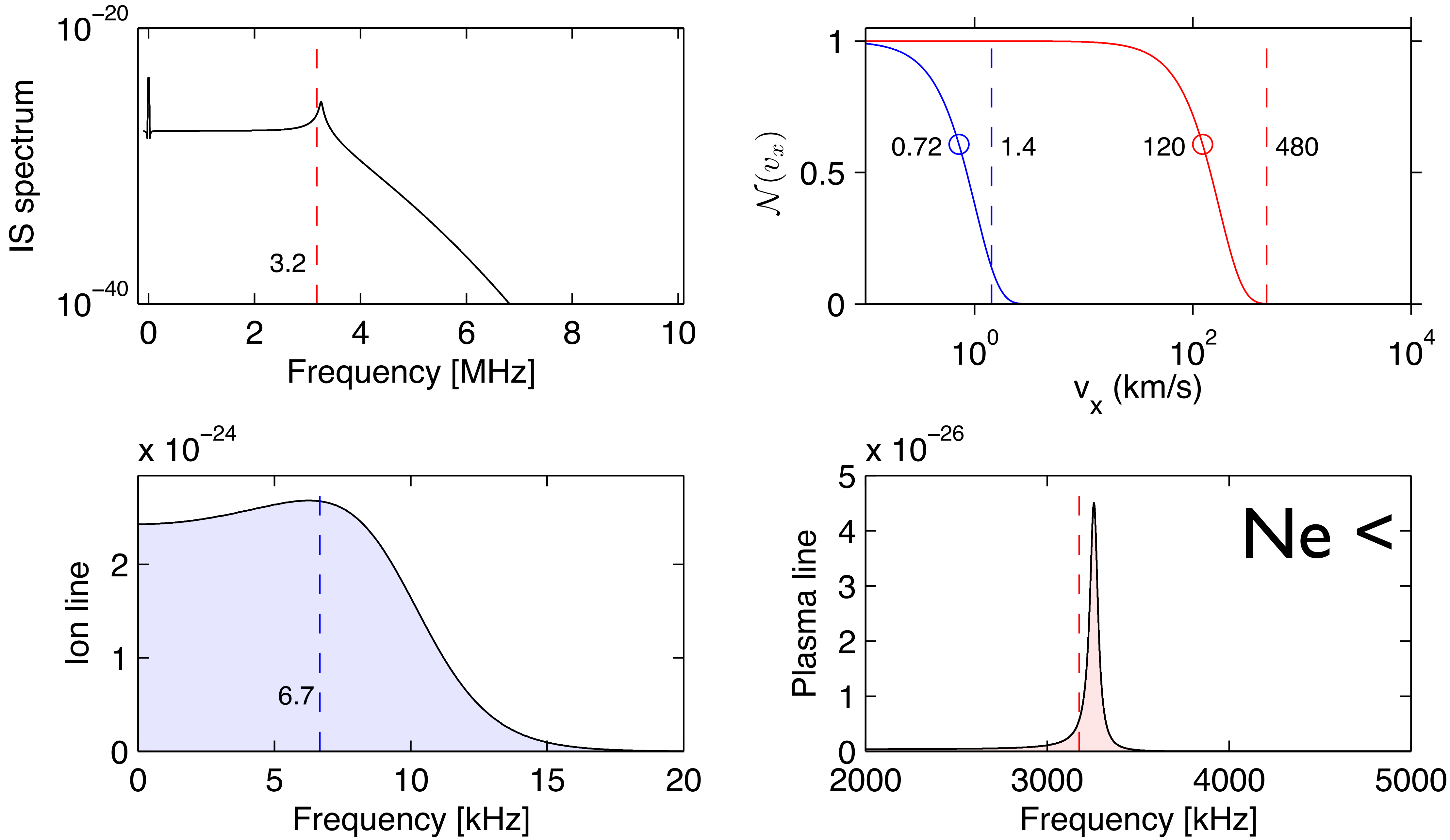
Example of an IS spectrum

$$N_{\mathrm{e}} = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_{\mathrm{e}} = 2000\,\mathrm{K} \quad T_{\mathrm{i}} = 1000\,\mathrm{K}$$



