New high accuracy determination of range and range rate of satellites from EISCAT radar data taken during 2010 SSA CO-VI campaign

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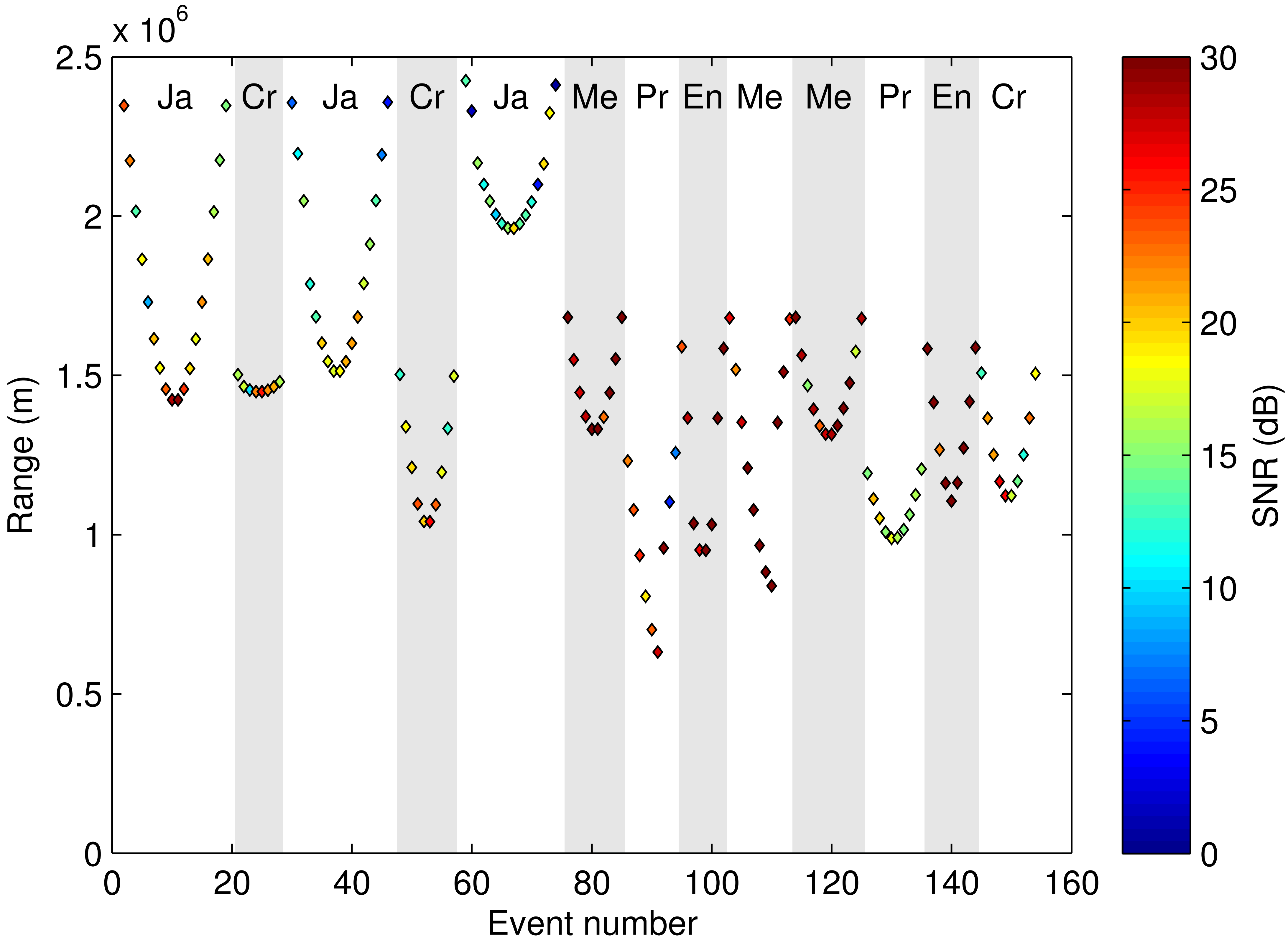
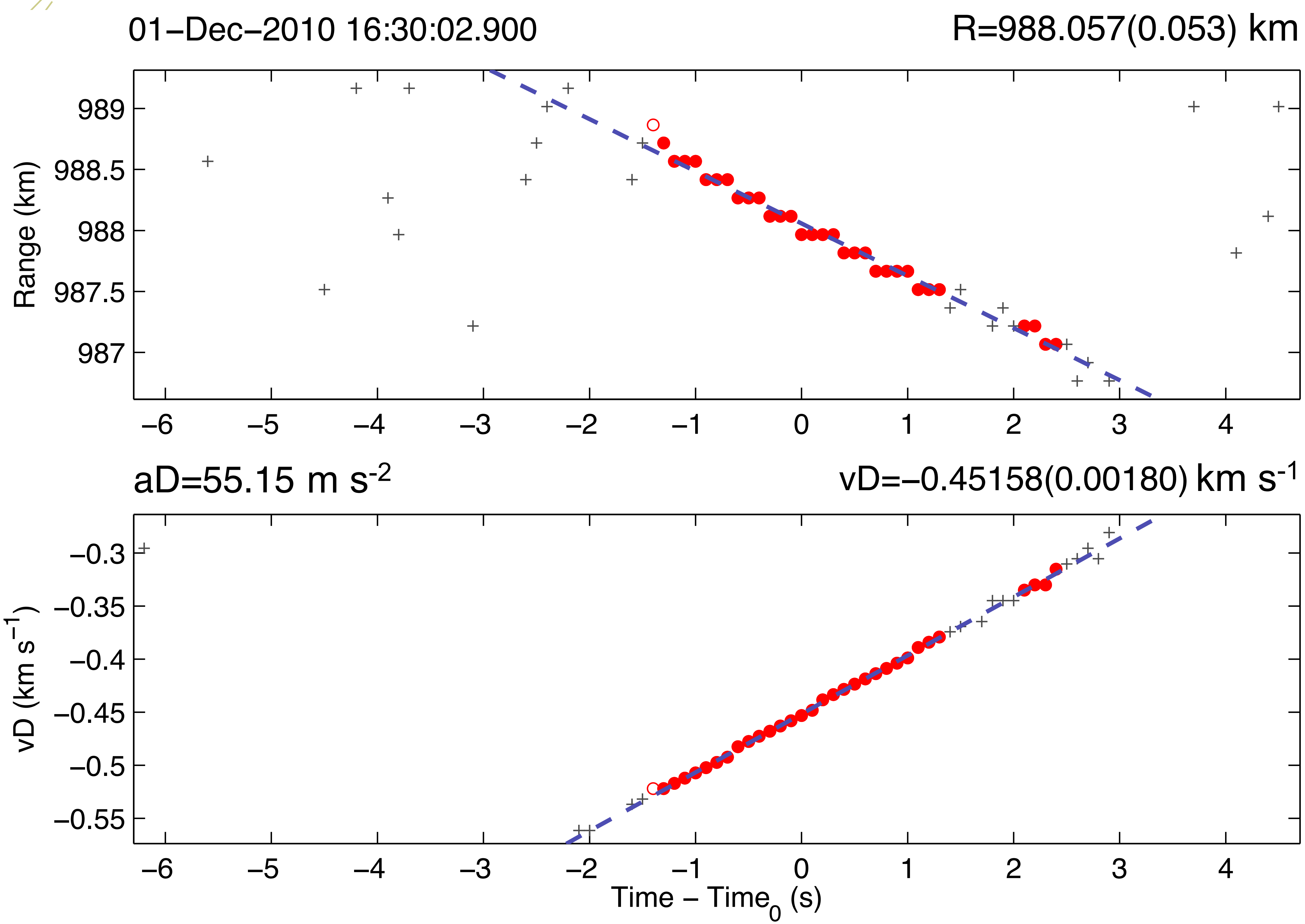
EISCAT range measurements 1-Dec-2010

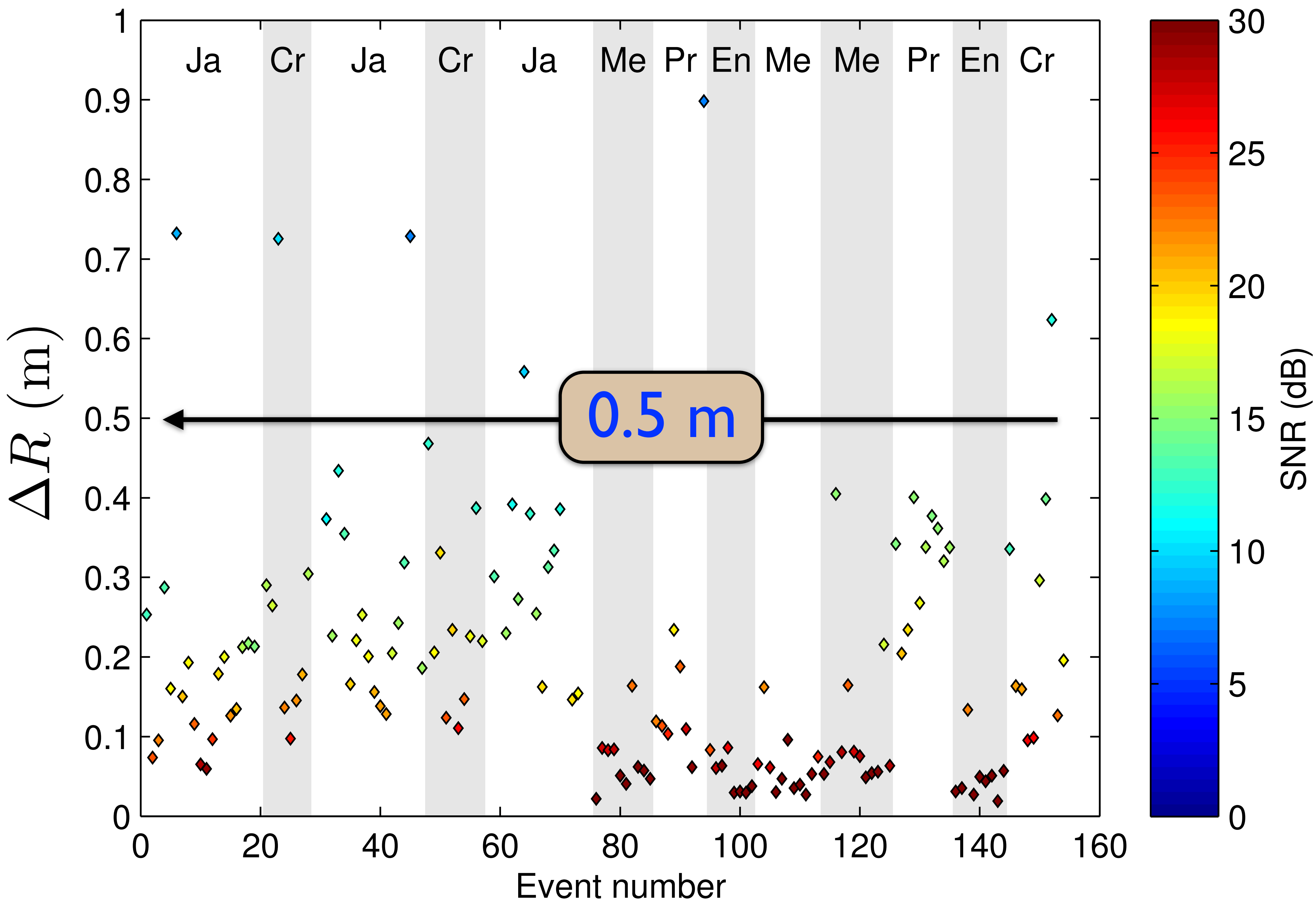


Original analysis 2010

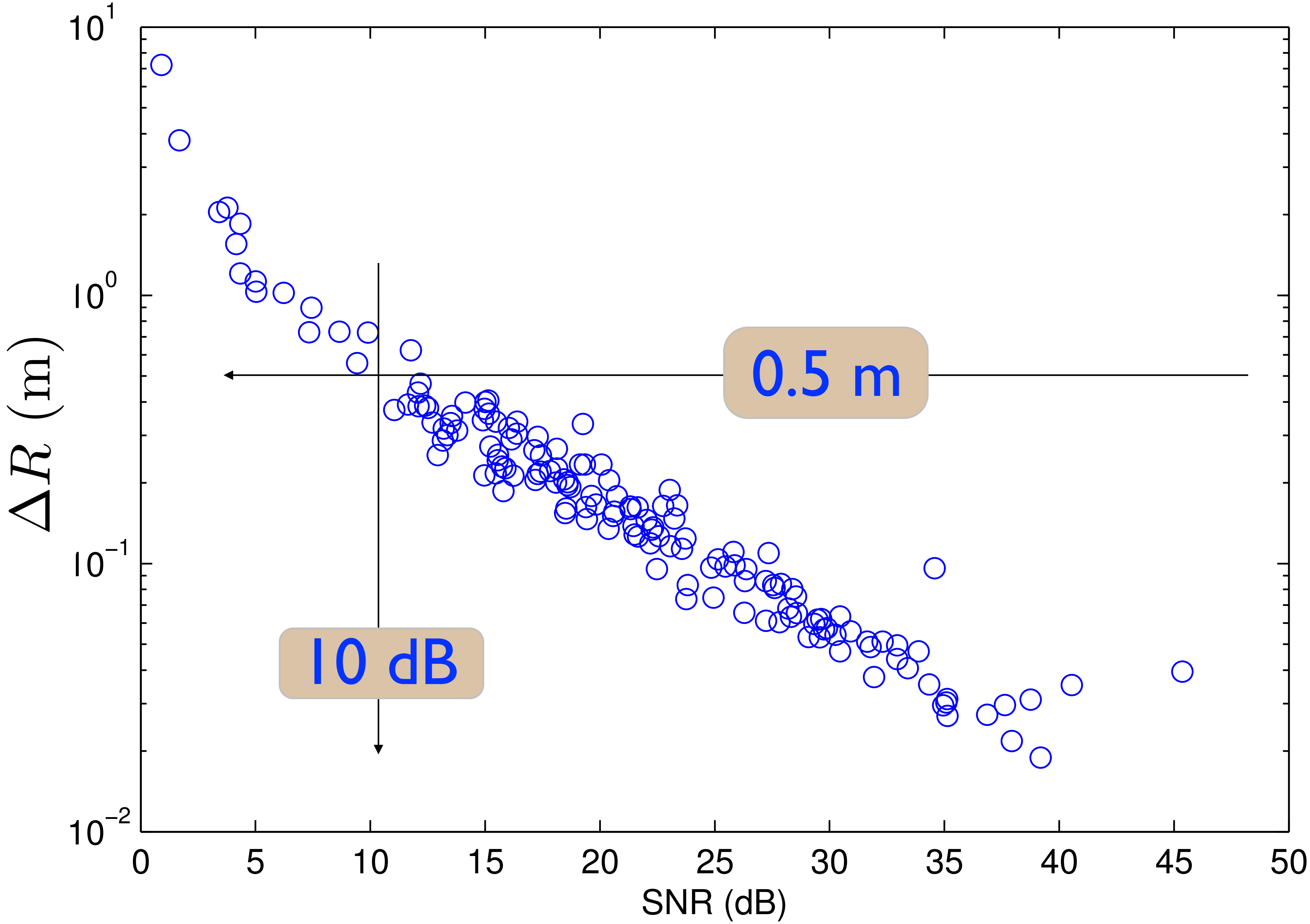
Proba-1
max SNR
18.1 dB



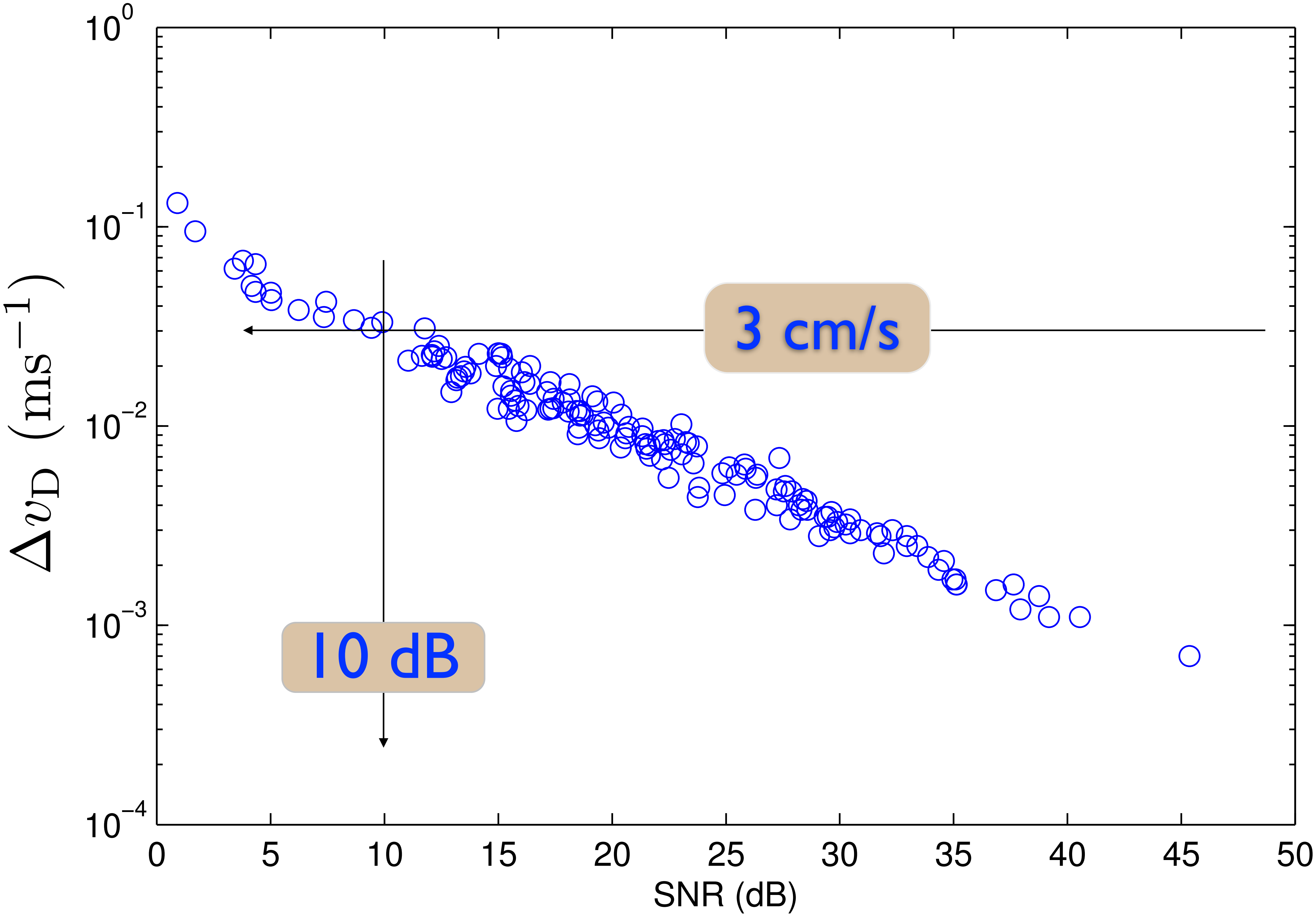
New analysis: 1-sigma error of range



New: 1-sigma error in range v. SNR



New: 1-sigma error in range rate v. SNR



ANALYSIS: Overview (1/2)

1. Coarse initial estimates of R and V_D for each pulse during a beam passage

Maximize the Match Function (MF) via a grid search

$$\mathrm{MF}(R,\omega) = \frac{1}{M}\left|\sum_m z_m\,\varepsilon(t_m - \frac{2R}{c})\,\exp i\omega t_m\right|^2$$