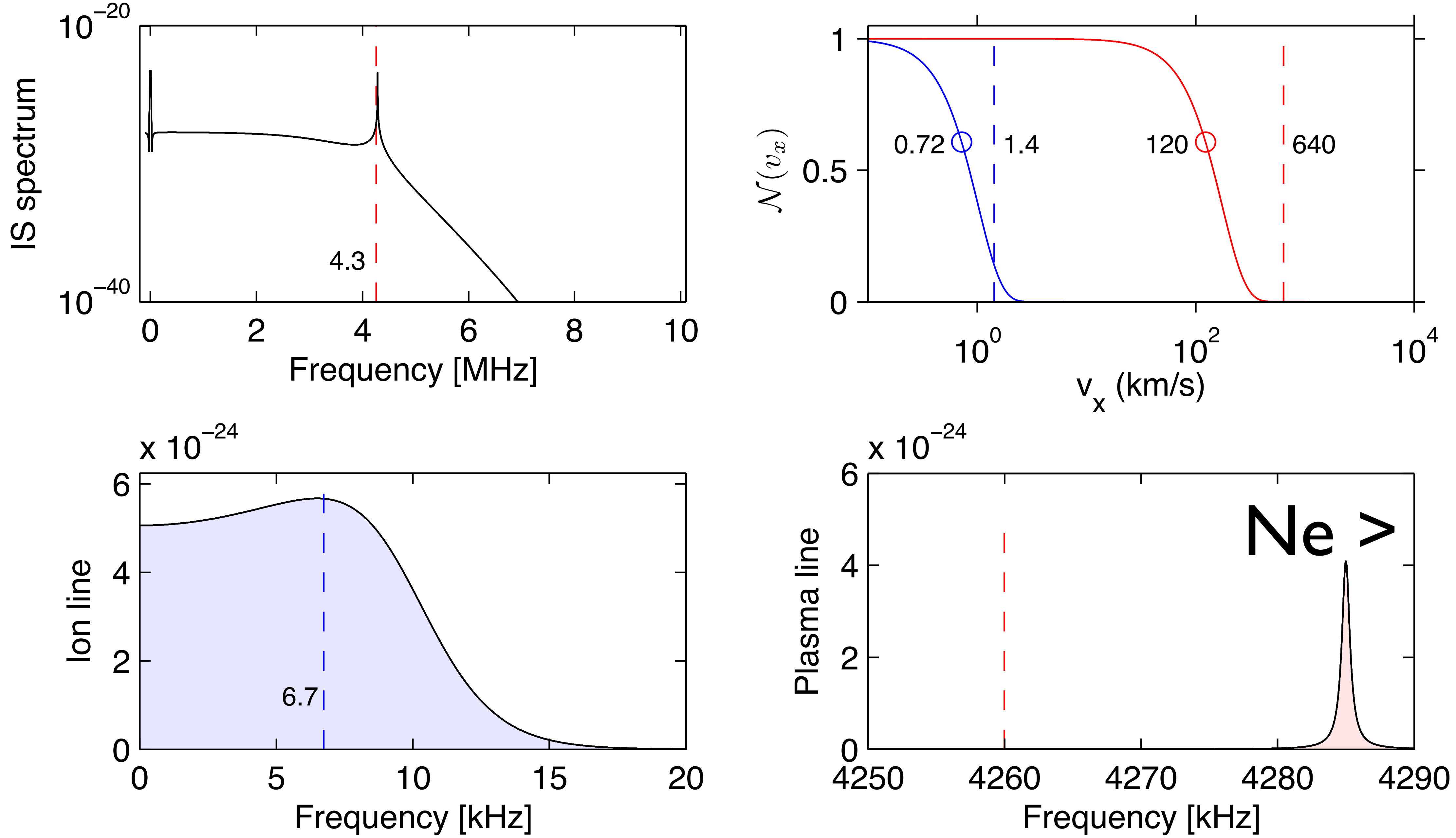
Example of an IS spectrum

$$N_\mathrm{e} = 1.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_\mathrm{e} = 1000\,\mathrm{K} \quad T_\mathrm{i} = 1000\,\mathrm{K}$$



Example of an IS spectrum

$$N_e = 2.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_e = 1000\,\mathrm{K} \quad T_i = 1000\,\mathrm{K}$$



Example of an IS spectrum

$$N_\mathrm{e} = 5.0 \times 10^{11}\,\mathrm{m}^{-3} \quad T_\mathrm{e} = 1000\,\mathrm{K} \quad T_\mathrm{i} = 1000\,\mathrm{K}$$