z_m : Measured complex-valued signal samples

M : Number of samples from a pulse

t_m : Sampling times

$\varepsilon(t)$: Transmitted phase code pattern (also measured)

$\exp(i\omega t)$: Doppler-factor

ANALYSIS: Overview (2/2)

2. Single-pulse analysis

R and V_{D} estimated with high resolution from each pulse separately

Error estimates σ_R and σ_V for each pulse

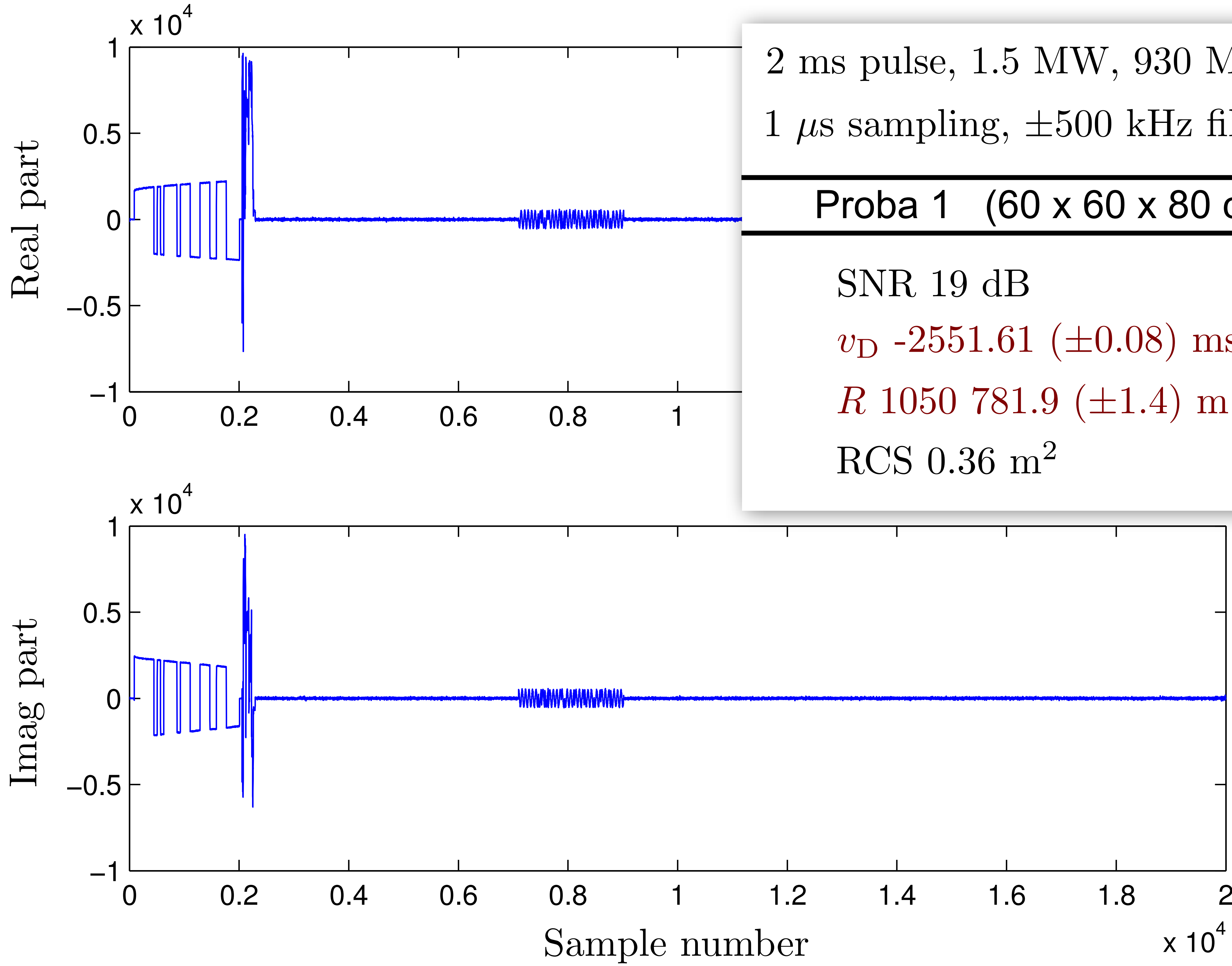
3. Combined analysis

Fit the single-pulse data to Taylor-polynomials of $R(t)$ and $V_{\mathrm{D}}(t)$

Weighted least-squares fitting, using σ_R^2 and σ_V^2 as weights

Standard theory gives error estimates for the fitted $R(t_0)$ and $V_{\mathrm{D}}(t_0)$

Data vector from a single 20 ms transmission-reception cycle



2 ms pulse, 1.5 MW, 930 MHz

1 μs sampling, ±500 kHz filter

Proba 1 (60 x 60 x 80 cm)

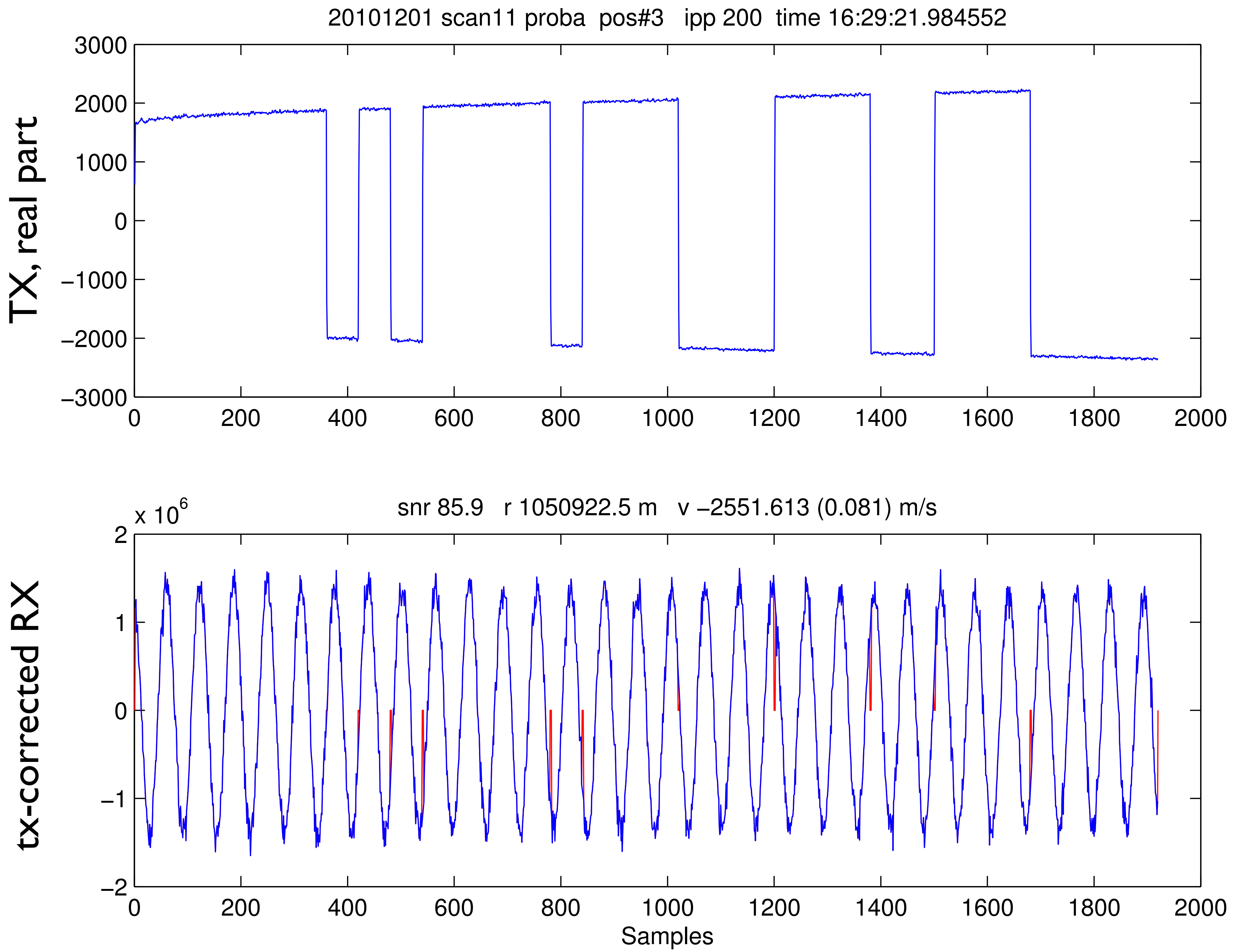
SNR 19 dB

v_D -2551.61 (±0.08) ms^{-1}

R 1050 781.9 (±1.4) m

RCS 0.36 m^2

Doppler analysis of a single pulse (1/2)



Doppler analysis of a single pulse (2/2)

$$h(\omega) \equiv \frac{|\sum_m z_m \exp i\omega t_m|^2}{M}$$

Schuster periodogram
(power spectrum)

$$p(\omega|\boldsymbol{z}) \propto \exp \frac{h(\omega)}{\sigma^2_{\text{noise}}}$$

Posterior probability density

$$\widehat{\omega} = \arg\max h(\omega)$$

Doppler-frequency estimate

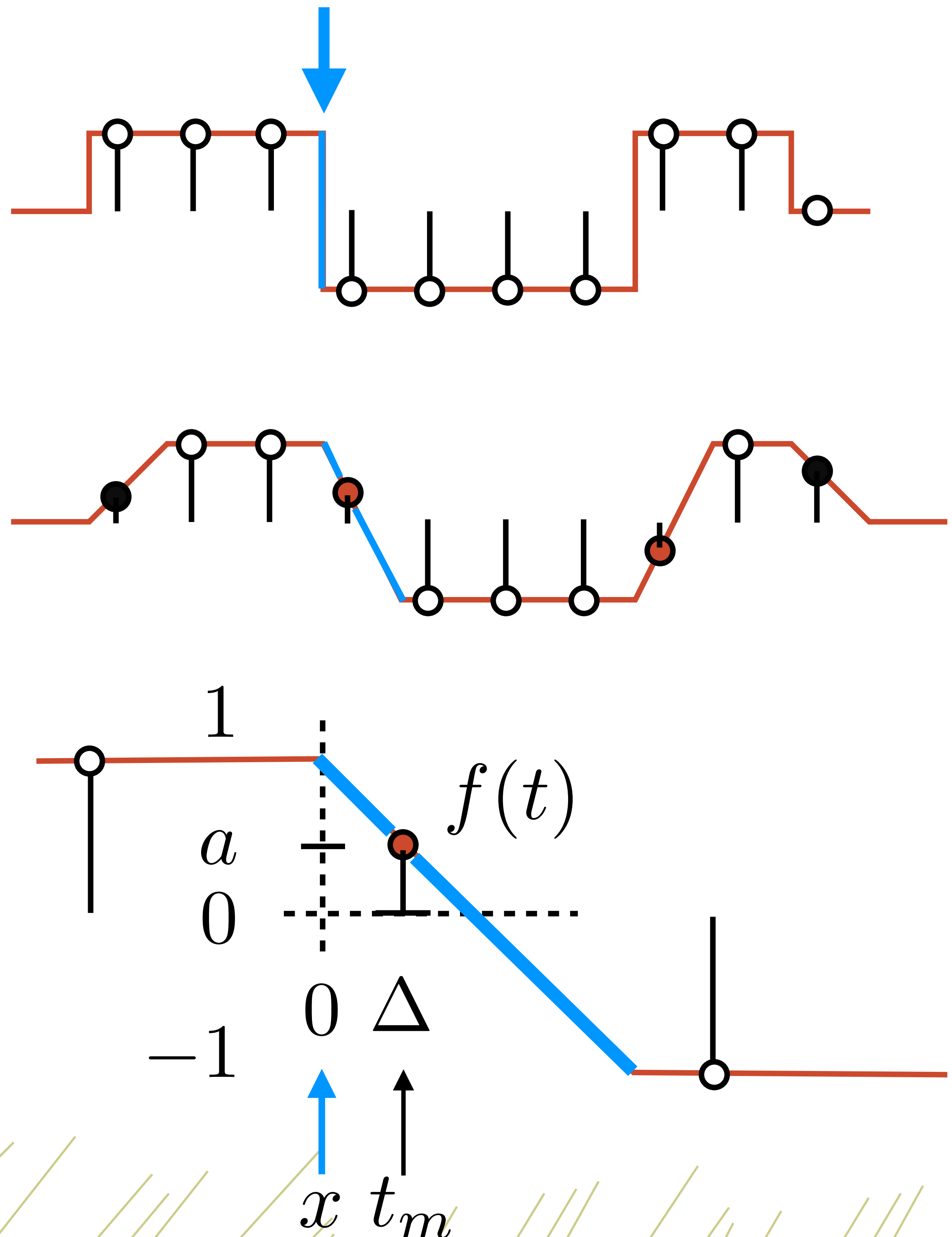
$$\sigma^2_\omega \approx \frac{6}{M \times \text{SNR} \times L^2}$$

Doppler-freq error estimate

Pulse length

Number of samples

Range analysis of a single pulse



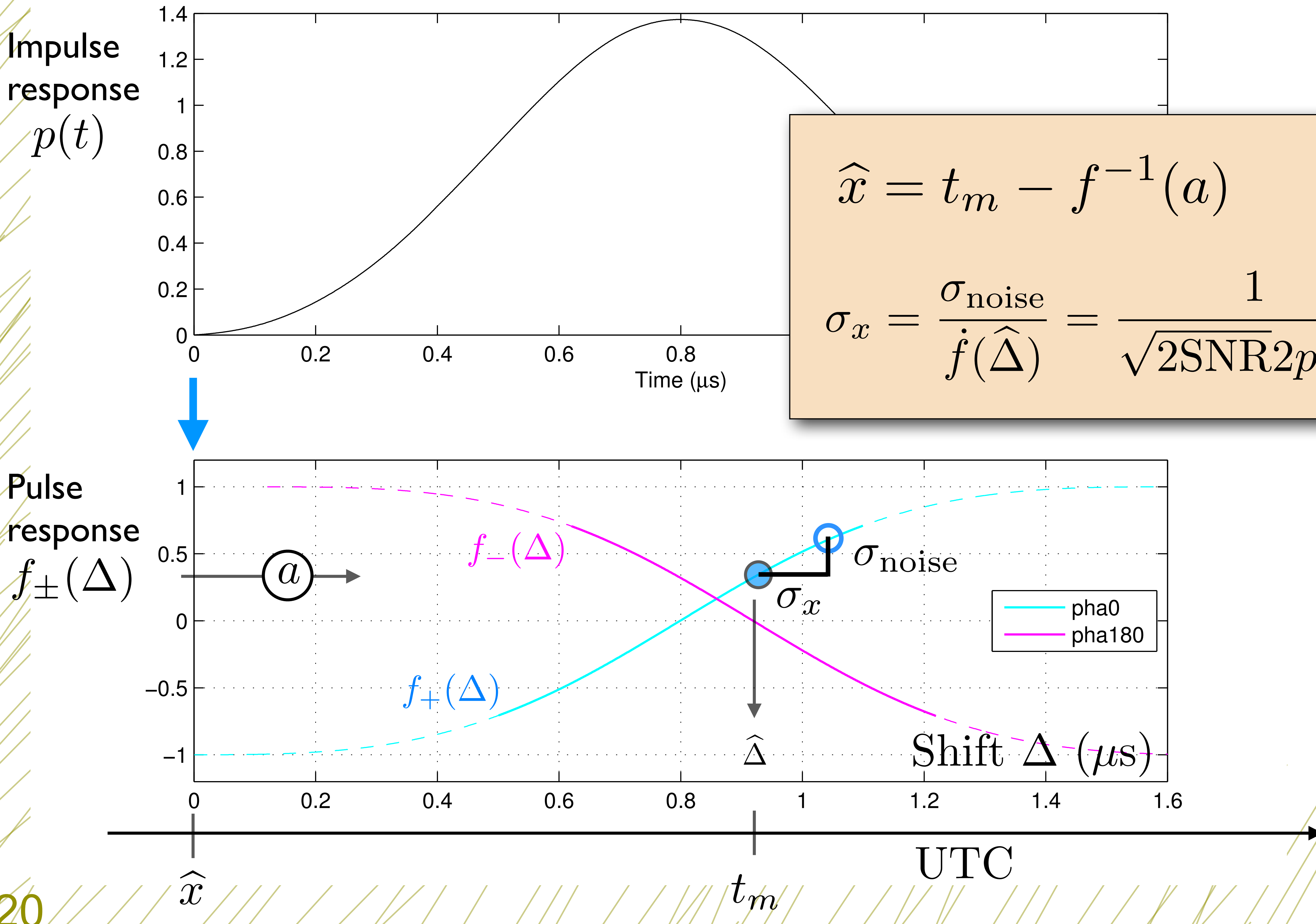
Ideal pulse (can't be accurately timed)

Filtered pulse

Phase flip sample's measured amplitude (a) allows one to estimate the time instant (x) of the flip:

$$x = t_m - f^{-1}(a)$$

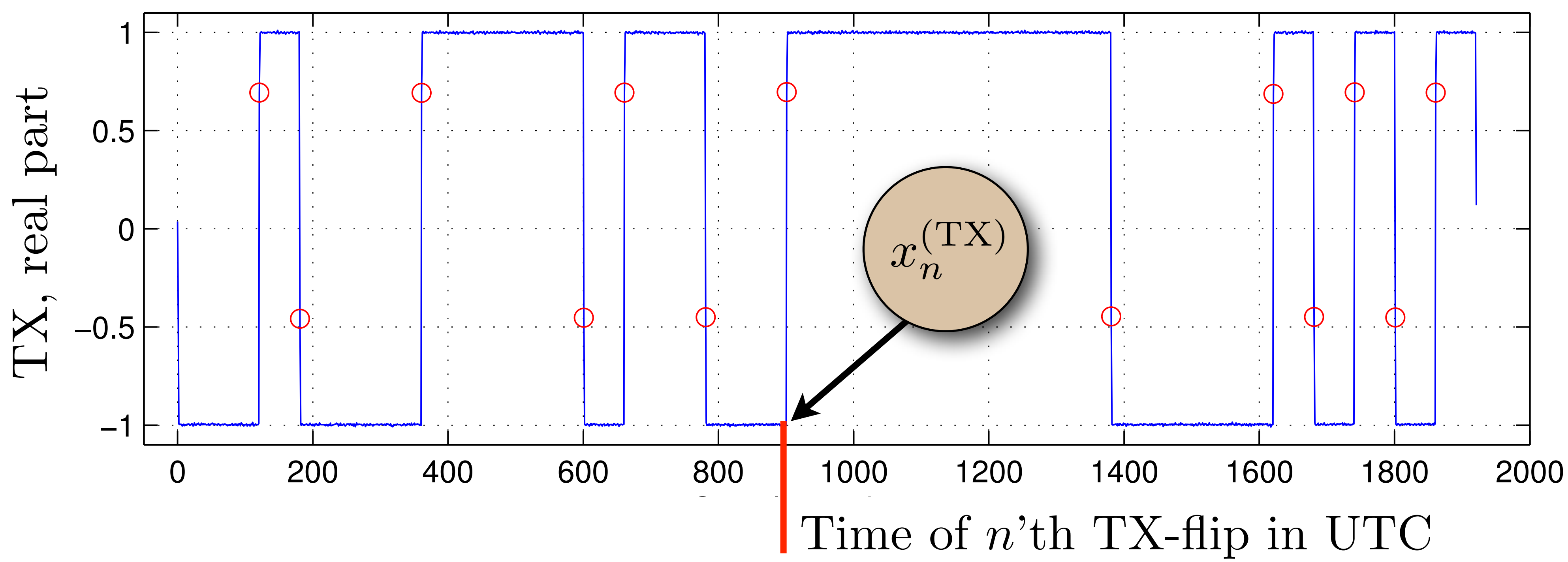
Timing a single phase flip

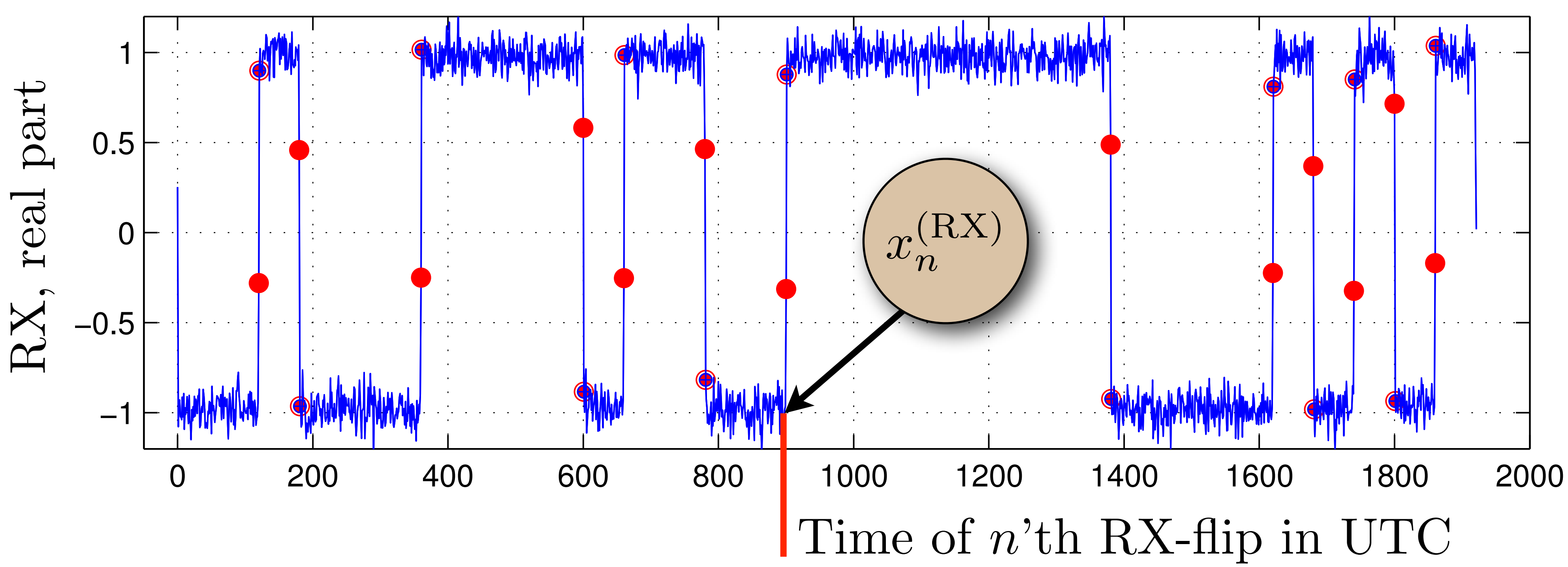


$$\hat{x} = t_m - f^{-1}(a)$$

$$\sigma_x = \frac{\sigma_{\text{noise}}}{\dot{f}(\hat{\Delta})} = \frac{1}{\sqrt{2\text{SNR}}2p(\hat{\Delta})}$$

There are 10-21 flips per pulse





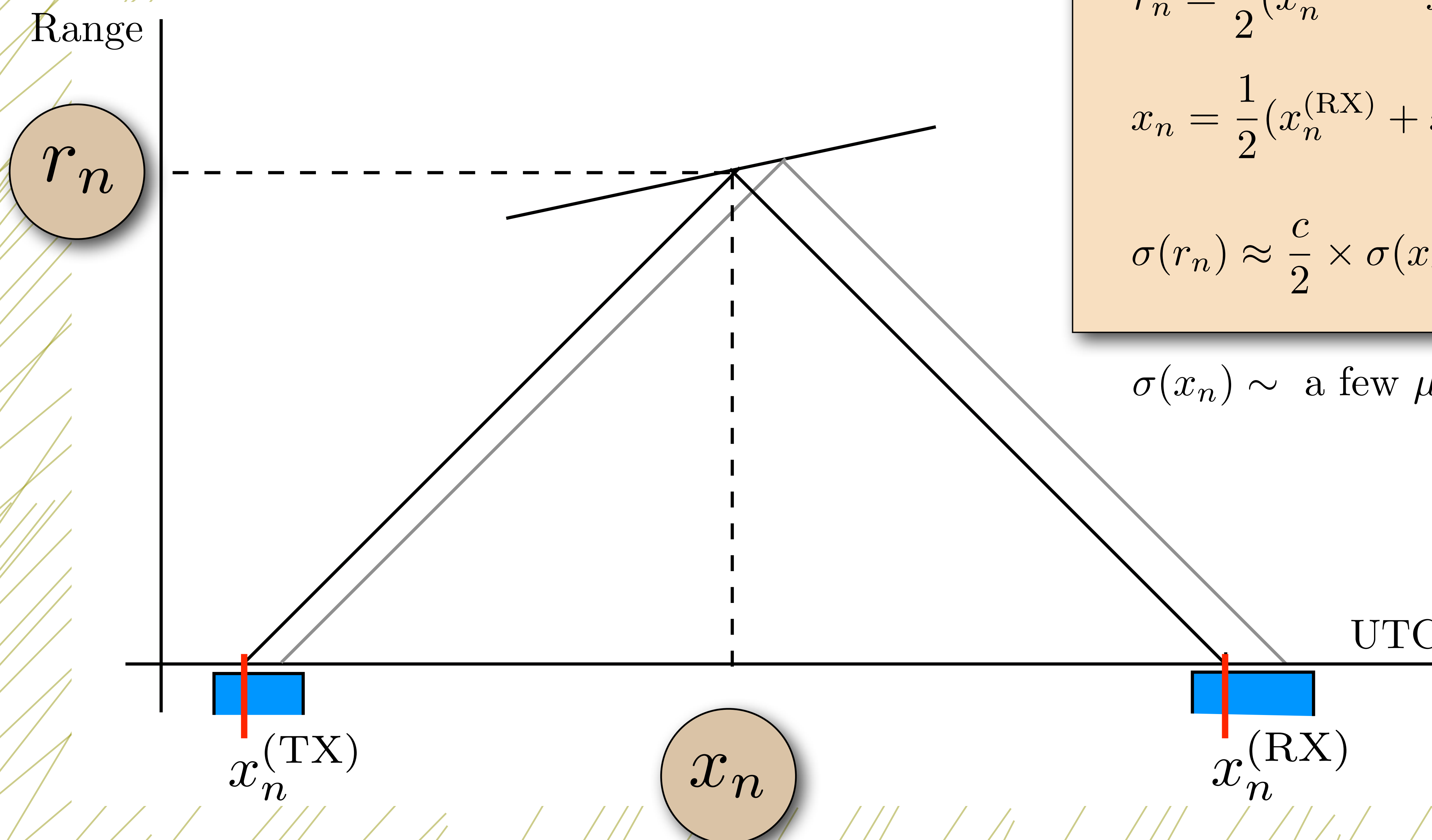
Each flip gives a Range and a Time

$$r_n = \frac{c}{2}\left(x_n^{(\mathrm{RX})} - x_n^{(\mathrm{TX})}\right)$$

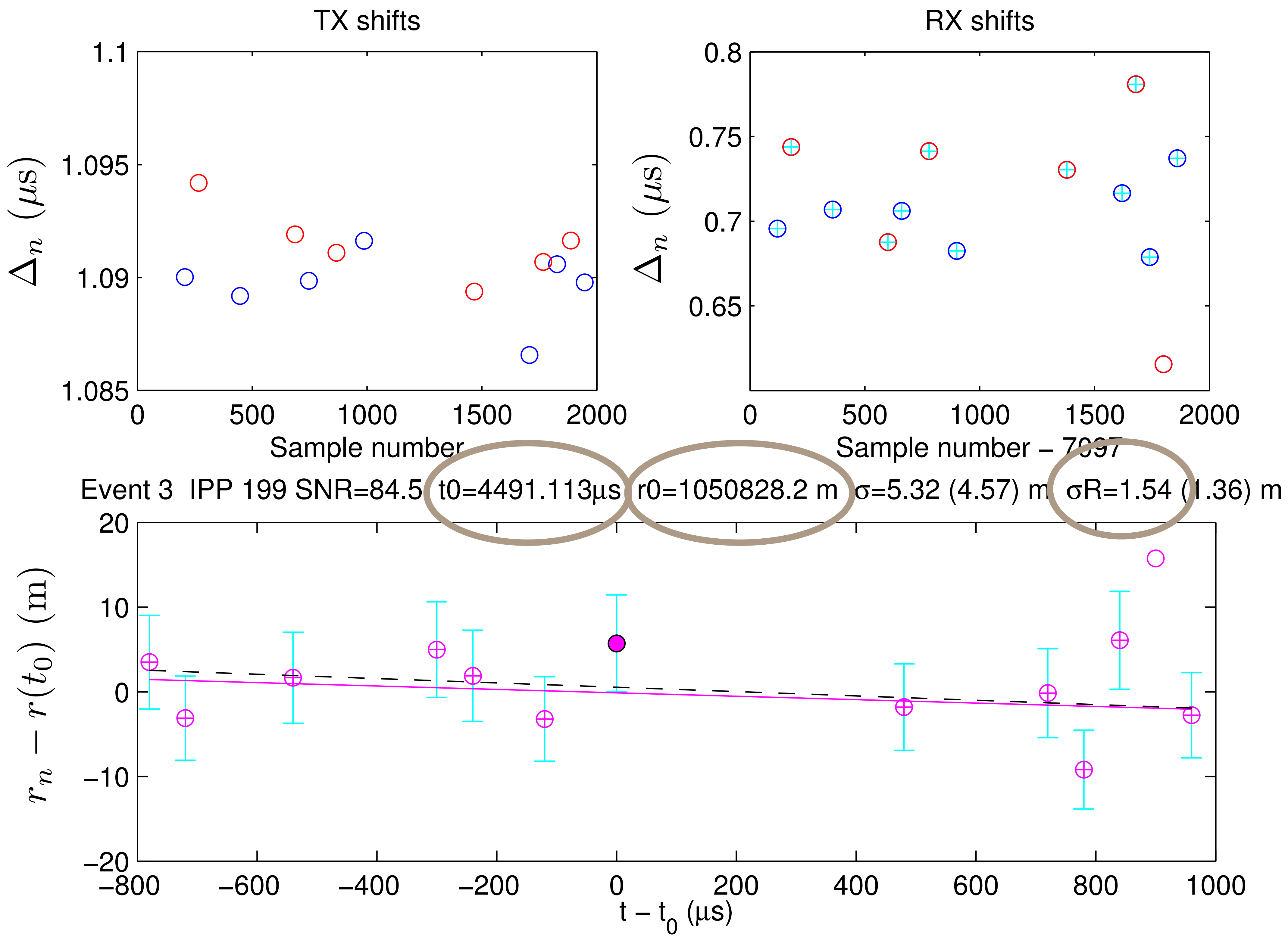
$$x_n = \frac{1}{2}\left(x_n^{(\mathrm{RX})} + x_n^{(\mathrm{TX})}\right)$$

$$\sigma(r_n) \approx \frac{c}{2} \times \sigma\left(x_n^{(\mathrm{RX})}\right)$$

$\sigma(x_n) \sim$ a few μs ≈ 0



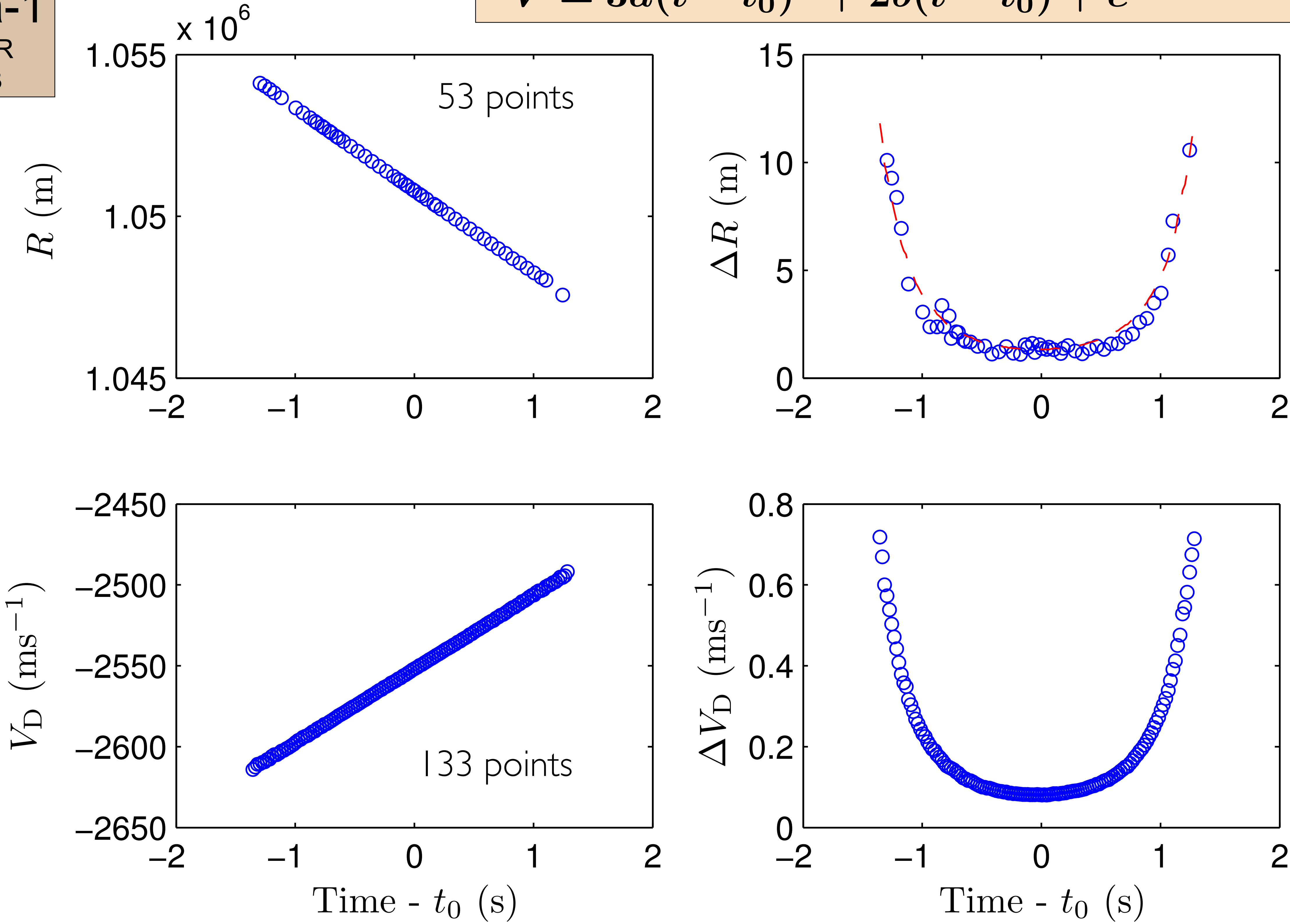
Weighted linear fit to the flip-wise r_n, x_n gives pulse-wise R and t, and an errorbar



Combining pulse-wise R's and V's by fitting

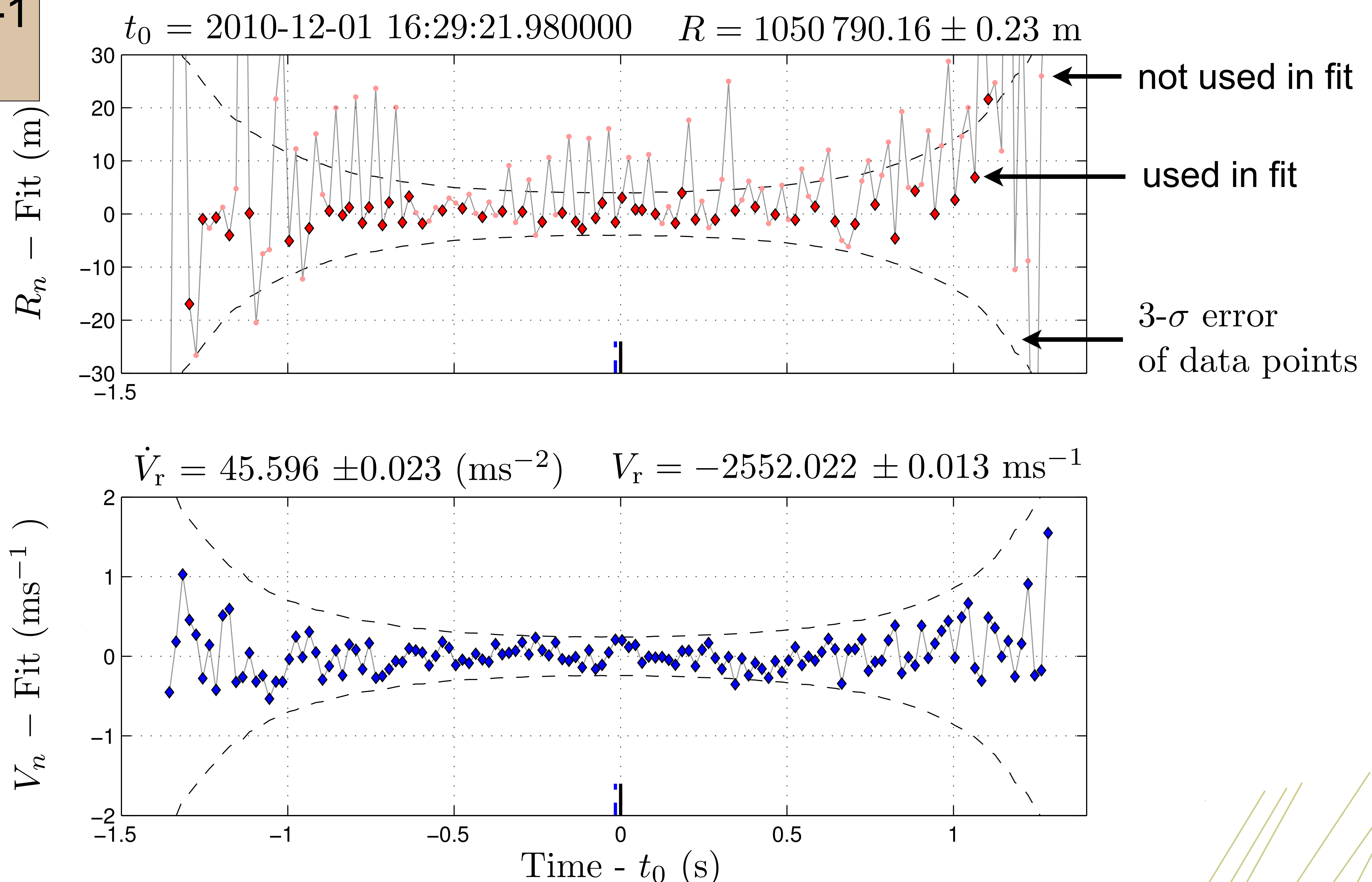
$$R = a(t - t_0)^3 + b(t - t_0)^2 + c(t - t_0) + d$$
$$V = 3a(t - t_0)^2 + 2b(t - t_0) + c$$

Proba-1
max SNR
19.3 dB



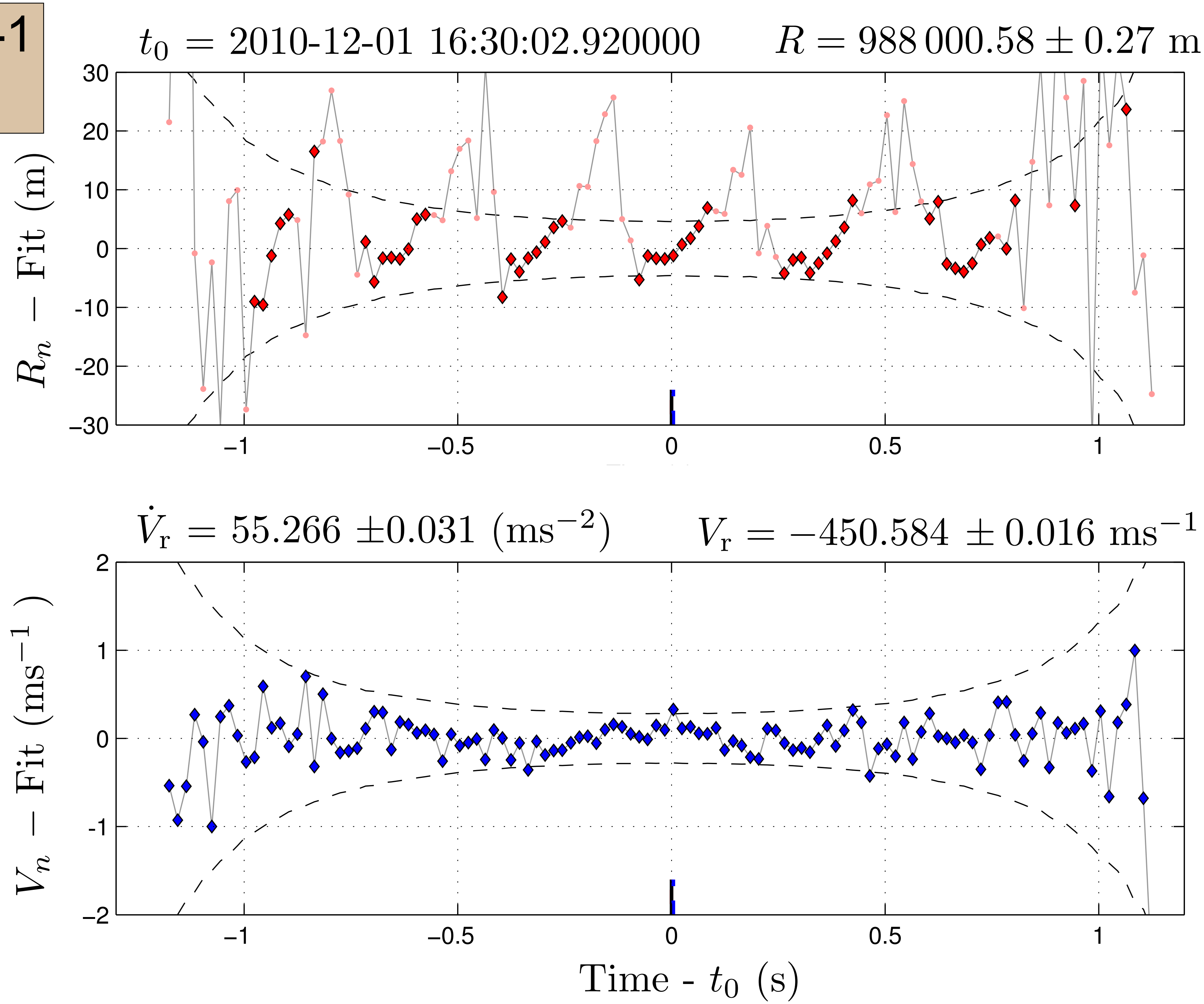
R and V_D fit residuals (good case)

Proba-1
max SNR
19.3 dB



R and V_D fit residuals (less good case)

Proba-1
max SNR
18.1 dB



Conclusion

- An improvement by two orders of magnitude in statistical accuracy (limited by SNR). For satellites:
  - Error in range ~ 10 cm
  - Error in range rate ~ 1 cm s^{-1}
- Serious systematic errors remain
  - Site and antenna geometry not fully accounted for
  - Signal model needs to be improved
- Should be applicable to debris targets, but
  - Coherent integration needed (==> computational problems, but work by J. Vierinen shows that these are manageable).